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Viscous dissipation effects on the flow of a radiating gas between concentric elliptic cylinders

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Abstract

The solution of a boundary layer flow problem often neglects the effects of viscous dissipation. However, the present treatment incorporates these effects with a view to assessing their global contributions to velocity and temperature distributions in the flow field. Hence, fluid motion induced between two differentially heated concentric elliptic cylinders is investigated under transient condition and significant viscous dissipation. When the temperatures of the cylinder are large enough for radiative heat transfer to be significant. The solution approach is via an explicit finite difference algorithm on a PC 1512 micro-computer. The numerical results obtained for the two cases show that the velocity and the temperature of the fluid are increased as a result of increase in thermal internal energy of the fluid caused by viscous dissipation.

1.0 Introduction

The problem of radiative transfer is of importance in temperature phenomena prevalent in hypersonic flight, liquid metal fast breeder reactors, re-entry, problems, just to mention a few. Little attention has been devoted to the effects of viscous dissipation in flow problems as most flow situations considered do not warrant the retention of dissipation term in the energy equation.

However, the influence of viscous dissipation on momentum and energy transport may be quite significant when the stream velocity of the fluid flow is very high. In particular, viscous dissipative effects play an important role in natural convention flow fields of extreme size or extremely low temperature or in high gravity which are various situations prevalent in physiology and engineering. Notable workers in this area of investigations include Y. Joshi and B. Gebhard [4]. D.L. Turcotte, A.T. Hsul, K.E. Torrance and C. Schubert [5] to mention a few.

The present work therefore studies the influence of viscous dissipation on laminar convention flow of a radiating gas between two vertical concentric elliptic cylinders.

Two cases considered in this work are:

(i) Optically thin and
(ii) Thick gas limits

A finite difference algorithm is developed for the cases since the resultant equations are not amendable to analytic treatment.

2.0 Mathematical Formulation/Problem Formulation

In this section, we consider unsteady, incompressible, induced flow between two differentially heated concentric elliptic cylinders of infinite. The semi-minor axis of the inner cylinder is denoted by

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the length L_0 (say), while the semi-major axis or the same cylinder is denoted by the length L_0'. The semi-minor axis and the semi-major axis of the outer cylinder are denoted by the length L_1 and L_1' respectively. At time $t' < 0$, the cylinders are maintained at a fixed temperature T0 (say) for equilibrium conditions to prevail. At time $t' = 0$, the outer cylinder is raised to a temperature $T_1 = T_0 + \gamma$. This analysis assumes that T_1 and T_0 are sufficiently large enough for radiative heat transfer to be significant. If we consider the asymptotic flow (i.e flow parallel and uniform at far distance from the origin), then for the axisymetric problem the velocity and radiative flux components in the elliptic cylindrical coordinates (r', v', w') are given by $(0, 0, V_w')$ *and* $(q_{r'}, 0, 0)$ respectively.

Thus, if we denote the temperature of the fluid by T, the scale factor h_r the fluid pressure by p, the thermal conductivity by k, the gravitational acceleration by g, the heat capacity by C_p, the fluid density by ρ, the fluid viscously by a and we further assume that the only force acting is the body force, the equations of momentum and energy under the usual Boussinesq approximation can be written as:

$$\rho_0 \frac{\partial V_w}{\partial t} = \frac{\mu}{(hr)^2} \frac{\partial^2 v_w}{\partial r^2} - \beta\rho_0 g(T - T_0) \tag{2.1}$$

and

$$\rho_0 C_p \frac{\partial T}{\partial t} = \frac{k}{(hr)^2} \frac{\partial^2 T}{\partial r^2} + \frac{1}{hr^2}\frac{\partial}{\partial r}(q_r hr) + 4\mu\left(\frac{1}{h_r} \frac{\partial v_w}{\partial r'}\right) \tag{2.2}$$

The subscript 0 refers to the conditions in the static fluid at time $t' < 0$. When γ is small, the radiative flux term in (2.1) may be replaced by the optically thin gas approximation as given by Cogley, Vicenti and Gilles [8].. That is:

$$\frac{1}{(h'r)^2} \frac{\partial}{\partial r'} (q_r' h_r') = 4(T - T_w) \int_0^x \left(a\lambda \frac{\partial B\lambda}{\partial T}\right)_w d\lambda \tag{2.3}$$

where the subscript w denotes condition at a wall $\lambda\beta$ *and* a denote frequency, plank's function and absorption coefficient respectively.

When γ is arbitrary, we adopt the Rosseland differential approximation for optically thick grey gas given by:

$$q_r' = \frac{16\delta}{3\alpha} T^3 \frac{\partial T}{\partial r}, \tag{2.4}$$ (1.4)

where, δ is the Stephan –Boltzman constant and a is the absorption coefficient.

To expedite analysis, we now introduce the following non-dimensional qualities.

$$r = \frac{r'}{r_0};\ t = \frac{t_w' v}{r_0^2};\ r_w = \frac{v_w' r_0}{r};\ \theta = \frac{T - T_0}{T_1 - T_0}\ \textit{Optically thin}$$

$$T/T_0\ \textit{optically thick}$$

$$r = \mu/\rho_0;\ pr = \frac{\mu c C_p}{k};\ qr = \frac{q_r' h_r'}{KT}$$

$$\frac{\beta g r_0^3}{r^2}(T_1 - T_0)\ \frac{4r_0^2}{k}\int_0^z \left(a\lambda \frac{\hat{c}\beta\lambda}{\hat{c}T}\right) d\lambda\ \textit{Optically thin} \tag{2.5}$$

$$Gr = \frac{\beta g r_0^3 T_0}{r^2};\quad N = \frac{16\delta T^3}{3ak}\ \textit{Optically thick}$$

Here, is the kinematic coefficient of viscousity, *Gr* is the free convection parameter or Grashor number, *Pr* is the Prandtl number, N is the radiation parameter and the subscript 0 is *N* corresponds to condition at the inner cylinder. Thus, making use of (2.5) in (2.1), (2.2) and coupled with the introduction of $h_r' = a \sinh r$ we obtain:

$$\frac{\delta V_w}{\delta t} = \frac{1}{a^2 \sin h^2 r} \frac{\delta^2 V_w}{\delta r^2} \mp Gr \begin{cases} \theta & Optically\, thin \\ \theta - 1 & Optically\ thick \end{cases} \quad (2.6)$$

and similarly, we obtain:

$$\Pr \frac{\partial \theta}{\partial t} = \frac{1}{a^2 \sin h^2 r} \frac{\partial \theta}{\partial r^2} \mp \begin{cases} N\theta \\ \frac{N}{a \sin h^2 r} \frac{\partial}{\partial r}(a \sin h\, r\, \theta^3) \end{cases}$$

$$+ \frac{4\mu v^2}{ka^2 r_0^2 \sin h^2 r} \left(\frac{\partial V_w}{\partial r} \right) = \begin{cases} \left(\frac{1}{T_1 - T_0} \right) & Optically\ thin \quad (2.7) \\ \frac{1}{T_0} & Optically\ thick \quad (2.7^*) \end{cases}$$

subject to the condition

(i) Optically thin

$$t = 0 : \theta(r,t) = V_w \ (r, t) = on \ r = r_0$$

$$t > 0 : \theta(r,t) = \theta; \ on\, r = r_0$$

$$\theta(r, t) = 1; \ on \ r = r_1$$

$$V_w \ (r, t) = 0; \ on \ r = r_0 \quad (2.8)$$

$$V_w \ (r,t) = 0; \ on \ r = r_1$$

(ii) Optically thick

$$\theta \ (r, 0) = \theta \ (r_0, t) = 1 \quad (2.9)$$

$$\theta(r, t) = \theta_1$$

$$V_w \ (r,0) = V_w \ (r_0,t) = V_w(r,t) = 0$$

3.0 Solution Techniques Algorithm

In this section, we consider a forward finite difference technique in time and a central difference in space, also denoting time by superscript j and position by subscript i, then provided the higher order derivatives are not considerably large. In order to solve equations (2.6) and (2.7), a reasonable approximation of the form:

$$\theta^{j-1} = \frac{\Delta t}{a^2 \Pr(\Delta r)^2 \sinh^2 r_i (\theta_{i+1}^j + \theta_{i-1}^j}$$

$$+ \left(1 - \frac{N\Delta t}{\Pr} - \frac{2\Delta t}{a^2 \Pr(\Delta r)^2 \sinh^2 r_i}\right) \theta_i^j \quad (3.1)$$

$$+ \frac{\Delta t \mu v^2}{ka^2 \Pr r_0^2 \ (T - T_0) \ (\Delta r)^2 \sinh^2 r_i} (V_{w,i+1}^j - v_{w,i-1}^j)$$

and $V_{w,i}^{j-1} = \frac{(\Delta t)}{a^2 (\Delta r)^2 \sinh^2 r^l} \left(V_{w,i+1}^j - V_{w,i=1}^j\right).$

$$- \left(1 - \frac{(\Delta t)}{a^2(\Delta r)^2 \sinh^2 r^i} V_{w,i+1}^j - \Delta t Gr \theta_i^j \right) \quad (3.2)$$

are used for optically thin problem with the following conditions:

θ(i , 0) = θ(i_0, j)

θ(i , j) = I

$$v_w = v_w(i_1, j) = 0 \quad (3.3)$$

Also, for an optically thick problem, a reasonable approximation to solve (1.6*) and (1.7*) simultaneously are of the form:

$$\theta^{j+1} = \frac{\Delta t}{a^2 \Pr(\Delta r)^2 \sinh^2 r} \left(\theta_{i+1}^j + \theta_{i-1}^j\right) - 1 - \frac{2\Delta t}{a^2 \Pr(\Delta r)^2 \sinh^2 r_i^j}$$

$$= \frac{N\Delta t}{a^2 \Pr(\Delta r)^2 \sinh^2 r_i^4} \left(r_{i+1}^j - r_{j-1}^j - 2r_i^j\right) = \frac{N\Delta t}{a^2 \Pr(\Delta r)^2 \sinh^2 r_i} \left(\rho_{i=1}^j - \rho\right) \quad (3.4)$$

$$= \frac{\Delta t}{a^2 KTr^2 \Pr(\Delta r)^2 \sinh^2 r_i} \left(V_{j-1} - V_{j-1}\right)^2$$

and $$V_{w,j}^{j-1} = \frac{\Delta t}{a^2 (\Delta r)^2 \sinh^2 r_i} \left(V_{w,j-1}^j - V_{w,j-1}^j\right)^2$$

$$1 - \frac{2\Delta t}{a^2 (\Delta r)^2 \sinh^2 r_1} V_{w,i}^j - \Delta t Gr\left(\theta_i^j - 1\right) \quad (3.5))$$

are used for optically thick problem with the following conditions

$$\theta(i, 0) = \theta(i_0, j) = 1$$
$$\theta(i_1, j) = \theta_1 \quad (3.6)$$
$$V_w(i, 0) = V_w(i_0, j) = V_w(i_1, j) = 0$$

where $N^4 = \dfrac{N\theta_1^4}{4}$ and θ_t is an arbitrary constant temperature not equal to zero and

$$\Gamma' = a \sinh' r \frac{\theta^4}{\theta_1^4}; \ \rho' = a \cosh' r \frac{\theta^4}{\theta_1^4}$$

3.0 Numerical Result 1

In this section, we give the results obtained using the solution techniques algorithms for solving optical thinness problem with the following constants.

(i)	k = 0.0104	(ii)	Pr = 0.71	(iii)	$T_1 = 227$
(iv)	$T_0 = 225$	(v)	a = 1.0	(iv)	$r_0 = 1.0$
(vii)	N = 0.5	(viii)	Gr = 5	(ix)	t = 0.0056

Temperature and Velocity Distribution for Optical Thinness Problem

Position(r)	**Temperature(θ)**	**Velocity(V_w)**
1.0	0.0	0
1.1	0.130719644	2.98847×10^{-3}
1.2	0.225455135	5.76209×10^{-3}
1.3	0.319637038	8.52150×10^{-2}
1.4	0.415009183	1.12880×10^{-2}
1.5	0.511311277	1.40607×10^{-2}
1.6	0.608315154	1.68381×10^{-2}
1.7	0.705848915	1.96188×10^{-2}
1.8	0.804689542	2.23960×10^{-2}
1.9	0.982777945	2.4984×10^{-2}
2.0	1.0	0

Position(r)	**Temperature(θ)**	**Velocity(V_w)**
1.0	0	0
1.1	0.136473938	3.02378E - 03
1.2	0.230101929	5.79285E - 03
1.3	0.323314628	8.54596E - 03
1.4	0.417946324	1.13075E - 02

1.5	0.513668338	1.40763E - 02
1.6	0.610213564	1.68507E - 02
1.7	0.707386512	1.96289E - 02
1.8	0.806235998	2.24064E - 02
1.9	0.998526143	2.50885E - 02
2.0	1	0

Table: 2 μ = 7.50, v = 5.00

4.1 Numerical Result 2:

Here, we present the results obtained when our solution techniques algorithm is employed for solving optically thick problem with the following constants.

(i)	$t = 0.00294$	(ii)	$k = 0.0104$	(iii)	$P_1 = 0.71$
(iv)	$T_1 = 227$	(v)	$T_6 = 225$ (iv)	(iv)	$a = 1.0$
(vii)	$\tau_0 = 1.0$	(viii)	$N = 0.5$ (ix)	(ix)	$Gr = 5$

4.2 Temperature and Velocity Distribution for Optical Thinness Problem

Position(r)	**Temperature(θ)**	**Velocity(V_w)**
1.0	1	0
1.1	8.01138725	2.40193 E - 02
1.2	12.4253219	3.23450 E - 02
1.3	16.1758781	3.43331 E - 02
1.4	17.3509248	3.21236 E - 02
1.5	12.3837787	2.77051 E - 02
1.6	6.13788653	2.40230 E - 02
1.7	3.915149	2.18423 E - 02
1.8	3.18813422	2.02551 E - 02
1.9	2.63733663	1.78150 E - 02
2.0	5	0

Table: 2 μ = 7.25: v= 4.75

Position(r)	Temperature(θ)	Velocity(V_w)
1.0	1	0
1.1	8.05604208	2.40732 E - 02
1.2	12.4859803	3.23786 E - 02
1.3	16.2579069	3.43560 E - 02
1.4	17.4352455	3.21359 E - 02
1.5	12.421119966	2.77085 E - 02
1.6	6.14218136	2.40230 E - 02
1.7	3.91540058	2.18424 E - 02
1.8	3.18852044	2.02558 E - 02
1.9	2.6391788	1.78205 E - 02
2.0	5	0

Table: 4 μ = 14.50, v = 9.50

5.0 Conclusion

Results at various dynamic and kinematic visciousities are presented in tables 1 to 4. The radiation parameter N and Grashof number Gr both assumed constant values 0.5 and 5 respectively in all cases. It is obviously clear that a slight increase in viscous dissipation (i.e dynamic viscousity and kinematic viscousity) leads to corresponding increase in both temperature and velocity of the fluid in the case of optical thinness problem. While an appreciable increase in viscous dissipation leads to a slight increase in both temperature and velocity of the fluid, for optical thickness problem.

From the result of the present analysis, it is evidently clear that viscous dissipation causes local temperature and velocities to increase throughout the flow region. Emphatically speaking viscous dissipation causes the thermal internal energy of the fluid to increase throughout the flow region, and thus acts essentially as a heat generating source. In fact, viscous dissipation inhibits heat transfer from the hot wall. In all the numerical discussions in this paper, the prandtl number Pr. is taken as 0.71, which corresponds to that of air. The other quantities N. Gr. K . . . etc. are chosen to simulate physically realistic situations.

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