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Radiation effect of magnetohydrodynamic flow of gas between concentric spheres

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Abstract

Time independent flow of fluid between two concentric rotating spheres permeated by magnetic filed is studied. Prevailing mode of heat transfer is radiation while optically thin limit case is considered. The mathematical model of the problem with the induced magnetic field is developed and the resulting differential equations were solved using perturbed numerical technique. It is found that the magnetic field has no effect on the temperature distribution. However, when the magnetic field is introduced a decrease in velocity is obtained with an increase in either radiation parameter or Reynolds number.

1.0 Introduction

The problem of heat transfer by free convection in an enclosure is in itself of practical importance. Such problem plays a significant role, for example, in the re-entry problem of intercontinental ballistic missiles. The problem of heat transfer in electrical conducting fluids permeated by electromagnetic fields has also been studied. Such studies are of importance in connection with geophysical, astrophysical and engineering problems such as the motion of the inter stellar gas, origin of earth magnetism, plasma jet, the design of magneto hydrodynamic (MHD) generator, cross-field accelerators, shock tubes & pumps. The works of the following researches are important in the present investigation.

Romig [4] presented a comprehensive review of the problem of heat transfer in electrically conducting fluids subjected to an electromagnetic filed. Gershuni and Zhukhovitsky [1] presented an analysis of convection flow in a vertical channel when the wall temperature is constant. Yu [5] extended the work in [1] by considering the case when the wall temperature varies linearly with the vertical distance. The flows in both cases [5] and [1] are subjected to a transverse magnetic field.

The studies mentioned above, however, do not take into account heat transfer by radiation, which is significance when one is concerned with space applications, liquid metal fast breeder reactors and higher operating temperature. Grief et al [8] obtained the solution for the problem of steady radiating laminar convective flow in a vertical heated channel. He adopted a differential approximation for the radiative flux given by Cogley and Vincenti [6]. The effect of radiation on magnetohydrodynmic channel flow with heat transfer, however does not seems to have received much attention.

Viskanta [3] investigated the forced convection flow in a horizontal channel permeated by uniform vertical magnetic field taking radiation into account to study the effects of magnetic field and radiation on the temperature distribution and the rate of heat transfer in the flow but did not discuss the influence of radiation on the induced magnetic field. Gupta and Gupa [9] considered the effect of radiation on combined free and forced convection flow of an electrically conducting fluid inside an open ended vertical channel permeated by a uniform transverse magnetic field. Closed form solutions for the velocity, temperature and the induced magnetic field are obtained when the wall temperature vary linearly with the vertical distance. Recently the effect of radiation on temperature and velocity in electro hydrodynamic forth flow process was analysed in [13]. Furthermore, Adesanya and Ayeni

[14] also studied radiative effect on hydro magnetic flow of a radiation reacting gas with variable thermal conductivity in a vertical channel.

In the aforementioned investigations only the physical model consisting of a channel is considered. However, a defect in this model is that in most cases an additional assumption of large width of the plates has to be made so that edge effects are negligible. Better models for avoiding edge effects are cylinder and sphere and in fact, the best is the sphere.

Some researchers such as Bestman [10] considered a steady flow of a radiating gas between concentric rotating spheres. He assumed that the rotational Reynolds number of the flow to be small. The solution is effected by classical perturbation scheme. Consequences of important radiative effects on the heat transfer characteristics are discussed quantitatively. In this chapter, therefore, the problem of steady laminar convection to magneto hydrodynamic flow of a radiating gas between concentric spheres is studied. Since the problem of radiative transfer involves complicated integral expression we shall assumed that the gas is optically thin non-grey and is near equilibrium so that Cogley, Vincenti and Gilies differential approximation could be employed. The temperature of the sphere are high enough so that radiation transfer is significant, though, the difference in sphere's temperature is assumed small as in [10] so that the free convection parameter is correspondingly small.

The mathematical formulation of the problem, is discussed, this is followed by the perturbation scheme, where the leading and higher approximate solution are presented. Finally the results are discussed.

2.0 Mathematical Formulation

Considering a static fluid at a given Temperature T_s, density ρ_s and pressure P_s that is introduced between two concentric rotating spheres. The inner sphere has a radius of r_0, rotating an angular velocity of ω_0 and maintains a constant temperature T_0. While the outer sphere is of radius r_1 rotating at an angular velocity of ω_1 and is maintained at constant temperature T_1. If $(U_r\ U_\theta, U_w)$ are the velocity component and (H_r, H_0, H_w) are the magnetic intensity components in the spherical coordinate system (r , θ, w), then for axisymetric motion, the continuity equation is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 U_r\right)+\frac{1}{r\sin\theta}\left(U_\theta \sin\theta\right)=0 \tag{2.1}$$

The corresponding Navier – Stokes equations are

$$\begin{aligned}(\overline{U}.grad)U_r - \frac{U_\theta^2}{r} - \frac{\mu}{4\pi\rho s}\left\{(H.\ grad)\ H_r - \frac{\overline{H}_\theta^2}{r}\right\} &= \\ \frac{-\partial\pi_s}{\partial r} + v\left(\nabla^2 U_r - \frac{2}{r^2}\frac{\partial U\theta}{\partial v} - \frac{U_r}{r^2}\right)\end{aligned} \tag{2.2}$$

$$\begin{aligned}(\overline{U}.grad)U_w + \frac{U_\theta U_r}{r} - \frac{\mu}{4\pi\rho_s}\left\{(H.\ grad)\ H_\theta + \frac{H_\theta H_r}{r}\right\} &= \\ -\frac{1}{r}\frac{\partial\pi_s}{\partial\vartheta} + v\left(\nabla^2 U_\vartheta - \frac{2}{r^2}\frac{\partial u_r}{r\theta} - \frac{U_0}{r^2}\right)\end{aligned} \tag{2.3}$$

$$(\overline{U}.grad)U_w - \frac{\mu}{4\pi\rho_s}(\overline{H}.\ grad)H_w = v\nabla^2 U_w \tag{2.4}$$

while the magnetic equations are

$$(\overline{U}.grad)\ H_r - (\overline{H}.\ grad)U_r = \eta\left(\nabla^2 H_r - \frac{2}{r^2}\frac{\partial H_\theta}{\partial\theta} - \frac{H_r}{r^2}\right) \tag{2.5}$$

$$(\overline{U}.grad)H_\theta - (\overline{H}.\ grad)U_\theta + \frac{1}{r}[U_\theta H_r - U_r H_\theta] = \eta\left(\nabla^2 H_\theta - \frac{2}{r^2}\frac{\partial H_\theta}{\partial\theta} - \frac{H_\theta}{r^2}\right) \tag{2.6}$$

$$(\overline{U}.grad)H_w - (\overline{H}.grad)U =_w \eta\nabla^2 H_w \tag{2.7}$$

Furthermore, the associated Energy equation is

$$P\,\overline{U}.\nabla\theta = \nabla^2\theta \, - \, \nabla.\vec{q}R \tag{2.8}$$

From the above equations

$$\overline{U}.\nabla = U_r\,\frac{\partial}{\partial r} + \frac{U_\theta}{r}\,\frac{\partial}{\partial\theta} \tag{2.9a}$$

$$\overline{H}.\nabla = H_r\,\frac{\partial}{\partial r} + \frac{H}{r}\,\frac{\partial}{\partial\theta} \tag{2.9b}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta} + \frac{\cot\theta}{r}\frac{\partial}{\partial\theta} \tag{2.9c}$$

μ is the viscosity, $\vec{q}_R$ is the radiative flux vector, $\eta = \frac{1}{4\pi\rho}$ is the resistivity, C_ρ is the specified heat at a constant pressure, k is the thermal conductivity, $\overline{U}$ is the velocity vector field, $\overline{H}$ is the magnetic vector field.

$$K = \frac{\mu}{4\pi\rho} \qquad \text{and} \qquad \pi_s = \frac{P_s}{\rho_s} + \frac{\mu\left|\overline{H}\right|^2}{8\pi\rho_s}$$

Using these definitions and introducing the following Dimensionless quantities

$$r = \frac{r'}{r_0};\ \left(U_r, U_\theta, U_w\right) = \frac{r_0 w}{\upsilon}\left(U'_r, U'_\theta, U'_w,\ \right);$$

$$\theta = \frac{T - T_s}{T - T_s};\quad p = \left(p' - p_s\right)\frac{r_0^2}{\rho 0 v^2};\quad R = \frac{r_1}{r_0};$$

$$\left(H_r, H_\theta, H_w,\right) = \frac{r_0 w}{v}\left(H'_r,\ H'_\theta, H'_w,\right);\ \Omega = \frac{w_1}{w_0};\ p_r = \frac{\mu_c P}{K}$$

$$G_r = \beta\,\frac{gr^2\left(T_1 - T_s\right)}{v^2}$$

$$M = \frac{T_1 - T_0}{T_0 - T_s},\ \mathrm{Re} = \frac{r_0\ w_0}{v},\ N = \frac{K_{r0}}{K}$$

$$K = 4\int_0^\infty\left(\alpha v\,\frac{\partial\beta v}{\partial T}\right)_0\partial N \qquad \ldots \tag{2.10}$$

where G_r is the free convection parameter or Grashof number. P_r is the Prandlt number, Re is the Reynolds number and N is the radiation parameter.

We have

$$\frac{1}{r^2}\frac{\partial}{\partial r'}\left(r'^2\ U'_r\right) + \frac{1}{r'\sin\theta}\frac{\partial}{\partial\theta}\left(U'_\theta\ \sin\ \theta\right) = 0 \tag{2.11}$$

$$\mathrm{Re}\left(U'_r\,\frac{\partial U'_r}{\partial r} + \frac{U'_0}{r}\frac{\partial U'_r}{\partial\theta} - \frac{U'^2_\theta\ +\ U'^2_w}{r}\right) - K\left[H_r\ \frac{\partial H_r}{\partial r}\ +\ \frac{H_\theta}{r}\ \frac{\partial H_r}{\partial\theta}\ -\ \frac{H_\theta^2\ +\ H_w^2}{r}\right]$$

$$= -\frac{\partial\pi_p}{\partial r} + \upsilon\left[\left(\nabla^2 - \frac{2}{r^2}\right)U_r\ \frac{-2}{r^2}\ \frac{\partial U_\theta}{\partial\theta}\ - \frac{2u_\theta\ \cot\theta}{r^2}\ \right] - \ G\theta\ \cos\theta \tag{2.12}$$

$$\mathrm{Re}\left(U_r\ \frac{\partial U_\theta}{\partial r} + \frac{U_0}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{u_r u_0}{r} - \frac{U_w^2\ \cot\theta}{r}\right) - K\left[H_r\ \frac{\partial H_\theta}{\partial r}\ +\ \frac{H_\theta}{r}\ \frac{\partial H_\theta}{\partial\theta} - \frac{H_r H_\theta}{r}\ -\ \frac{H_w^2\ \cot\theta}{r}\right]$$

$$= -\frac{1}{r}\frac{\partial \pi}{\partial \theta} + \upsilon\left[\left(\nabla^2 \quad \frac{1}{r^2 \sin^2\theta}\right) U_\theta \; - \frac{2}{r^2}\frac{\partial U_\theta}{\partial \theta}\right] + G\theta \sin\theta$$

$$\mathrm{Re}\left(\frac{U_r \partial U_w}{\partial r} + \frac{U_0}{r}\frac{\partial U_w}{\partial \theta} + \frac{U_r U_w}{r} + \frac{U_r U_w}{r}\cot\theta \right) - \tag{2.13}$$

$$K\left[H_r \frac{\partial H_w}{\partial r} + \frac{H_\theta}{r}\frac{\partial H_w}{\partial \theta} + \frac{H_\theta H_w}{r} + \cot\theta \right] = \upsilon\left(\nabla^2 \quad \frac{1}{r^2}\sin^2\theta\right) U_w$$

$$U_r \frac{\partial H_r}{\partial r} + \frac{U_0}{r}\frac{\partial H_r}{\partial \theta} - \frac{H_0^2 - H_w^2}{r} - H_r \frac{\partial U_r}{\partial r} - \frac{H_\vartheta}{r}\frac{\partial U_r}{\partial \theta} \quad \frac{H_\theta}{r}\frac{\partial U_r}{\partial \theta} + \frac{U_\theta^2 + U_w^2}{r}$$

$$= \eta\left[\left(\nabla^2 \quad \frac{2}{r^2}\right) H_r \; - \frac{2}{r^2}\frac{\partial H_r}{\partial \theta} - \frac{2H_r}{r^2}\cot\theta\right] \tag{2.15}$$

$$\mathrm{Re}\left(U_r \frac{\partial H_\theta}{\partial r} + \frac{U_0}{r}\frac{\partial H_\theta}{\partial \theta} + \frac{U_0 \; H_\theta}{r}\frac{U_w^2 \cot\theta}{r} \right) - \left(H_r \frac{\partial U_\theta}{\partial r} - \frac{H_\theta}{r}\frac{\partial U_\theta}{\partial \theta} + \right. \tag{2.16}$$

$$\frac{H_w^2 \cot\theta}{r} - \frac{H_r \; u_\theta}{r} = \left[\left(\nabla^2 - \frac{1}{r^2 \sin^2\theta}\right) H_\theta + \frac{2}{r^2}\frac{\partial U_r}{\partial \theta}\right]$$

$$\mathrm{Re}\left(U_r \frac{\partial H_w}{\partial r} + \frac{U_0}{r}\frac{\partial H_w}{\partial \theta} \right) - H_r \frac{\partial U_w}{\partial r} - \frac{H_\theta}{r}\frac{\partial U_w}{\partial \theta} + \frac{1}{r}\left(U_r H_w - H_r U_w\right) + \tag{2.17}$$

$$\frac{\cot\theta}{r}\left(U_r \; H_w - H_r U_w\right) = \left(\nabla^2 - \frac{1}{r^2 \sin^2\theta}\right) H_w$$

$$\Pr \mathrm{Re}\left[U_r \frac{\partial}{\partial r}\left(\frac{T - T_S}{T_1 - T_S}\right) + \frac{U_\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{T - T_S}{T_1 - T_S}\right)\right] = \nabla^2\left(\frac{T - T_S}{T_1 - T_S}\right) - \frac{Kr_\theta^2}{k}\left(\frac{T - T_S}{T_1 - T_S}\right) \tag{2.18}$$

Eliminating the terms involving π in equation (2.12) and (2.13) and simplifying gives

$$\mathrm{Re}\frac{1}{r}\left(\frac{\partial}{\partial r}\left\{ r\left[U_r \frac{\partial U_\theta}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_\theta}{\partial \theta} + \frac{U_r \; U_\theta}{r} - \frac{U_w^2}{r}\right)\right)\cot\theta - k\left[H_r \frac{\partial H_\theta}{\partial r} + \right.$$

$$\left.\frac{H_\theta}{r}\frac{\partial H_\theta}{\partial \theta} + \frac{H_r H_\theta}{r} - \frac{H_w^2 \cot\theta}{r}\right]\bigg\}\bigg] - \mathrm{Re}\frac{1}{r}\frac{\partial}{\partial \theta}\left(\left[u_r \frac{\partial U_r}{\partial r} + \frac{U_\theta}{r}\frac{\partial U_r}{\partial \theta} - \frac{U_\theta^2 + U_w^2}{r}\right]\right. \tag{2.19}$$

$$\left. - K\left[H_r \frac{\partial H_r}{\partial_r} + \frac{H_\theta}{r}\frac{\partial H_r}{\partial \theta} - \frac{H_0^2 + H_w^2}{r}\right]\right) + Gr\left[\sin\theta \frac{\partial \theta}{\partial r} - \frac{1}{r}\cos\theta \frac{\partial \theta}{\partial \theta}\right]$$

$$= \left[\nabla^2 - \frac{1}{r^2 \sin^2\theta}\right]\left(\frac{1}{r}\frac{\partial}{\partial r}\left(rU_\theta\right) - \frac{1}{r}\frac{\partial U_r}{\partial_\theta}\right) - \frac{2}{r^2}\left(\frac{\partial^2 U_\theta}{\partial r^2} - \frac{2\partial U_\theta}{\partial r} - \frac{\partial^2 U_r}{\partial r \partial \theta}\right)$$

The corresponding boundary conditions are

$$\begin{aligned} &U_r = U_\theta = 0;\; U \;= Sin\,\theta; \qquad \theta = 0 \\ &H_r = H_\theta = 0;\; H_w = Sin\,\phi \qquad on \quad r = 1 \;; \\ &U_r = U_\theta = 0;\; U \;= \Omega Sin\,\theta; \qquad \theta = m \\ &H_r = H_\theta = 0;\; H_w = \gamma Sin\,\phi \qquad on \quad r = R \end{aligned} \tag{2.20}$$

Thus, equation (2.11), (2.14) - (2.19) subject to (2,20) are mathematical problem to be solved.

3.0 Perturbation analysis of the boundary value problem

These above equations are non-linear and not amenable to analytical treatment. Thus asymptotic expansion will be more applicable. For small Re we seek the solution of the form

$$U'_r = R_e\ U_{r1},(r,\theta) + h.0.t$$
$$U'_\theta = R_e\ U_{01},(r,\theta) + h.0.t$$
$$H_r = R_e\ H_{r1},(r,\theta) + h.0.t$$
$$H'_\theta = R_e\ H_{\theta 1},(r,\theta) + h.0.t \quad (3.1)$$
$$U'_w = U_{w0},(r,\theta) + R_e U_{w1}(r,\theta) + h.0.t$$
$$H'_w = H_{w0},(r,\theta) + R_e U_{w1}(r,\theta) + h.0.t \quad (3.2)$$

while for θ we have $\theta = \theta^{(0)}$ Re $\theta^{(1)}$ (r, θ) + h.o.t (3.3)
(where h.o.t means "higher order temrs")..

Now on substituting (3.1) to (3.3) into (2.11) and (2.14) – (2.19) and collecting terms of the same order we have, for 0(1)

$$\left(\nabla^2 - \frac{1}{r^2 \sin^2\theta}\right) U_{w0} = 0 \quad (3.4a)$$
$$\left(\nabla^2 - \frac{1}{r^2 \sin^2\theta}\right) H_{w0} = 0 \quad (3.4b)$$
$$\left(\nabla^2 - N\right)\theta_0 = 0 \quad (3.4c)$$

while for 0(*Re*) we have the following distinct equation viz:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(rU_{r1}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(U_{\theta 1}\sin\theta\right) = 0 \quad (3.5)$$

$$\mathrm{Re}\sin\theta\frac{\partial\theta_0}{\partial r} = v\left[\left(\nabla^2 - \frac{1}{r^2\ \sin^2\ \theta} - \frac{\partial}{\partial r}\right)\left(\frac{1}{r}\ \frac{\partial}{\partial r}(rU_{\theta 1}) - \frac{1}{r}\frac{\partial U_{r1}}{\partial\theta}\right)\right.$$
$$\left. - \frac{2}{r^2}\left(r\frac{r\partial^2 u_{\theta 1}}{\partial r^2} + \frac{2\partial U_\theta}{\partial r} - \frac{\partial^2 U_{r1}}{\partial r\,\partial\theta}\right)\right] \quad (3.6)$$

$$\mathrm{Re}\left[U_{r1}\frac{\partial U_{w\theta}}{\partial r} + \frac{U_{\theta 1}}{r}\frac{\partial_{w0}}{\partial\theta}\frac{U_{r1}}{r}\ U_{w0} + \frac{U_{\theta 1}\ U_{w0}}{r}\ \cot\theta - K\right]$$
$$\left[H_{r1}\frac{\partial H_{w\theta}}{\partial r} + \frac{H_{\theta 1}}{r}\frac{\partial H_{w0}}{\partial\theta} - \frac{H_{r1}H_{w0}}{r} + \frac{H_{\theta 1}H_{w0}\cot\theta}{r}\right] = v\left(\nabla^2 - \frac{1}{r^2\sin^2\theta}\right)U_{w1} \quad (3.7)$$

$$\eta\left[\left(\nabla^2 - \frac{2}{r^2}\right)H_{r1} - \frac{2}{r}\frac{\partial H_{\theta 1}}{\partial\theta} - \frac{2H_{\theta 1}\ \cot\ \theta}{r^2}\right] = 0 \quad (3.8)$$

$$\eta\left[\left(\nabla^2 - \frac{2}{r^2\sin^2\theta}\right)H_{\theta 1} + \frac{2}{r}\frac{\partial H_{\theta 1}}{\partial\theta}\right] = 0 \quad (3.9)$$

$$\mathrm{Re}\left[U_{r1}\ \frac{\partial H_{w\theta}}{\partial r} + \frac{U_{\theta 1}}{r}\frac{\partial_{w0}}{\partial\theta} - H_{r1}\frac{\theta_{w0}}{\partial\theta} + \frac{1}{r}\left(U_{r1}\ H_{w0} - H_{r1}\ U_{w0}\right)\right]$$

$$+\cos\left(U_{r1}\ H_{w0} - H_{\theta 1}\ U_{w0}\right) = \eta\left(\nabla^2 - \frac{1}{r^2\ \sin^2\theta}\right)\ H_{w1} \tag{3.10}$$

$$R_e\ P_r\left[U_{r1}\ \frac{\partial\theta_0}{\partial r} + \frac{U\theta_r}{r}\ \frac{\partial\theta_0}{\partial r}\right] = \left(\nabla^2 - N\right)\theta_1 \tag{3.11}$$

With the corresponding boundary conditions 0 (1)

$$\begin{aligned} &H_{w0} = U_{w0} = \sin\theta;\ \theta_0 = \theta^{(0)}\ on\ r = 1 \\ &\theta_0 = 0,\ on\ r = 1,\ \theta_0 = m\ on\ r' = R \\ &U_{w0} = \Omega\ \sin\theta;\ H_{w0} = \gamma\ \sin\theta\ on\ r = R \end{aligned} \tag{3.12}$$

0(R)

$U_r = 0 = U_\theta = H_r = H_\theta$ on $r = 1$

4.0 Analytical solution of leading order problem

We notice equation (3.4c) can be written as

$$\frac{d^2\theta_0}{dr^2} + \frac{2}{r}\ \frac{d\theta_0}{dr} - N\theta_0 = 0 \tag{4.1}$$

subject to $\theta_0 = 0$ on r = 1 and θ_0 = m on r = 1

Solving (33) via series method solution we have

$$\begin{aligned} \theta_0 &= \frac{1}{r}\left[\sum_{n=0}^{\infty} a_2\ r^{2n} + \sum_{n=0}^{\infty} a_{2n+1}\ r2^{n+1}\right] \\ \theta_0 &= \frac{1}{r}\left[\sum_{n=0}^{\infty}\frac{N^n r^{2n} a_0}{(2n)!} + \sum_{n=0}^{\infty}\frac{N^n a_1 r^{2n+1}}{(2n+1)!}\right] \\ \theta_0 &= \frac{1}{r}\left[A\cosh\ N^{\frac{1}{2}} r + B\sinh N^{\frac{1}{2}} r\right] \end{aligned} \tag{4.2}$$

where $A_0 = a_0$ and $B = \frac{a_1}{N^{\frac{1}{2}}}$. Applying the boundary conditions we have

$$\theta_0 = \frac{R_m}{r}\frac{\left(Sinh\, N^{\frac{1}{2}} r\ Cosh N^{\frac{1}{2}} - Sinh N^{\frac{1}{2}}\ \cosh N^{\frac{1}{2}} r\right)}{Sinh\, N^{\frac{1}{2}} r\ Cosh N^{\frac{1}{2}} - Sinh N^{\frac{1}{2}}\ \cosh N^{\frac{1}{2}} r} \tag{4.3}$$

which when written as Bessel function becomes

$$\theta_0 = m\ \frac{J_0\left(iN^{\frac{1}{2}} r\right) Y_0\left(iN^{\frac{1}{2}} r\right) - J_0\left(iN^{\frac{1}{2}} r\right) Y_0\left(iN^{\frac{1}{2}} r\right)}{J_0\left(iN^{\frac{1}{2}} R\right) Y_0\left(iN^{\frac{1}{2}}\right) - J\left(iN^{\frac{1}{2}}\right) Y_0\left(iN^{\frac{1}{2}} R\right)} \tag{4.4}$$

where J_n (r) is Bessel function of first kind and Y_n (r) is a Bessel function of second kind or order n.

Now to solve equation (3.4a) we assume this form of solution

$$U_{w0} = h_0(r)\sin\theta \tag{4.5}$$

On substituting (4.5) into (3.4a) and solve via series solution we have

$$\frac{d^2 h_0}{dr^2} + \frac{2}{r}\frac{dh_0}{dr} = 0 \qquad (4.6)$$

which gives

$$h_0 = A_0 r + B_0 r2 \qquad (4.7)$$

where A_0 and B_0 are arbitrary constant. On applying the bounding conditions $h_0(r) = 1$ when $r = 1$ and $h_0(r) = \Omega$ when $r = R$ we have

$$h_0(r) = Mr + N/r^2 \qquad (4.8)$$

where

$$M = \frac{\Omega R^2 - 1}{R^3 - 1} \quad and \quad N = \frac{R^2(R - \Omega)}{R^3 - 1}$$

Equation (3.4b) is similar to (3.4a) thus

$$K_0^{(r)} = \Pr + Q/r^2 \qquad (4.9)$$

where

$$P = \frac{\gamma R^2 - 1}{R^3 - 1} \quad and \quad Q = \frac{R^2(R - \gamma)}{R^3 - 1}$$

and γ is an arbitrary constant

5.0 Solution of the Higher Order problem

Solving (3.5) to (3.11) we assume the following

$$U_{r1} = f1(r)\cos\theta; \quad U_{01} = g1(r)\sin\theta$$
$$H_{r1} = N1(r)\cos\theta; \quad H_{01} = M1(r)\sin\theta$$
$$U_{w1} = h1(r)\cos\theta\sin\theta; \quad H_{w1} = K1(r)\sin\theta\cos\theta \qquad (5.1)$$
$$\theta_1 = \Gamma_1(r)\cos\theta$$

Substituting (5.1) into (3.5) to (3.11) we have

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 f_1(r)\right) + \frac{2g_1(r)}{r} = 0 \qquad (5.2)$$

$$\operatorname{Re}\frac{d\theta_0}{dr} = \left[\frac{d}{dr^2} + \frac{2}{r} - \frac{d}{dr} - \frac{2}{r^2}\right] \qquad (5.3)$$

$$\left[\frac{1}{r}\frac{d}{dr}\left(rg_1(r) + H_1(r)\right) - \frac{2}{r^2}\left(\frac{rd^2 g_1(r)}{dr} + \frac{2dg_1(r)}{dr} + \frac{df_1(r)}{dr}\right)\right]$$

$$\operatorname{Re}\left[\frac{1}{r}\frac{d}{dr}(rh_0)f_1 + \frac{2g_1h_0}{r} - K\left(\frac{1}{r}\frac{d}{dr}(rK_0)n_1 + \frac{2M_1K_0}{r}\right)\right] \qquad (5.4)$$

$$= \left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{6}{r^2}\right)h_1(r)$$

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + \frac{4}{r^2}\right)n_1(r) - \frac{2m_1(r)}{r} - \frac{2m_1(r)}{r^2} = 0 \qquad (5.5)$$

$$\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{4}{r^2}\right)n_1(r) - \frac{2n_1(r)}{r^2} - \frac{2n_1(r)}{r^2} = 0 \qquad (5.6)$$

$$\frac{1}{r}\frac{d}{dr}\{rk_0(r)\ f_1(r) - \frac{1}{r}\frac{d}{dr}(rh_0(r))n_1(r) + \frac{2g_1(r)k_0(r)}{r} - \frac{2m_1(r)k_0(r)}{r}\} = \Omega\left(\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{2}{r^2} - N\right)\Gamma_1 \qquad (5.7)$$

Equation (5.2) to (5.7) are subjected to the following boundary condition

$$\begin{aligned} &f_1(1) = g_1(1) = 0; \qquad n_1(1) = m_1(1) = 0 \\ &\Gamma_1(1) = h_1(1) = k_1 = 0 \\ &f_1(R) = g_1(R) = \Gamma_1(R) = h_1(R) = k_1(R) = 0 \\ &n_1(R) = m_1(R) = 1 \end{aligned} \qquad (5.8)$$

Equations (5.1) to (5.7) subject to (5.8) are solved numerically using a scheme for solving ordinary differential equation in particular Runge Kutta method is adopted here and these equations are reduced to first order O.D.E. from second order.

6.0 Discussion

In the flow of any high temperature gas, the temperature distribution plays a significant role. However, as a result of free convention in the limit of flow, the polar velocities are induced.

In the numerical analysis carried out the Prandtl number (Pr) is taken to be 0.71, M = 0.1. Tables 1 to 3 show respectively the velocity distribution without magnetic field effect, velocity distribution with the introduction of magnetic field and temperature distributions.

It was observed from Table 1 that when the radiation parameter N is increased, it causes in general an increase in the velocity. Obviously an increase in the Reynolds number causes an increase in the velocity. On the other hand an increase in the distance between the concentric spheres causes a decrease in the velocity distribution

From Table 2 it is observed that the introduction of magnetic field causes a decrease in the velocity varying all the parameter like radiation parameter, Reynolds number and distances between the spheres as done in Table 1

Table 3, concerns the temperature distribution. For the maximum temperature distribution, $\theta = \theta_0 + Re\Gamma_1$ was used.

From the table, it is observed that an increase in the radiation parameter N causes an increase in the temperature distribution. In same way an increase in Reynolds number (Re) causes a sharp increase in the temperature. But an increase in the distance between the concentric spheres causes a decrease in the temperature distribution.

7.0 Conclusion

The steady state magnetohydrodynamic flow of a radiating gas between concentric spheres was studied with the view of ascertaining the influence of magnetic field on the velocity and temperature distribution. The temperature of spheres is assumed large to allow for radiation while the difference in the temperatures is assumed small. The steady flow of the radiating gas between the concentric spheres is discussed under the optically thin gas limit. Also, all the physical variables, unlike what we have in [14] are assumed constant. Furthermore, the gas flow is not a two-phase flow as the one treated in [13].

The analysis carried out, in the present work, showed that when the magnetic field is introduced a decrease in velocity is obtained with an increase in the distance between the spheres. This is at variance with result obtained in [10] where it was observed that as the distance between the spheres is increased the velocity increases. The difference between the present result and that of [10] can be accounted for by the absence of magnetic field in [10]. Similarly, it is further observed, in this present work, that with an increase in either radiation parameter N or Reynolds number Re causes a decrease in velocity. Finally, although the magnetic field does not have direct effect on the temperature distribution but it causes a reduction in the flow of the gas.

3.0 Tables of Results

Table 1: Velocity Distribution without Magnetic Field Effect

	N =0.2,R=1.5,Re=10 U_w	N =0.2,R=1.5,Re=20 U_w	N =1.0,R=1.5,Re=10 U_w	N =0.2,R=2.0,Re=10 U_w
1	0.707106781	0.707106781	0.707106781	0.707106781
1.1	1.543194517	1.546823099	1.543194527	1.185647444
1.2	2.272835413	2.289311813	2.272836063	1.612055608
1.3	2.933865733	2.975204133	2.933876833	2.00817225
1.4	3.551278754	3.6290557754	3.551343254	2.388630547
1.5	4.142076765	4.262787	4.142314	2.76363838
1.6	4.717888607	4.8752161	4.71859607	3.140450428
1.7	5.286327705	5.448012205	5.287932705	3.524100259
1.8	5.851568534	5.937308034	5.854981534	3.917681266
1.9	6.415205682	6.471343782	6.421798502	4.322839737

Table 2: Velocity Distribution with Magnetic Field Effect

	N =0.2,R=1.5,Re=10 U_w	N =0.2,R=1.5,Re=20 U_w	N =1.0,R=1.5,Re=10 U_w	N =0.2,R=2.0,Re=10 U_w
1	0.707106781	0.707106781	0.707106781	0.707106781
1.1	1.541003087	1.542440207	1.54100308	1.183455999
1.2	2.26400521	2.271651413	2.264005913	1.1610917708
1.3	2.913590883	2.934654433	2.913601983	2.9878974
1.4	3.514400654	3.555301154	3.514465254	2.351752397
1.5	4.08312	4.144875	4.0833587	2.70468288
1.6	4.631114107	4.701668107	4.631785607	3.053675978
1.7	5.165774205	5.206904205	5.167378705	3.438052759
1.8	5.691085934	5.716343034	5.694499034	3.757198766
1.9	6.221347847	6.369757782	6.2279400782	4.128981828

Table 3: Temperature Distribution

	N =0.2,R=1.5,Re=10 θ	N =0.2,R=1.5,Re=20 θ	N =1.0,R=1.5,Re=10 θ	N =0.2,R=2.0,Re=10 θ
1	0.5	0.5	0.5	1
1.1	0.577557634	0.577829515	0.56857816	1.128059459
1.2	0.651632176	0.653199748	0.636157895	1.253612561
1.3	0.72326873	0.727167506	0.703379685	1.37767484
1.4	0.793237845	0.800540897	0.765823123	1.501013675
1.5	0.862125214	0.873960489	0.83832603	1.624228089
1.6	0.930798301	0.947951566	0.907887148	1.747788043
1.7	0.999380073	1.022958593	0.978219093	1.872066171
1.8	1.066443866	1.099368601	1.050152545	1.997351176
1.9	1.134787768	1.177527259	1.12395958	1.123851258

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