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On the fluctuating filtrate

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Abstract

We show by the application of the 'stick-slip' phenomenon, that, the filtrate through a porous medium could be oscillatory under low-intensity driving forces. The frequency of the oscillation is dependent on the nature of the porous medium and the external driving forces. It is, therefore, possible to characterize the medium by the use of low amplitude external driving forces.

1.0 Introduction.

An important mechanism for physical and biological processes is the flow of fluids through porous media. Such processes include filtration, catalysis, soil drainage, and flow in reservoirs (aquifers and oil formation).

We define (broadly) the filtrate as a fluid which has passed through a porous medium, the flow being dependent on the physical and chemical properties of the fluid and porous medium.

It has been shown [1] that oscillatory filtration can result from blockage of the channels by solid particulates. The striations created by the blocking particulates can modulate the flow of the filtrate, leading to oscillation. We, also, hereby show that oscillations can arise from the enhanced interaction between the fluid and the filter medium, under low intensity driving forces.

2.0 Theoretical considerations

The flow of fluid through a porous medium is governed by Darcy's law

$$Q_x = \mathrm{A}_x \cdot K_x \cdot \frac{dh}{dx} \tag{2.1}$$

where Q_x is the discharge rate (in the direction, x), perpendicular to an area, A_x, of the medium, and $\frac{dh}{dx}$ is the hydraulic gradient. The permeability, K_x, is a 'constant', which depends on the physical and chemical properties of the fluid and medium. The permeability is widely variable, due to a host of factors [2]. Important factors include the chemical constitution of the interstitial surfaces and that of the contained pore fluids [3], the rugosity of the interstitial surfaces and the turtosity of the channels. The surface interaction creates a concentration gradient, which extends from the interface into the bulk of the liquid. This often divides the fluid particles into distinct regions, each with its own dynamics. The particles of the fluid tend to bind on the surfaces, i.e., sticking occurs. When the concentration of sticking particles is large, limiting friction is attained, and slip occurs. We therefore have a stick-slip phenomenon, leading to oscillation.

The sticking fraction, S, is proportional to the ratio of the partition functions of the solid, and the fluid. That is

$$S \alpha \frac{f_s}{f_l} \qquad (2.2)$$

where f_s and f_l are the partition functions for the solid and fluid respectively. A model for the stick-slip phenomenon is shown in Figure 1, where V, is the fluid velocity due to the hydraulic gradient. K_f is a viscosity factor due to interactions between the layers of the moving fluid and the 'adsorbed' particles, which gives rise to viscous force F_v, and F_s is the static friction between the 'adsorbed' particles and the surface. Sliding occurs when

$$F_v = F_s \qquad (2.3)$$

The equation of motion of the particles is then

$$M\frac{d^2y}{dt^2} + F_v = F_d \qquad (2.4)$$

where y is the distance moved in time t by the particle after sliding, F_d is the dynamic friction and

$$F_v = F_v(y) \qquad (2.5)$$

For the sustenance of stick-slip behaviour, $F_s > F_d$. If we make the transformation

$$\left.\begin{aligned} \phi &= \frac{F_s}{F_d} \\ \tau_t &= t\left(\frac{K_f}{M}\right)^{1/2} \\ Y &= \frac{F_v}{F_s} \end{aligned}\right\} \qquad (2.6)$$

Equation (2.3) becomes

$$Y = 1 \qquad (2.7)$$

and equation (2.4) is

$$\frac{d^2Y}{d\tau_t^2} + Y = \frac{1}{\phi} \qquad (2.8)$$

The general solution of equation 2.8 is [4]

$$Y = \frac{1}{\phi} + \left(1 - \frac{1}{\phi}\right)\cos\tau_t \qquad (2.9)$$

3.0 Result and discussion

Equation 2.9 contains a sinusoidal term, which indicates that the flow is oscillatory. In Figure 2, we plot the dependence of Y and $\frac{dY}{d\tau_t}$ on τ_t for ϕ = 1.25 and ϕ = 1.50. Since the mass, M, of 'adsorbed' particles is dependent on the sticking fraction and the viscosity factor, and K_f is a fluid parameter, it then follows that the term $\left(\frac{K_f}{M}\right)^{1/2}$ is a fluid-medium property. From equation (2.6), the period of oscillation, T, is the time interval from $t = 0$ to $t = T$, corresponding to $\tau_t = 0$ to $\tau_t = \pi$. Therefore,

$$T = \pi\left(\frac{K_f}{M}\right)^{-1/2} \qquad (3.1)$$

The frequency of oscillation, f, is given by

$$f = \frac{1}{T} = \frac{1}{\pi}\left(\frac{K_f}{M}\right)^{1/2} \qquad (3.2)$$

By measuring the frequency of oscillation of the filtrate, it is possible to have a qualitative knowledge of the fluid-medium interactions.

4.0 Conclusion

The flow of fluid in porous media has been investigated under low intensity transport. Stick-slip behaviour could result, leading to oscillation in the filtrate. It is possible to use the information from the frequency of oscillation to characterize the fluid-medium system.

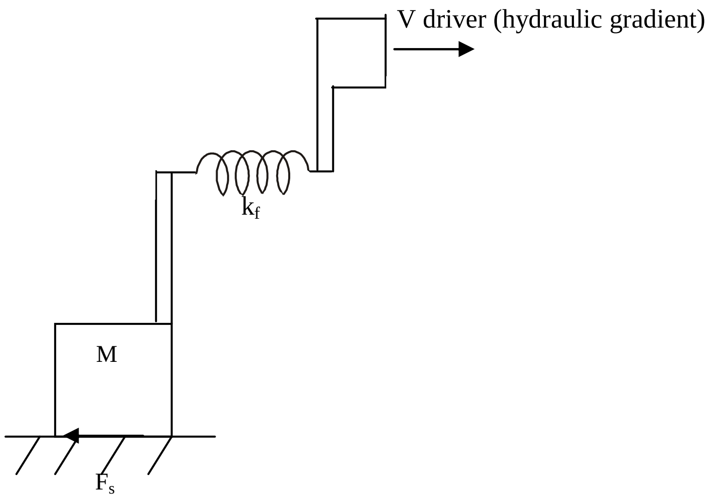

Figure 1: The stick–slip mechanism

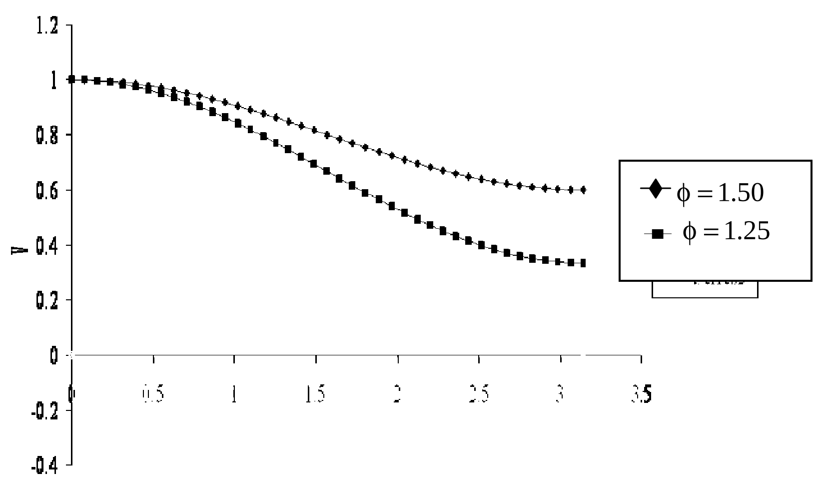

Fig.2: Slip displacement and Oscillation amplitudes

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