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Effect of queue discipline on the performance of a queueing system

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Abstract

The effect of three queue discipline namely first in, first out (FIFO), last in first out (LIFO) and service in random order (SIRO) on some measures of performance of a single sever queue are examined. The measures of performance are average waiting time and queuing time. The comparison of the systems were carried out by writing an appropriate program in BASIC to simulate the queue discipline. This is due to the versatile nature of simulation and the fact that it is extremely difficult to obtain numerical results mathematically for the single server queue when the queue discipline is not FIFO and the arrival process is non-stationary. The approach adopted is to generate arrival time and service times for n customers through a single sever queuing system under each queue discipline. The measures of performance were calculated for each system using appropriate expressions. The result show that the average queuing time and average waiting time are higher when the queue discipline is LIFO, whereas the total idle time for all the systems were found to be same in most cases.

Keywords: Queues, waiting time, Queuing time & Service time.

1.0 Introduction

In a single queuing system, there is only one server. Customers arrive at a service point and if the sever is busy on the customer's that is in the system, the new customer joins the queue. When there is a queue of waiting customers, then the server has to decide which customer to serve next whenever he is free. Queue discipline is the rule for deciding which customers is selected for service out of all those waiting for service. The common queue disciplines are First In, First out (FIFO), Last in Last out (LIFO), and service in random order (SIRO). In FIFO customers are served in order of arrival; in LIFO, customers are served in reverse order of arrival. In SIRO, customers are served at random, i.e. customers waiting for service have equal probabilities to be served next in some cases, we may have a priority system i.e. Priority Service (PS). There are two main types of priority systems viz: pre-emptive priority. In a pre-emptive priority system, a high priority customer takes immediate precedence on arrival over a customer of low priority. The customer whose service is interrupted can only return to the service point only when there are no higher priority customers in the system. In non pre-emptive priority system, a customer receiving service must complete his service before leaving the system. But once a server is free, the admission of a customer with lower priority is considered.

Several methods have been used to discuss queue disciplines. Neuts (1981) used a numerical method to calculate the performance measures of single server queue disciplines. Omosigho and Worthington (1985, 1987) had to use the discrete time queuing model to study the single server queue with non-stationary arrival process. Li (1997) used an approximation method for the analysis of G/G/I(the first G mean general arrival, the second G general service and I number of servers in the system)queues . Omosigho and Samuel(2003) used computer simulation to study the effect of FIFO and LIFO on the mean queueing and waiting time taking a sample of 100 customer. This paper considers the extension of Omosigho and Samuel (2003) work to finite capacity n, and by comparing their work with SIRO single server queueing systems with non-stationary arrival process. Not that Omosigho and Samuel (2003) did not generalize their approach to take n customers. Which mean that for you to sample 200, 300, and so on you need to remodel their system again. Hence in this paper, we gave a general system that take n customers. That is given n customers the model will be able to provide the performance of the system.

Most queueing problems are complex in nature, and they are difficult to handle using analytic methods. In these cases an event driven computer simulation are used. An event-driven computer simulation is one in which the state of the simulated system is updated from one discrete event to the next. An event is defined as any occurrence that changes the state of the system or will affect the state of the system at some time in the future.

In this paper, we shall formulate an event-driven computer simulation to examine the effect of FIFO, LIFO and SIRO on the mean queueing time and the mean waiting time. By waiting time, we mean the time spent in the system by a customer from the instant of entering the system to the moment when his service is completed. By queueing time, we mean waiting time minus service time. Thus queuing time of a customer is the time spent in the system from the instant of entering the system to the moment service is started.

In the results reported in this paper, the arrival process is non-stationary in the sense that the rate is time dependent.

2.0 Method of comparison

The comparison of the three systems was carried out by writing an event-driven computer program in BASIC to simulate the queue disciplines, FIFO, LIFO & SIRO. This is due to the versatile nature of simulation and the fact that it is very difficult to obtain mathematical solutions for queueing system when the queue discipline is different from FIFO. The approach adopted is to take a sample of n customers. The customers are numbered as they arrive. For each customer, the arrival time and service time is generated using pseudo-random numbers. The customers are then passed through a single server queuing system under the queue discipline in force and the measures of performance are calculated. Three queue discipline namely FIFO, LIFO and SIRO were used. The results from the systems were compared.

It is known (see Cox and Smith (1961)) that samples from exponentially distributed random variables with mean λ can be obtained by using the expression

$$x = -\lambda \log_e(\lambda). \qquad (2.1)$$

where λ is a pseudo-random number. The single server queueing system studied in this paper can be described as follows. The service times of individual customers are independent and identically distributed exponential random variable with mean μ. The inter-arrival times of the customers are independent and identically distributed exponential random variables with mean λ. The relationship between the rate of arrival λ, the number of customers in the system L and the time a customer is in the system *W* is given by the expression L=λW. This is known as Little’s law. If they are equal we have a stationary inter-arrival time distribution. Equation 1 is used to generate the inter-arrival times and the service times for n customers using appropriate parameters.

3.0 Mathematical formulation

The following symbols will be used throughout the study.

n	customer’s number
a_n	time intervals between the arrival of the nth and $(n + 1)^{th}$ customer
A_n	arrival epoch of the n^{th} customer
b_n	service time of the nth customer
S_n	epoch of completion of service for the nth customer
I	idle period
W_n	waiting time of the nth customer
ρ	traffic intensity of the system
Q_n	queuing time of the nth customer

Now, $a_1 = A_2 - A_1$, $a_2 = A_3 - A_2$, Clearly,

$$a_n = A_{n+1} - A_n, n = 1, 2, \cdots \qquad (3.1)$$

Also under FIFO

$$Q_1 = 0,_3\ Q_n = \max\{Q_n + b_n - a_n, 0\}, n = 1,2,\cdots \quad (3.2)$$

$$W_n = Q_n + b_n \quad (3.3)$$

See Cox (1961) for details.

The idle period of the server is calculated as follows: If $a_n > W_{n-1}$, then the server is idle. The idle time in this case is

$$A_n - S_{n-1} \quad (3.4)$$

The variance of the waiting time, idle time and queueing time were computed by the formula

$$V = \tfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \quad (3.5)$$

where V is the variance, x_i (i = 1, 2, …, n) are real numbers and the mean is defined by $\bar{x} = \frac{(x_1 + x_2 + ... + x_n)}{n}$

For LIFO, the customer's arrival time is ordered and last customer is employed in the use of the equations above. For SIRO the customer are served in random order by also using the equations above

Equation 2 to 8 was used to develop an event-driven computer program to simulation the three systems.

4.0 Computer program

For the program to be more convenient we use a set of computer subroutines to generate the arrival and service time for each customer and to compute the epoch of service completion for the different queue disciplines (FIFO, LIFO and SIRO). Also the subroutines would be used to compute all the necessary parameters for the queue disciplines.

The customers were served under the different queue disciplines. And the parameters for measuring performance (mean waiting time, mean queuing time, mean epoch of service completion, mean idle time and variance of the waiting time, queuing time, idle time) for the different queue disciplines were calculated. The program output was validated by careful manual calculations and found to be correct.

5.0 Results

Taken n =200 the following results were obtained.

***Experiment* 1**

Table 1: Results for the 1st set of 200 customers sampled

Parameters	Queue Disciplines		
	FIFO	**LIFO**	**SIRO**
Mean waiting Time	4.72	4.93	4.76
Mean Idle Time	0.51	0.51	0.51
Mean Queuing Time	2.06	2.26	2.09
Mean Service Completion	328.48	328.68	328.51
Variance of Waiting Time	6.48	35.58	8.20
Variance of Idle Time	0.91	0.91	0.91
Variance of Queuing time	7.49	38.50	9.04

***Experiment* 2**

Table 2: Result for 2nd set of 200 customers sampled

Parameters	FIFO	LIFO	SIRO
Mean Waiting Time	7.00	7.01	6.89
Mean Idle Time	0.29	0.29	0.29
Mean Queuing Time	4.42	4.43	4.30
Mean Service Completion Time	291.09	291.10	290.98
Variance of Waiting Time	18.60	161.21	21.45
Variance of Idle Time	0.66	0.66	0.66
Variance of Queuing Time	20.49	163.07	23.93

The results obtained from both experiments show that both mean queueing time and mean waiting time are higher for LIFO in all cases. This can be explained as follows. When there is congestion, many customers are forced to wait when the queue discipline is LIFO. This tends to increase the total queuing time and hence average queueing time. Once the average queueing time is high, the average waiting time will be equally high since waiting time is the sum of queueing time and service time. In fact, if W is waiting time, Q is queueing time and S is service time, then

$$E(W) = E(Q) + E(S) \qquad (5.1)$$

In all the systems E(S) are all the same.

6.0 Conclusion

Here we have been able to remodel Omosigho and Samuel (2003) to take n customer which have solve the problem of remodeling when we sampe more than 200 customers in their system. We used simulation to study the effect of queue discipline on the single server queue with non-stationary Poisson arrival process and exponential service time. The results show that both waiting time for the LIFO queueing system is higher than that of FIFO and SIRO queueing systems. The results demonstrate that the LIFO system tend to increase the queueing time for the customers while reducing it for other. Those customers with increased queueing time, when ever they are people, tend to perceive the system as unfair. Where possible, controllers of queueing system should adopt the FIFO and SIRO queue discipline so to be fair to their customers.

References

[1] Cox D.R. and Walter L, Smith (1961) QUEUES, Chapman and Hall, London.

[2] Jingwen L I, (1997) An Approximation Method For The Analysis Of G/IG/I Queues. Operation Research Vol 45, No.1, Pp 140-144.

[3] Micheal, P. (1984). Computer Simulation In Management Science. John Wiley and Sons Inc. New York.

[4] Naylor, T.H. (1966) Et Al. Computer Simulation Techniques. John Wiley and Sons Inc. New York.

[5] Ojobor, S.A (1999). The Effects of Queues Disciplines on the Performance of a Queueing System .M.Sc. Thesis University of Benin, Nigeria. 56pp.

[6] Omosigho S.E. and Worthington D.J. (1985). "The Single Server Queue With Inhomogeneous Arrival Rate And Discrete Service Time Distribution", European Journal Of Operation Research 22, 397-407.

[7] Omosigoho S.E. and Worthing D.J. (1988)."An Approximation Of Known Accuracy For The Single Server Queue With Inhomogeneous Arrival Rate And Discrete Service Time Distribution", European Journal Of Operational Research 33, 304-313.

[8] Omosigho S.E. and Samuel O.A. (2003). "Effect of Queue Discipline on the Single Server Queue. Nigerian Journal of Science and Environment" Vol.3 .Pp 82