*Journal of the Nigerian Association of Mathematical Physics*
*Volume 10 (November 2006),* **497 - 498**
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# A new poof of multiple solutions of combustions problems

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**Abstract**

**We revisit the combustion problem** $\frac{1}{r^n}\frac{d}{dr}\left(r^n\frac{d\theta}{dr}\right)+\lambda\exp(\theta)=0$**, for the plane (*n* = 1), cylinder (*n* = 2) and sphere (*n* = 3) vessels. Using polynomial approximations. We show that the problem has two (2) solutions.**

## 1.0 Introduction.

In the theory of laminar flames [2], the energy equation before appreciable consumption of the reactants is of the form

$$\nabla^2\theta + \delta e^{\theta} = 0 \qquad (1.1)$$

where $\theta=\frac{E}{RT_0^2}(T-T_0)$, $T$ is the temperature, $T_0$ is the initial temperature, $E$ is the activation energy and $R$ is the Universal gas constant and $\delta$ is the scaled Damkohler number.

## 2.0 Previous solution

For the infinite slab the problem reduces to subject to $\theta(-1)=\theta(1)=0$ (2.2)

Problem (2.1) has closed form solution

$$\theta = 2\ln\left[\exp\left(\theta_m/2\right)\operatorname{sec}h\, cx\right], \qquad (2.3)$$

with $c^2=\frac{\delta}{2}\exp(\theta_m)$, where $\theta_m=\theta_0$. Using equation (2.2) in equation (2.3), we obtain $\sqrt{\delta/2}=\exp\left(-\theta_m/2\right)\cosh^{-1}\left(\exp\left(\theta_m/2\right)\right)$. So there exist 2 solution for $\theta m$ when $0<\delta<\delta_c$. Here $\delta_c=0.878$.

In the case of the cylindrical vessel

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\theta}{dr}\right)+\delta e^{\theta}=0 \qquad (2.4)$$

A substitution, $r = \exp x$ *and* $\phi=\theta+2x$ reduces the problem to $\frac{d^2\phi}{dx^2}+\delta e^{\phi}=0,$ (2.5)

and as before we obtain two solutions. On the other hand, in the case of spherical vessel

$$\frac{d^2\theta}{dr^2}+\frac{2}{r}\frac{d\theta}{dr}+\delta e^{\theta} \qquad (2.6)$$

and the problem has no closed form solution

## 3.0 New Method

In view of equation (2.5) it suffices to consider only equations (2.1) and (2.6)

**Planar case** $$\frac{d^2\theta}{dx^2}+\delta\exp\ \theta=0,\quad \theta(-1)\ =\theta(1)\ =0. \qquad (3.1)$$

By symmetry $$\frac{d\ \theta}{dx}(0)=0 \qquad (3.2)$$

We seek an approximate polynomial solution $V(x)=C_1(x+1)^2(x-1)+C_2(x+1)(x-1)^2$ (3.3)
which satisfies the boundary conditions, where $C_1$ and $C_2$ are constants to be determined by the condition of the problem. Thus $$\frac{d^2V}{dx^2}+\delta\exp V\ =R(x). \qquad (3.4)$$

using (3.2), we obtain $C_1=-C_2$ and take $x=-\frac{1}{3}$ or $\left(x\ =\ \frac{1}{3}\right)$,

$$R(-1/3)=+4C_1+\delta\ \exp-\left(\frac{16}{9}C_1\right) \qquad (3.5)$$

For $R(-1/3)\ >0$ (as expected) (3.5) has two solutions, confirming the existence of two solutions for the planar case. From $C_1=C_2$, and (3.5) the polynomial $V(x)=C_1(x+1)^2(x-1)+C_2(x+1)(x-1)$, becomes $V(x)=2C_1(x^2-1)$, $V_{max}=V(0)=-2C_1$ for the upper solution and $V(x)=-2C_2(x^2-1)$, $V_{max}=V(0)=2C_2$. for the lower solution.

**Table1: The coefficients $C_1$ and $C_2$**

| δ | $C_1$ | $C_2$ |
|---|---|---|
| 0.1 | -2.61589911988 | 2.61589911988 |
| 0.3 | -1.78200287232 | 1.78200287232 |
| 0.5 | -1.33017158736 | 1.33017158736 |
| 0.7 | -0.953832340092 | 0.953832340092 |
| 0.82 | -0.643106126611 | 0.643106126611 |
| 0.827 | -0.586440374325 | 0.586440374325 |
| 0.827728742 | -0.562502095784 | 0.562502095784 |
| 0.82772874263574 | -0.5625000047 | 0.5625000047 |

**Spherical case**

$$\frac{d^2\theta}{dr^2}+\frac{2}{r}\frac{d\theta}{dr}+\delta e^{\theta} \qquad (3.6)$$

$$\theta(0)=\ finite,\ \theta(1)=0 \qquad (3.7)$$

$$V(r)=C_1(r+1)^2(r-1)+C_2(r+1)(r-1)^2 \qquad (3.8)$$

$$V(0)=C_1-C_2 \qquad (3.9)$$

$$V(1)=0 \qquad (3.10)$$

$$V^1(0)=0\Rightarrow C_1=-C_2 \qquad (3.11)$$

Now as before $$R(r)=\frac{d^2(V)}{dr^2}+\frac{2}{r}\frac{d(V)}{dr}+\delta\exp(V) \qquad (3.12)$$

$$12C_1+\delta\ \exp\left(-{}^{63}\!/_{62}C_1\right)=R\left({}^{1}\!/_{8}\right) \qquad (3.13)$$

$$V(x)=C_1(x+1)^2(x-1)+C_2(x+1)(x-1)^2 \qquad (3.14)$$

The problem has two solutions.

## 4.0 Conclusion

Polynomial approximation confirms existence of 2 distinct solutions for planar, cylindrical and spherical vessels.

**References**

[1] Aregbesola, Y. A. S Numerical solution of Bratu problem using the method of weighted residual, Electronic Journal of South African mathematical Association (2003)

[2] Buckmaster J. D. and. Ludford, G. S Lectures on mathematical combustion, SIAM, Belfast (1983).