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Thermal neutron counts and derivated charts

[+]*Olusogo O. Odusote and Ayomide O. Balogun*
Department of Physics, Olabisi Onabanjo University, Ago-Iwoye, Nigeria.

Abstract

The neutron diffusion equation was solved under a "single cylinder one group (thermal neutrons)" approximation. The resulting equation was applied with a mixing index, ν, for various formation matrices and porosities. The ratio of counts from two different detectors was plotted as a function of porosity for these formations. These plots are useful for wireline log interpretations.

1.0 Introduction.

The determination of reservoir properties is very essential in the prospecting for crude oil and gas. One of the tools employed is neutron logging, where the slowing down behaviour of the neutrons through matter yields the necessary information, i.e., the porosity, ϕ, of the formation. This parameter is an indication of the likely amount of oil/gas stored in the reservoir. This work presents the slowing down process of thermal neutrons in a "one-cylinder" geometry, using a mixing parameter, ν. The resulting equations are used to derive data sets and charts, from which field logs could be interpreted.

2.0 Theoretical consideration

During the passage of neutrons through matter, their number must be conserved. The neutron balance equation states that for a given volume,

Time rate of change of neutron density	=	Rate of Production	–	Leakage rate	–	Absorption rate

This leads to the Boltzman transport equation, which, provided the angular distribution of the neutron velocity vector is isotopic, simplifies to (in the "one-group" approximation) the diffusion equation.

$$D\nabla^2\Phi - \Sigma_a\Phi + S = \frac{\partial n}{\partial t} \qquad (2.1)$$

$\frac{\partial n}{\partial t}$ is the time rate of change of neutron density, Φ is the neutron flux, S is the rate of neutron production, which in the case of this report is the source strength of the sonde, D is the diffusion coefficient for flux, and Σ_a is the macroscopic neutron absorption cross section of the medium.

At steady state, $\frac{\partial n}{\partial t} = 0$, therefore,

$$\nabla^2\Phi - \frac{\Phi}{L^2} + \frac{S}{D} = 0 \qquad (2.2)$$

where $\frac{1}{L^2} = \frac{\Sigma_a}{D}$, and L is the diffusion length, which is given by [1],

$$L = \tfrac{1}{2}\bar{r}_t \qquad (2.3)$$

and $\bar{r}_t$ is the average distance (measured in straight line) a neutron travels between the points of

+ Author for correspondence; e-mail: sog_odusote@yahoo.com

thermalization and absorption. In a source free medium, equation (2.2) becomes

$$(\nabla^2 - L_s^2)\Phi = 0 \qquad (2.4)$$

and $L_s^2 = \frac{1}{L^2}$.

2.1 Solution of the diffusion equation in cylindrical geometry

Many authors have solved the diffusion equation as an exercise in nuclear reactor design [2,3,4]. The geometries employed in such solutions are in spherical coordinates, which is not appropriate for solving the problem in geophysical prospecting application involving boreholes. Consider the problem in cylindrical geometry, using the "single-cylinder" approximation. In right cylindrical coordinates (r, θ, z), and assuming azimuthal symmetry of Φ, we have $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ (2.5}

If the dependence of neutron flux on coordinates is separable, then $\Phi(r, z) = R(r)Z(z)$, and equation. (2.4) becomes

$$\frac{\partial^2\Phi}{\partial r^2} = R''(r)Z(z) \text{ and } \frac{\partial^2\Phi}{\partial z^2} = R(r)Z''(z) \qquad (2.6)$$

Substituting (2.5) and (2.6) into (2.4) yields $\frac{1}{R}\left(\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r}\right) + \frac{1}{z}\frac{\partial^2\Phi}{\partial z^2} - L_S^2 = 0$.

i.e.

$$f(r) + f(z) + const = 0 \qquad (2.7)$$

where $f(r) = \frac{1}{R}\left(\frac{\partial^2 R}{\partial r^2} + \frac{1}{r}\frac{\partial R}{\partial r}\right)$, $f(z) = \frac{1}{z}\frac{\partial^2\Phi}{\partial z^2}$ and $L_s^2 = const$. Let $f(r) = -\alpha^2$ and $f(z) = \gamma^2$, where . α *and* γ are constants. We then have that $r^2\frac{\partial^2 R}{\partial r^2} + \frac{\mathrm{r}\partial \mathrm{R}}{\partial \mathrm{r}} + \mathrm{r}^2\alpha^2 R = 0$ (2.8)

If $u = \mu r$, then eqation (2.8) becomes $u^2\frac{\partial^2 R}{\partial u^2} + \frac{\mathrm{u}\partial \mathrm{R}}{\partial \mathrm{r}} + \mathrm{u}^2 R^2 = 0$ (2.9)

Equation (2.9) is Bessel's differential equation of the 1st kind, and its solution is the Bessel function of order zero. The general solution is $R_m(r) = \mathrm{A_m} J_\circ(\mu_m r) + \mathrm{B}_m Y_\circ.(\mu_m r)$. *A* and *B* are constants and $J_\circ$, $Y_\circ$ are zero-order Bessel functions of the first and second kind respectively.

Considering variation along *z*-axis, equation (2.7) is $-\mu^2 + \frac{1}{\mathrm{z}}\frac{\partial^2 Z}{\partial z^2} = L_s^2$, i.e., $\frac{1}{\mathrm{z}}\frac{\partial^2 Z}{\partial z^2} = \gamma_m^2$, where $\gamma_m^2 = L_s^2 + \mu_m^2$. Therefore

$$\frac{\partial^2 Z}{\partial z^2} - \gamma_m^2 Z = 0 \qquad (2.10)$$

The only physically admissible solution of equation (2.10) is

$$Z = \text{const.}\exp(-\gamma_\mathrm{m} z) \qquad (2.11)$$

Therefore,

$$\Phi = \left[\mathrm{AJ}_\circ(\mu r) + \mathrm{BY}_\circ(\mu r)\right]\text{const.}\exp(-\gamma_\mathrm{m} z) \qquad (2.12)$$

But $\underset{r\to 0}{Y_\circ(\mu r)} \approx -\infty$

$$\therefore \qquad \Phi = \mathrm{AJ}_\circ(\mu r)\,\text{const.}\exp(-\gamma_\mathrm{m} z)$$

The general solution for the flux distribution is $\Phi = \sum_{\mathrm{m}=0}^{\infty}\mathrm{A_m J_\circ}(\mu_m r)\exp(-\gamma_\mathrm{m} z)$ (2.13)

Let $\mu_m r_\circ = \alpha_\mathrm{m}$; $J_\circ(\alpha_\mathrm{m}) = 0$, where $r_\circ$ is the effective radius of the reservoir i.e. the radius at which the flux vanishes, which is the diffusion length, *L*. Then $\mu_m = \frac{\alpha_\mathrm{m}}{L}$ (2.14)

The α_m are determined by the zeros of the function $J_\circ(\alpha_m)$. If we have a point source,

$$S = \mathrm{S}\delta(r) \tag{2.15}$$

The neutron current through z = 0 plane is given by $I_m = -\mathrm{D}\nabla_z\Phi$ which from equation (2.13) gives

$$I_m = -\mathrm{D}\nabla_z A_m J_\circ(\alpha_m)\exp(-\gamma_m z)\Big|_{z=0}^{r=0} = D\gamma_m A_m \tag{2.16}$$

Expanding $\delta(r)$ in a Fourier – Bessel series [5], we obtain $\delta(r) = \sum_m C_m J_0(\alpha_m)$

The neutron current is also given by

$$I_m = \tfrac{1}{2}S\delta(r) = \tfrac{1}{2}S\sum_m \mathrm{C_m} J_0(\alpha_m)\Big|_{r=0} = \tfrac{1}{2}S\sum_m C_m \tag{2.17}$$

Therefore, from equation (2.16) and (2.27) $\qquad D\gamma_m \mathrm{A_m} = \tfrac{1}{2}S\sum_m \mathrm{C_m}$ (2.18)

Also from (2.6), $\qquad \int_0^L 2\pi r\,\delta(r) J_\circ\left(\frac{\alpha_m r}{L}\right)dr = 1$ (2.19)

i.e. $\qquad 2\pi r\sum_m C_m \int_0^L r J_\circ^2\left(\frac{\alpha_m r}{L}\right)dr = 1$ (2.20)

and $\qquad \int_0^L r J_\circ^2\left(\frac{\alpha_m r}{L}\right)dr = \tfrac{1}{2}L^2\left[J_\circ(\alpha_m) + \mathrm{L}^2 J_\circ'^2(\alpha_m)\right]$ (2.21)

Substituting (2.21) into (2.20), we have $2\pi\mathrm{C_m}\,\tfrac{1}{2}L^2\left[J_\circ(\alpha_m) + \mathrm{L}^2 J_\circ'^2(\alpha_m)\right] = 1$. But, $J_\circ(\alpha_m) = 0$,

$\therefore$

$$\mathrm{C_m} = \frac{1}{\mathrm{L}^4 \pi J_\circ'^2(\alpha_m)} \tag{2.22}$$

From the theory of Bessel functions [7], $J_\circ'(\mu_m r) = -\mu_m J_1(\mu_m r)$, therefore $\mathrm{C_m} = \dfrac{1}{\pi\alpha_m^2 \mathrm{L}^2 \mathrm{J}_1^2(\alpha_m)}$

Then equation (2.18) yields $\qquad A_m = \dfrac{\mathrm{S}}{2\mathrm{D}\pi\mathrm{L}^2}\sum_m \dfrac{1}{\gamma_m \alpha_m^2 J_1^2(\alpha_m)}$ (2.23)

This from equation (2.23) yields $\qquad \Phi = \dfrac{\mathrm{S}}{2\mathrm{D}\pi\mathrm{L}^2}\sum_{m=0}^{\infty} \dfrac{\mathrm{J}_\circ(\mu_m r)\exp^{-\gamma_m Z}}{\gamma_m \alpha_m^2 J_1^2(\alpha_m)}$ (2.24)

This equation gives the neutron flux on the axis of the borehole when the detector is at a distance, *z*, from the source.

3.0 Calculation methods

On the axis of the borehole, r = 0, and, $\mathrm{J_0(0)} = 1$. The ratio of the neutron count at detector positions z_1 and z_2 is computed from equation (2.24), where the series is taken to fifth order. This yields

$$\frac{\Phi(z_2)}{\Phi(z_1)} = \frac{\sum_{m=1}^{5}\left(\exp^{-\gamma_m z_2}\Big/\alpha_m^2 J_1^2(\alpha_m)\right)}{\sum_{m=1}^{5}\left(\exp^{-\gamma_m z_1}\Big/\alpha_m^2 J_1^2(\alpha_m)\right)} \tag{3.1}$$

The input parameters for the computation are listed in Tables 1 and 2. The material scattering parameters have been calculated using detailed methods, which include the Goertzel-Greuling procedure [8,9,10].

In well logging, the detectors of thermal neutrons must be at a distance from the source greater than 70cm. For this computation, we have used z_1 = 100cm and z_2 = 80cm. The porosity, Φ, of the medium affects the neutron scattering parameters, and the nature of the formation (whether single component, or, a mixture) is also important. Suppose the diffusion length, L, is the parameter of interest in a two component matrix having volume fractions f_1 and f_2 of components 1 and 2, with respective diffusion lengths L_1 and L_2. The diffusion length of the matrix, L_{mat}, is given by [11],

$$L_{max}^{\nu} = \mathrm{L}_1^{\nu} f_1 + \mathrm{L}_2^{\nu} f_2 \tag{3.2}$$

where ν is the mixing index. For a formation with porosity ϕ,

$$L_{max}^{\nu} = \mathrm{L}_1^{\nu}\phi + \mathrm{L}_2^{\nu}(1-\phi) \tag{3.3}$$

For an n-component matrix, $\qquad \nu = \sum_{t=1}^{n} \nu_t f_t$ (3.4)

4.0 Results and Discussion

Figure 1 shows typical neutron flux ratio plots as calculated from equation (3.1), for the indicated formation matrices. This was done for oil-bearing formations, in varying porosities from 0 to 0.5, which is the range encountered in actual logging operations.

The plots show normal trend with a decrease of flux ratio with increase in porosity. This is in agreement with the increased scattering power of the hydrogenous media with increase in porosity.

In actual well logging, the wire line tool measures the detector count ratios. With additional information from the analysis of cuttings, and from other measurements such as gamma ray or aluminium activation, the reservoir matrix type is determined. The appropriate chart could then be read directly to determine the porosity, which is an important reservoir parameter. The porosity is an indication of the oil reservoir capacity - an important economic factor.

5.0 Conclusion

The solution of the diffusion equation in cylindrical geometry has afforded the calculation of the detector count ratio in a realistic manner. The computed plots are of benefit to the wire line log interpreter.

Table 1: Bessel function roots and values

M	1	2	3	4	5
$J_1^2(\alpha_m)$	2.405	5.520	8.654	11.792	14.931
$J_1^2(\alpha_m)$	0.269	0.116	0.073	0.054	0.043

Table 2: Material neutron scattering parameters.

Material	Sandstone	Limestone	Dolomite	Anhydrite	Crude oil
Diffusion Length (cm)	18.25	13.26	15.16		2.10
Slowing down length (cm)	28.79	25.69	21.28	31.38	
Mixing index	-1.66	-1.76	-2.00	-1.65	

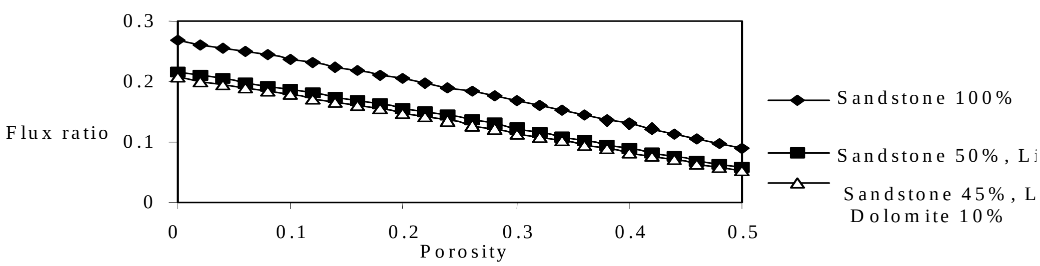

Figure.1: Flux ratio vs porosity

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