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Unsteady Magneto-Hydrodynamic (MHD) flow of a uniformly stretched vertical permeable surface I in the presence of heat generation/absorption and a first order chemical reaction.

****A. M. Okedoye, T. Adeniran, S. O. Adewale and R. O. Ayeni***
Department of Pure and Applied Mathematics Ladoke Akintola University of Technology Ogbomoso, Nigeria.
***e-mail: micindex@justice.com**

Abstract

Numerical results are presented for the transient and steady state Velocity, Temperature and Concentration fields. These results are obtained by solving the partial differential equations describing the conservation, momentum energy and species concentration by an explicit finite – difference method in time – dependent form. It was discovered that a maximum exist which confirm that maximum velocity and temperature occur in the body of the fluid and not in the surface as previously reported. A parametric study was conducted and the results were presented and discussed.

1.0 Introduction.

A study of boundary – layer behaviour on continuously moving solid surfaces has attracted the attention of several researchers. The analysis of magneto – hydrodynamics (MHD) flow of electrically conducting fluid finds application in different areas, such as the aerodynamic extrusion of plastic sheets, and the boundary – layer along a liquid film in condensation processes [2].

In order to understand the basic features of such a process, we consider a continuous flat plate which issues from a slot and moves with a constant velocity into a fluid which is at rest. As a result, the fluid adjacent to the plate moves and the region of penetration of the fluid motion into the ambient fluid depends on the Reynolds number of the flow. For large Reynolds number the region of penetration increases down stream of the slot, with the momentum and thermal boundary – layers originating from the slot and growing in the direction of the motion of the plane.

B. C. Sakiads [8, 9] in 1961 studies the boundary – adjacent to a continuous moving surface. He obtained solutions by approximate and exact methods of momentum boundary – layer equations, with no heat transfer on flat and cylindrical surfaces. And the corresponding heat transfer problems were considered theoretically by Tsou et. al. in [10] and experimentally by Griffin and Throne also in [3]. Vajravelu and Hadjinicolaou [11], reported on convective heat transfer in an electrically conducting fluid at a stretching surface with uniform free stream. Other example of studies dealing with hydro - magnetic flows can be found in the papers by Gray [5]. Michiyoshi et al. [6] and Funmizawa [4].

The study of heat generation or absorption effect, in moving fluids is important in view of several physical problems such as fluid undergoing exothermic or endothermic chemical reactions. In many chemical engineering processes, chemical reactions take place between a foreign mass and the working fluid which moves due to the stretching of a surface. A.J. Chamkha [2] examined the boundary – layer of an MHD flow when heat generation is linear in temperature. Ayeni et al [1] extended the problem posed to heat generation the is quadratic in temperature. Okedoye and Ayeni [6] report MHD flow of a uniformly stretched vertical permeable membrane in the presence of chemical and Arrhenius heat generation.

In this paper, we consider an unsteady case of [2] so that a previous steady case becomes special case of the present paper.

2.0 Governing equations

Consider a coupled heat and mass transfer by hydro - magnetic flow of a continuously moving vertical permeable surface in the presence of surface suction, heat generation/absorption effects, transverse magnetic field effects and chemical reactions. The flow is unsteady and two-dimensional and the surface is maintained at a uniform temperature and concentration species, and is assumed to be infinitely long, i.e the dependent variables are not dependent on the vertical or axial coordinate. The physical coordinates (x, y) are chosen such that the x – axis lies in the plane of the plate. It is also assumed that the applied transverse magnetic field is uniform and that the magnetic Reynolds number is small so that the induced magnetic field is neglected. In addition, there is no applied electric field and all of the Hall Effect, viscous dissipation and Joule heating are neglected, thermo - physical properties are assumed constant except the density in the buoyancy terms of the momentum equation which is approximated according to the Boussinesq approximation. With these assumptions, the equations that describe the physical situation are given by

$$\frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\upsilon \partial^2 u}{\partial y^2} + g\beta_T\left(T - T_\infty\right) + g\beta_c\left(C - C_\infty\right) - \frac{\sigma}{\rho}B_0^2 u \tag{2.2}$$

$$\rho\, cp\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + Q_0\left(T - T_\infty\right) \tag{2.3}$$

$$\rho\left(\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial y}\right) = D\frac{\partial^2 c}{\partial y^2} + \gamma\left(C - C_\infty\right) \tag{3.4}$$

where y is the horizontal or transverse coordinate, u is the axial velocity, v is the transverse velocity, T is the fluid temperature, C_∞ is the species concentration and ρ, g, β_τ. B_c, $\mu, \sigma, B_0, Q_0\, D$ and γ_n^n are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, dynamic viscosity, fluid electrical conductivity, magnetic induction, heat generation absorption coefficient, mass diffusion coefficient, the chemical reaction parameter and real number, respectively.

The physical boundary conditions for the problem are:

$$u(o,t) = u_w,\ v(0) = -v_w,\ T(0,t) = T_w,\ c(0,t) = c_w \tag{2.5}$$

$$as\ y \to \infty;\ u(y,t) \to 0,\ T(y,t) \to T_\infty,\ c(y,t) \to c_\infty \tag{2.6}$$

$$u(y,0) \to u_w,\ T(y,0) \to T_w,\ c(y,0) \to c_w \tag{2.7}$$

where u_w (a parameter dependent on time), $v_w > 0$, T_w and c_w are the surface velocity, suction velocity. Surface temperature and concentration, respectively.

3.0 Method of Solution

3.1 Non–Dimensionalisation

Integrating equation (2.1) subject to $v(0) = -v_w$ we have the solution

$$v(y) = -v_w \tag{3.1}$$

Using this solution, the momentum, energy, and species equations (equations. 2.2 – 2.4) can be non–dimensionalised using the following non–dimensional variables.

$$y^1 = y\frac{v_w}{v},\ u^1 = \frac{u}{u_w},\ \theta = \left(T - T_\infty\right)\frac{E}{RT_\infty^2},\quad c^1 = \frac{c - c_\infty}{c_w - c_\infty},\quad t^1 = \upsilon t \tag{3.2}$$

Hence, we obtain the following sets of equations corresponding to each of the cases described above:

$$\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_{rT}\theta + G_{rc}C - M^2 u = 0 \tag{3.3}$$

$$\Pr\left(\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y}\right) = \frac{\partial^2 \theta}{\partial y^2} + \Pr\varphi\theta = 0 \tag{3.4}$$

$$Sc\left(\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y}\right) = \frac{\partial^2 C}{\partial y^2} - \lambda\ ScC \tag{3.5}$$

and equations (2.5 – 2.7), gives $\quad u(o,t) = \theta(0,t) = C(0,t) = 1 \qquad (3.6)$

$$u(y,t) = \theta(y,t) = C(y,t) = 0 \quad as \quad y \to \infty \tag{3.7}$$

$$u(y,t) = \theta(y,0) = C(y,0) = 1 \tag{3.8}$$

where $\quad Grt = \frac{g\beta_\tau \varepsilon T_\infty v^2}{\upsilon v_w^2 u_w}$, $Grc = \frac{g\beta_c (c_w - c_\infty) v^2}{\upsilon v_w^2 u_w}$, $M^2 = \sigma B_0^2$, $\Pr = \frac{\mu c_p}{k}$,

$$\phi_0 = \frac{Q_0 v}{\mu c_p v_w^2}, \ \phi_1 = \frac{Q_1 v}{\mu c_p v_w^2}, \ \varepsilon = \frac{RT_\infty}{E}, \ b = (c_w - c_\infty)^n e^{1/\varepsilon}, \lambda = \frac{\gamma v}{v_w^2}$$

The above equations (2.3-3.5) are set of linear second – order boundary value problem. The derivatives in the system of equations can be approximated by $o(h^2)$ approximations

$$\frac{\partial \psi}{\partial t} = \frac{\psi_{i,j+1} - \psi_{i,j}}{2r}, \ \frac{\partial \psi}{\partial t} = \frac{\psi_{i,j+1} - \psi_{i,j}}{2h}, \text{ and } \frac{\partial^2 \psi}{\partial t^2} = \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-i,j}}{h^2}$$

where r and h are the step length in time and position y respectively. And ψ (here) represent $C(y,t)$, $\theta(y,t)$ or $u(y,t)$. Hence, we have

$$C(i, j+1) = \left(1 - \frac{r}{h} - \frac{4r}{Sc\,h^2} - 2r\lambda_0\right) C(i, j) + \frac{r}{h}\left(1 + \frac{2}{hSc}\right) C(i+1, j) + \frac{2r}{h^2 Sc} C(i-1, j) \tag{3.9}$$

$$\theta(i, j+1) = \left(1 - \frac{r}{h} - \frac{4r}{\Pr h^2} + 2r\phi\right)\theta(i, j) + \frac{r}{h}\left(1 + \frac{2}{h\Pr}\right)\theta(i+1, j) + \frac{2r}{h^2 \Pr}\theta(i-1, j) \tag{3.10}$$

$$u(i, j+1) = \left(1 - \frac{r}{h} - \frac{4r}{h^2} + 2rM^2\right) u(i, j) + \frac{r}{h}\left(1 + \frac{2}{h}\right) u(i+1, j) + \frac{2r}{h^2} u(i-1, j) + rGrt\theta(i, j) + rGrcC(i, j) \tag{3.11}$$

With the boundary conditions (3.6 – 3.8) becomes

$$u(o, j) = \theta(0, j) = C(0, j) = 1 \tag{3.12}$$

$$u(25, j) = \theta(25, j) = C(25, j) = 0 \tag{3.13}$$

$$u(i,0) = \theta(i,0) = C(i,0) = 1 \tag{3.14}$$

Equations (3.9 - 3.11) subject to boundary condition (3.12 – 3.14) are then solved by a standard tri-diagonal solver

4.0 Result and discussion

The investigation(s) on this problem is carried out using *Pr* = 0.71, Sc = 0.6, *Grt* = 1, *Grc* = 1 *k* = -0.2, $\phi = -0.4$ and *M* = 0.5 except where stated otherwise.

It should be noted that $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$ represent destructive, no and generative chemical reactions respectively. Also, $\phi \succ 0$, $\phi = 0$ and $\phi < 0$ indicates heat absorption, no heat generation/absorption and heat generation respectively.

Figures 2.1 to 2.12 presents axial velocity, temperature and concentration profiles for various combinations of the parameters λ , ϕ, *Pr* and *Sc* in the presence of magnetic induction *M*, thermal buoyancy and concentration buoyancy effects. Figure 1 show the velocity as a function of position y and time *t*(*u*(*y*, *t*)). Figure 2.1 show the profile against position y at different time. It can be clearly observed that for the choice of parameters enumerated above, a maximum occur at time *t* = 0.125, while a steady state is reached at position *y* =7. The presence of maximum establishes that maximum value velocity occurs in the body of fluid close to the surface and not the surface. Figure 2.2 show velocity profile at different time. It can seen that steady state along *t* – axis is reached *t* = 2. While figures 2.3 and 2.4 shows the effect of magnetic induction *M* on velocity. In fact magnetic induction opposes velocity.

In figures 2.5 to 2.8 shows temperature profile. Figure 2.5 show that as time increases, the temperature boundary layer decreases. It can also be seen from figure 2.6, a maximum in the value of temperature ϕ (y, 0.125). Figures 2.7 and 2.8 shows the effect of heat generation/absorption on temperature. Increase in heat absorption lowers the temperature boundary layer, while increase in heat generation brings about increase in temperature boundary layer accordingly.

Figures 2.9 to 2.12 illustrate concentration profile both against time and position separately. In figure 11, we displayed concentration $C(y,t)$ profile. Transient and steady – state temperature profile are shown in figure 2.9 and 2.10 for time t and fixed position y. The concentration decreases monotonically and reaches steady - state condition at $t = 2.0$ for fixed y and y = 10 for fixed t respectively. And figures 2.11 and 2.12 describes the effect of chemical reaction parameter λ . It could be seen that increase in λ results in decrease in concentration layer.

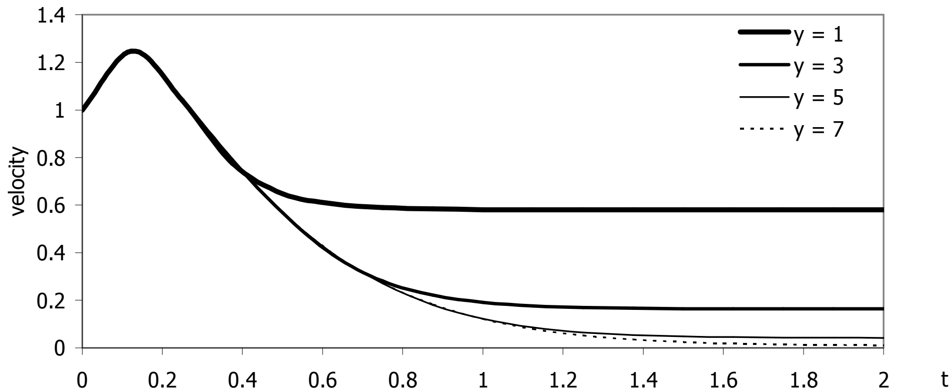

Figure 1: Velocity profile at different position y.

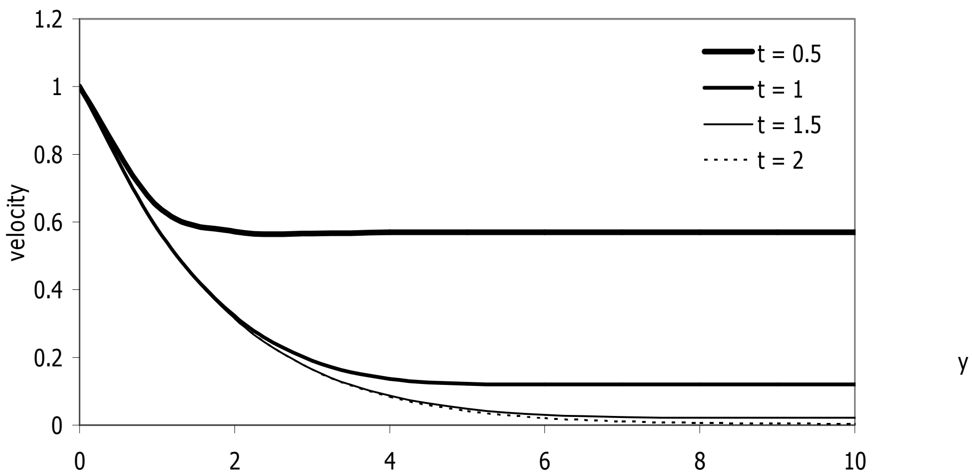

Figure 2: Velocity profile at different time t.

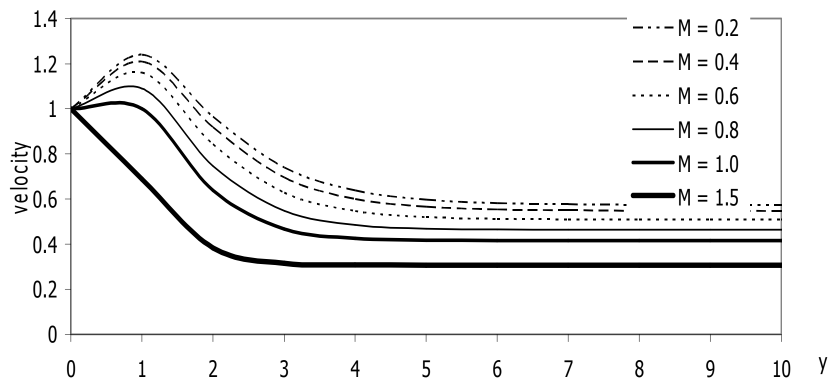

Figure 3: Effect of Magnetic induction on velocity at position y = 1.0

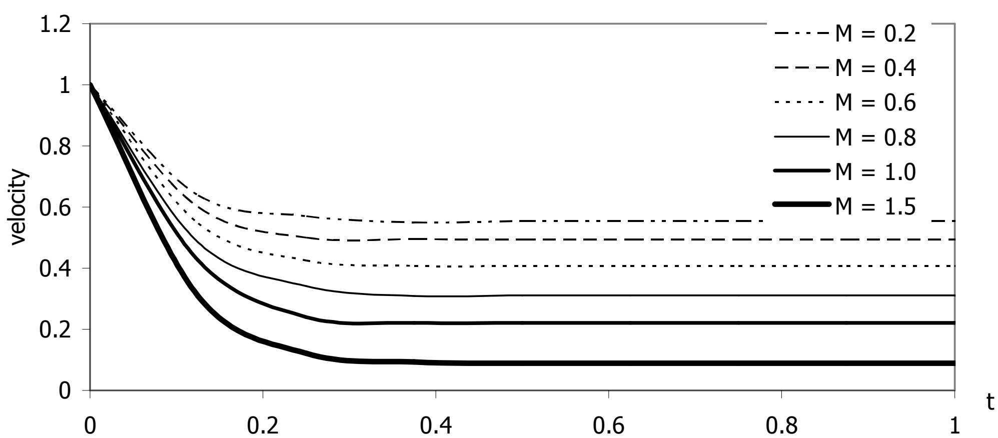

Figure 4: Effect of magnetic induction on velocity at position y = 1

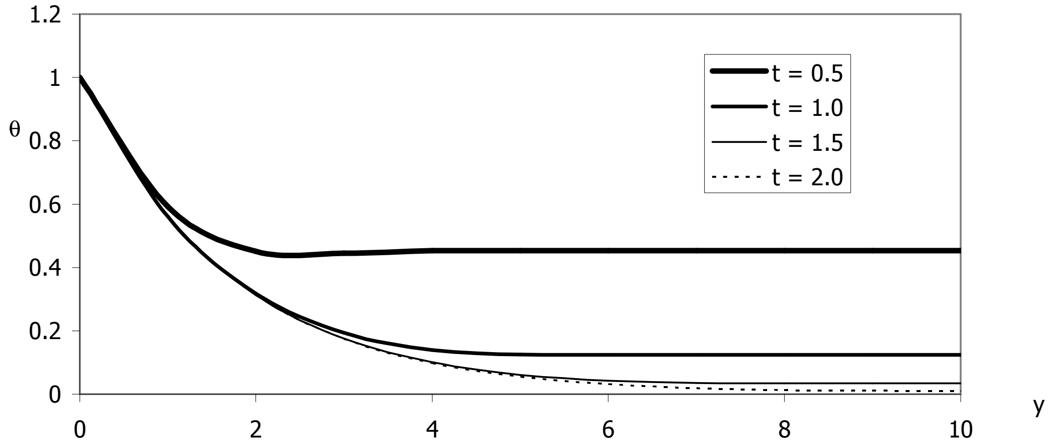

Figure 5: Temperature profile at different time t.

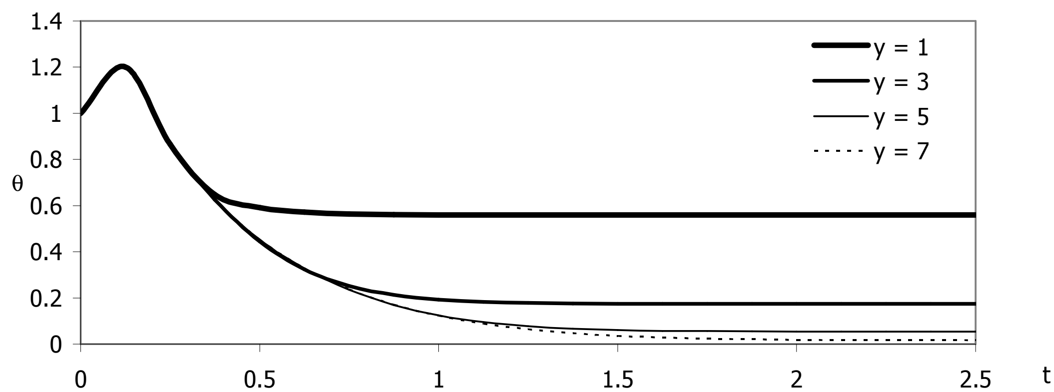

Figure 6: Temperature against time t at different position y.

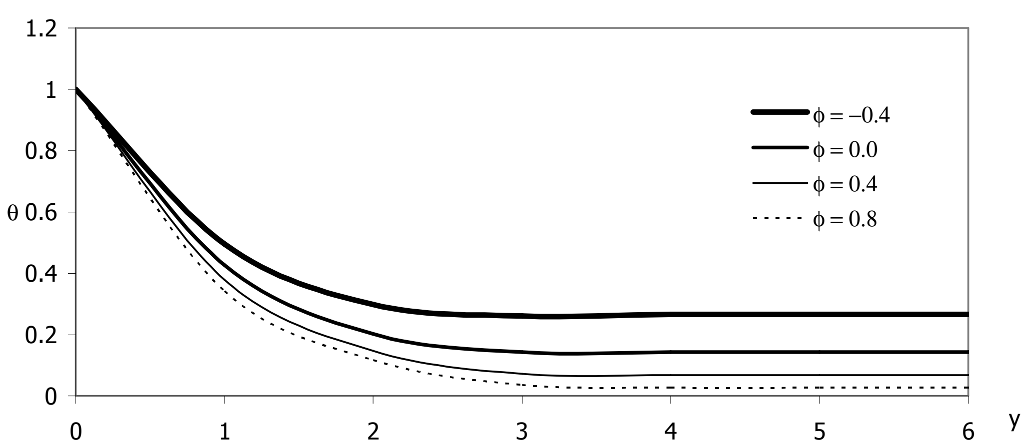

Figure 7: Effect of heat generation/absorption on temperature at time t = 0.5.

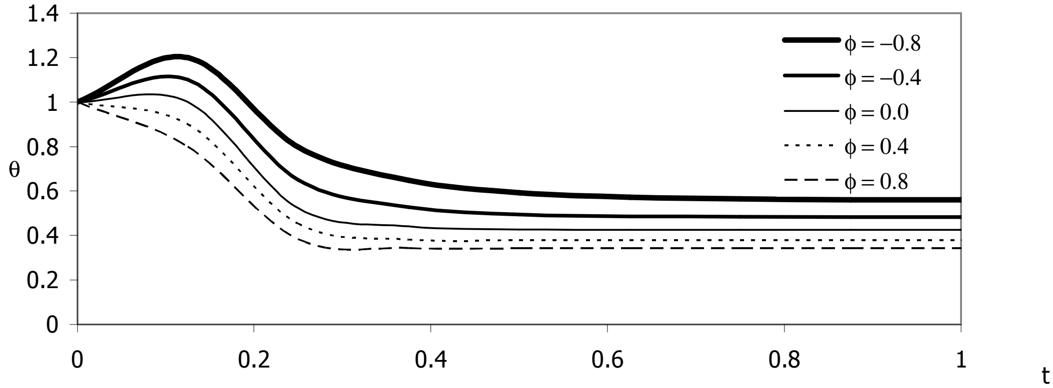

Figure 8: Effect of heat generation/absorption on temperature at positiony = 1.

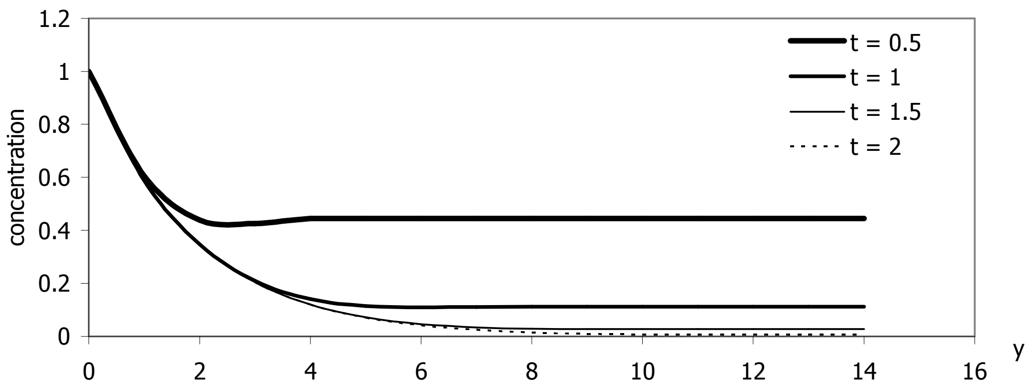

Figure 9: Concentration profile at different time t.

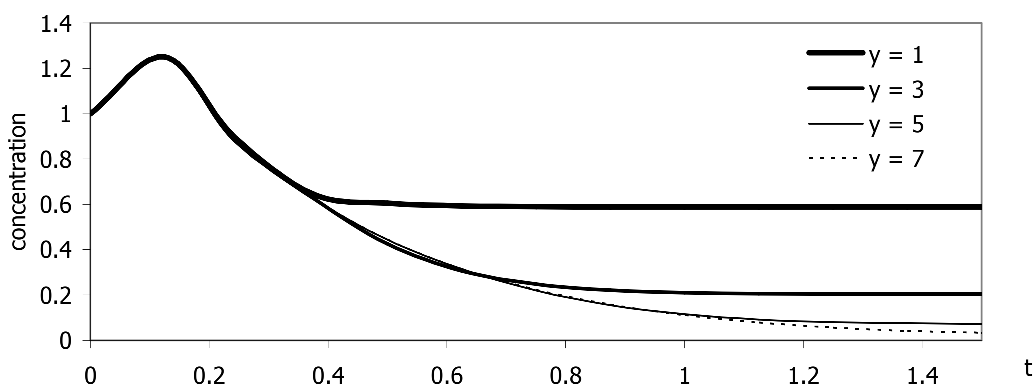

Figure 10: Concentration profile at different time t.

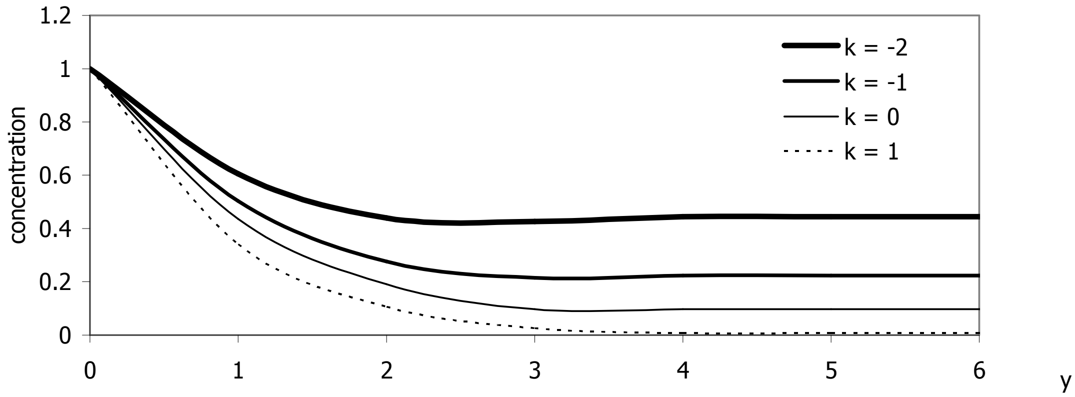

Figure 11: Effect of heat generation/absorption on temperature at time t = 0.5.

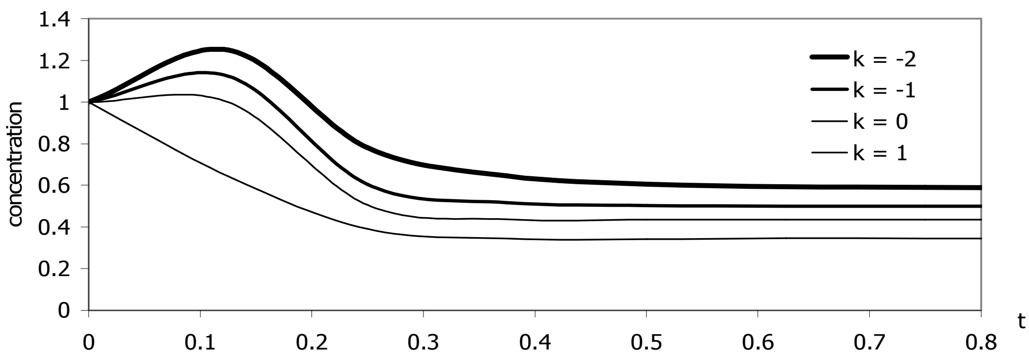

Figure 12: Effect of reaction parameter on concentration at position y = 1.

5.0 Concluding Remarks

Numerical solution for heat and mass transfer by laminar flow of an electrically conducting fluid on a uniformly moving vertical permeable surface in the presence of a magnetic field and a first – order chemical reaction were reported. Based on the obtained graphical results, the following conclusions were deduced:

[1] The fluid velocity increased as either time, position y or the strength of the magnetic field was increased and decreased as either of the thermal or concentration buoyancy effect were decreased.

[2] The fluid temperature increases during a generative chemical reaction and decreased during a destructive one. Also the presence of heat generation effects increases fluid temperature while the presence of heat absorption effect decreases it.

[3] The concentration of chemical species increases with increase in generative chemical reaction ($\lambda < 0$), and decrease with increase in destructive chemical reaction ($\lambda > 0$).

[4] The boundary layer of the velocity, temperature and concentration reduces as either time or position of the flow elements increases.

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