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On the possibility of multiplicity of temperature fields in a microwave heating cancer therapy

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Abstract

We investigate a steady temperature dependent perfusion during a cancer therapy. We show how the choice of perfusion could lead to more than one temperature fields which could lead to an undesired result.

1.0 Introduction.

Cancer continues to be a deadly disease and thus there have been many investigations on the ways by which the spread of cancerous tissues could be controlled. Microwave hyperthermia has been of great significance in the control or total elimination of tumor/cancer cells. Much of the progress of the investigations and the current knowledge could be found in papers [2] – [5], and [8]. However, in a recent paper Adebile and Ogunmoyela raised a concern on the existence of two temperature field when the perfusion is temperature dependent. It is this concern that is of much interest in this paper. Our results show how the perfusion term could be judiciously selected through valuable frequencies.

2.0 Mathematical formulation and method of solution

2.1 Mathematical formulation

The non–dimensional steady equations after the withdrawal of the microware heating is

$$\frac{d^2\theta}{dx^2} - a.w(\theta)(\theta - 1) = 0\ , \quad -1 < x < 1 \tag{2.1}$$

$$\theta(-1) = \theta(1) \tag{2.2}$$

Where $a.w(\theta)$ is the perfusion term, the perfusion term is crucial in the caner treatment as we shall see later.

2.2 Method of Solution

When a is taken as unity, $w(\theta)$ is of the form $(1+\theta)^m$, so that equation (2.1) becomes:

$$\frac{d^2\theta}{dx^2} - (1+\theta)^m(\theta - 1) = 0 \quad -1 < x < 1 \tag{2.3}$$

We are interested in a situation when m is a positive integer.
Thus

$$(1+\theta)^m = 1 + mC_1\theta + mC_2\theta^2 + \ldots + mC_m\theta^{m+1} \tag{2.4}$$

$$\frac{d^2\theta}{dx^2} = \theta + mC_1\theta^2 + mC_2\theta^3 + \ldots + mC_m\theta^m - 1 - mC_1\theta - mC_2\theta^2 - \ldots - mC_m\theta^m \tag{2.5}$$

Equation (2.5) together with equation (2.2) implies that maximum θ occurs at the origin. Following Ayeni et al. [6] we seek a solution of the form $V(x) = C_1(x+1)^2(x-1) + C_2(x+1)(x-1)$.

Let
$$R(x) = \frac{d^2V}{dx^2} - (V(x)+1)^2(V(x)-1) \tag{2.6}$$

Now $\frac{dV}{dx} = 0 \Rightarrow C_1 = -C_2$. This reduces to

$$V(x) = 2C_1(x^2-1) \tag{2.7}$$

At $x = \left(-\frac{1}{3}\right)$

$$V = -\frac{16C_1}{9} \tag{2.8}$$

$$\frac{d^2V}{dx^2} = 4C_1 \tag{2.9}$$

Hence
$$R\left(-\frac{1}{3}\right) = 4C_1 + \left(1 - \frac{16C_1}{9}\right)^m \left(1 + \frac{16C_1}{9}\right) \tag{2.10}$$

Numerical computation for $R\left(-\frac{1}{3}\right) = 0$

m = 0, C_1 = <u>-9/52</u>

m = 1, C_1 = <u>1.479487119</u>, <u>-0.2138621189</u>

m = 2, C_1 = <u>-0.2812500000</u>, 0.4218750000+0.6744138079*I,
0.4218750000-0.6744138079*I

m = 3, C_1 = <u>1.211598120</u>, -0.3808266994, 0.1471142896+0.4419610886*I,
0.1471142896-0.4419610886*I

m = 4, C_1 = <u>-0.4692166119</u>, 0.91068960891-0.3051113686*I,
0.91068960891+0.3051113686*I, 0.9872893451-0.4571732151*I,
0.9872893451+0.4571732151*I

m = 5, C_1 = <u>-0.5179252858</u>, 0.75977309541-0.2333346818*I,
0.75977309541-0.2333346818*I, 0.7208678376-0.5876192437*I,
0.7208678376+0.5876192437*I, <u>1.74234992</u>

m = 6, C_1 = <u>-0.5411580239</u>, 0.6908415536e-1-0.1905852410*I,
0.6908415536+0.1905852410*I, 0.5280676832-0.5969155703*I,
0.5280676832+0.5969155703*I, 1.079677173-0.3093607820*I,
1.079677173-0.3093607820*I,

m = 7, C_1 = <u>-.5521453189</u>, .6454615977e-1-.1620791721*I,
.6454615977e-1+.1620791721*I, .3964284772-.5661279376*I,
.3964284772+.5661279376*I, .9229025970-.4745537352*I,

.9229025970+.4745537352*I, 1.159390851

m = 8, C_1 = -.5574201737, .6100110853e-1-.1415599450*I,

0.61001108531+.1415599450*I, .3058654302-.5243806325*I,

.3058654302+.5243806325*I, .7728919117-.5519575260*I,

.7728919117+.5519575260*I, 1.107701636-.2312979521*I,

1.107701636+.2312979521*I

m = 9, C_1 = -.5599891222, .5801739118e-1-.1259924175*I,

.5801739118 +.1259924175*I, .2421693180-.4821334781*I,

.2421693180+.4821334781*I, .6460633314-.5805670628*I,

.6460633314+.5805670628*I, 1.008032851-.3844550302*I,

1.008032851+.3844550302*I, 1.151423339

m = 10, C_1 = -.5612530250, .5541583394e-1-.1137253225*I,

0.5541583394 +.1137253225*I, .1962639113-.4429756861*I,

.1962639113+.4429756861*I, .5429319093-.5828233194*I,

.5429319093+.5828233194*I, .8983335742-.4794770086*I,

.8983335742+.4794770086*I, 1.118931284-.1841246789*I,

1.118931284+.1841246789*I

m = 11, C_1 = -.5618789404, .5310345607e-1-.1037790650*I,

.5310345607 +.1037790650*I, .1624000502-.4078891253*I,

.1624000502+.4078891253*I, .4599476694-.5711711785*I,

.4599476694+.5711711785*I, .7940945516-.5348200401*I,

.7940945516+.5348200401*I, 1.050667267-.3200226685*I,

1.050667267+.3200226685*I, 1.146452952

So the problem has only one solution when $m = 0$, Two real solutions when m is odd and one real solution when m is even.

3.0 Discussion of results

This paper suggest that to avoid adverse consequences an even m is preferred. Moreover, maximum θ is $-2C_1$ and this is useful in the selection of even m depending on the maximum temperature that is preferred

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