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# Existence of a secondary flow for a temperature dependent viscous couette flow.

[1]***S. O. Adesanya,*** [3]***O.J Fenuga ,*** [4]***O. O. Otolorin.***
**Department of Mathematical Sciences ,**
**Olabisi Onabanjo University, Ago Iwoye, Nigeria.**
e-**mail: adesanyaolumide@yahoo.com**
[2]***R .O. Rufai***
**Department of Mathematical Sciences**
**Lagos State University, Ojo, Nigeria.**
[5] ***R. O Ayeni.***
**Department of Pure & Applied Mathematics,**
**Ladoke Akintola University of Technology, Ogbomosho, Nigeria.**

**Abstract**

**We model a viscous fluid flowing between parallel plates. The viscosity depends on temperature. We investigate the properties of the velocity and we show that the temperature and velocity fields have two solutions. The existence of two velocity solutions is new. This means that there exist secondary flows**

## 1.0 Introduction

The safety of working environment at explosive production plants stimulated a lot of research work devoted to thermal ignition and explosion {Merzhanor et al (1996), Frank – Kamenestkii (1987), Merzhanov and Duborvistskii (1996), Barzykn and Abramov (1981), Zel'dorich et al (1985) and Barzykn (1993)}. Amosov et al (1972, 1976) considered different models of ignition caused by the heat of mechanical friction. In the production plant Gainutdinov (2002) established that a thermal explosion or ignition is initiated, as a rule, by the frictional heat, and solutions of a non stationary heat-conduction equation subject to boundary conditions of the second kind for a semi-bounded body.

Mareschal et al (1998,1989), Koplik and Banavar (1995) showed that the Boltman equation are generally accurate at the macroscopic level expect under some extreme conditions. Since the Navier-Stokes equations are derived under the assumptions of local equilibrium and small gradient and flows with small Knudsen number (Yihao Zheng el al (2002).

The object of this paper is to consider the velocity of the flow when viscosity depends on temperature. In particular we investigate the couette flow.

## 2.0 Mathematical model

Momentum equation
$$\rho\left(\frac{\partial u}{\partial t}+V_0\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\ \mu\left(\frac{\partial u}{\partial y}\right)-\frac{\partial p}{\partial x} \quad (2.1)$$

Energy equation
$$\rho Cp\left(\frac{\partial T}{\partial t}+V_0\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+Qe^{E/RT} \quad (2.2)$$

with boundary and initial conditions
$$u(0,t)=0 \quad (2.3)$$
$$u(h,t)=U \quad (2.4)$$
$$u(y,0)=0 \quad (2.5)$$

$$T(h,t) = T \tag{2.6}$$

$$T(h,t) = T_1, T_1 \succ T_0 \tag{2.7}$$

$$T(y,0) = T_0 \tag{2.8}$$

where

u = velocity

$\rho$ = density

μ = viscosity

p = pressure

$v_0$ = constant vertical velocity

$c_p$ = specific heat

*k* = thermal conductivity

*T* = temperature

*x* = co-ordinate in the direction of flow

*y* = coordinate across the flow

*E* = Activation energy

*R* = Universal gas constant

*Q* = heat per unit mass during reaction

We assume that $\mu = \mu_0 e^{\alpha(T-T0)}$. We now introduce the following dimensionless parameters

$$\phi = \frac{u}{v}, \ \theta = (T-T_0)\,E/RT_O, \ \bar{y} = \frac{y}{h}, \ \bar{t} = \frac{t}{t_0}$$

We then get the dimensionless equations (after dropping of "-")

$$\frac{\partial\phi}{\partial t} + a\frac{\partial\phi}{\partial y} = \frac{\partial}{\partial y}\left(b e^{\lambda\theta} \frac{\partial\phi}{\partial y}\right) - g \tag{2.9}$$

$$\frac{\partial\theta}{\partial t} + a\frac{\partial\theta}{\partial y} = d\frac{\partial^2\theta}{\partial y^2} + f e^{\frac{\theta}{1+\varepsilon\theta}}$$

with $\theta(-1) = \theta(1) = 0.$ (2.10)

where $a = \frac{v_0 t_0}{h}$, $b = \frac{\mu_0 t_0}{\rho h^2}$, $g = \frac{\left(\frac{\partial p}{\partial x}\right)t_0}{\rho v_0}$, $d = \frac{k t_0}{\rho c_p h^2}$, $F = \frac{Q t_0 E}{\rho c_p R T_0^2} e^{\frac{E}{RT_0}}$

We now proceed to solve the equation (2.9) and (2.10) under the boundary conditions

## 3.0 Method Of Solution (Steady case)

Let , $a = 0, \ b = 1, \ \lambda = 0.5, \ \varepsilon = 0, \ d = 1, \ g = 1, \ f = \delta$

$$\frac{d^2\theta}{dy^2} + \delta e^{\theta} = 0 \tag{3.1}$$

$$\phi(-1) = 0, \ \phi(1) = 1 \tag{3.2}$$

The solution of (3.1) is known from Buckmaster and Ludford (1982) to be

$$\theta = 2\log_e\left(e^{\theta m} echcy\right) \tag{3.3}$$

with

$$c^2 = \frac{1}{2} f e^{\theta m} \tag{3.4}$$

and

$$\sqrt{\frac{f}{2}} = e^{-\theta m} \cosh^{-1}\left(e^{\frac{\theta m}{2}}\right) \tag{3.5}$$

When 0 < f < 0.878, θ$m$ has two values, in particular when $f$ = 0.5, θ$m$ has two values θ$m$ = 0.312 and 2.876. We now have $e^{\frac{\theta}{2}} = e^{0.156}\sec h0.58y$ and $e^{\frac{\theta}{2}} = e^{1.438}\sec h2.11y$ (3.6)

to be the two solutions for temperature. We now find the solution of equation (3.2) numerically for coutte flow

$$\frac{d}{dy}\left(e^{\frac{\theta}{2}}\frac{d\phi}{dy}\right) - 1 = 0$$

with

$$\phi(-1) = 0,\ \phi(1) = 1 \qquad (3.7)$$

when $e^{\frac{\theta}{2}} = e^{0.156}\sec h0.58y$. The velocity equation becomes $\frac{d}{dy}\left(e^{0.156}\sec h0.58y\frac{d\phi}{dy}\right) - 1 = 0$

The finite difference scheme is $\phi_{i+1} = \phi_i + \Delta h\,((y_i - l)\,e^{-0.156}\cosh 0.58y_i)$ (3.8)

where l is constant of integration. In the second case when $e^{\frac{\theta}{2}} = e^{1.438}\sec h2.11y$, The numerical scheme is $\phi_{i+1} = \phi_i + \Delta h\,((y_i - z)\ \ e^{-1.438}\cosh 2.11y_i)$, where z is constant of integration .

## 4.0 Results

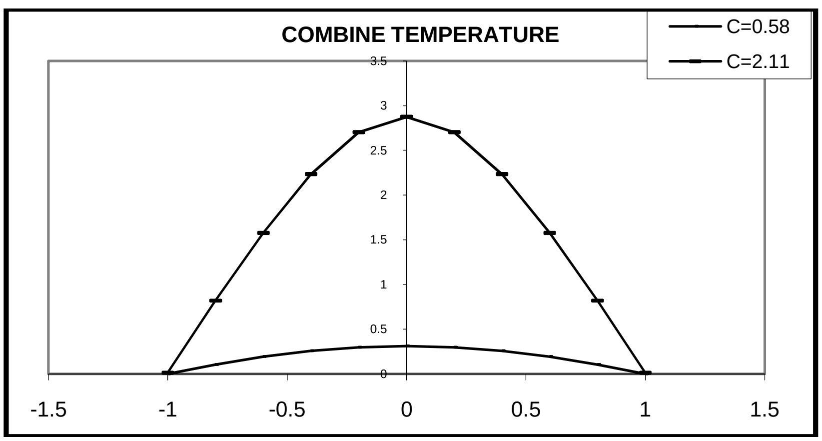


**Figure (a) Temperature profile.**

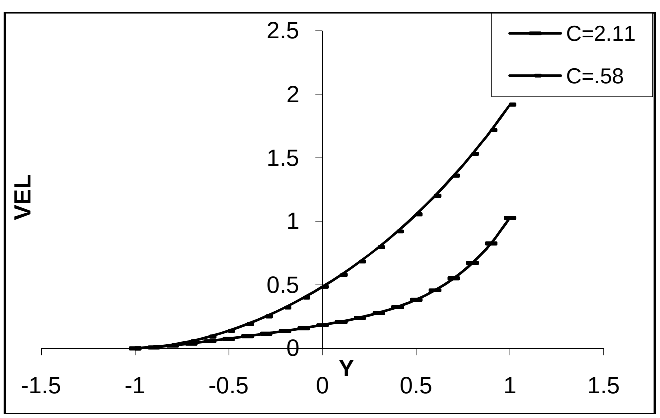


**Figure (b) Velocity profile.**

## 4.0 Discussion

The figure (a) shows that θ has two solutions and results is well known (Buckmaster and Ludford). The existence of two velocity solutions is just discovered here figure (b) .The first solution is stable but the second solution is not stable Physically, we interpret the existence of two solutions to mean that there exist secondary flows

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