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Impact of electric and magnetic fields in a resistant medium on the velocity of a particle subject to varying path angles

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Abstract

In this paper, we compare the impact of electric and magnetic fields in a resistant medium on the velocity of a particle subject to varying path angles by using numerical integration of finite difference method. The results show that the magnetic field has much impact on the velocity than the electric field.

1.0 Introduction

In a paper by Hayen [4], he subjected the particle on a projectile motion in a resistant medium to a uniform gravitational field. He used a drag force to act on the particle in the medium which is proportional to the square of the particle's speed. In another paper, Ayeni and Ayandokun [1], extended the model of Hayen to include an electric field with a general power n. They assumed that the initial motion is vertical and then studied the resulting unsteady problem. Recently, Fenuga and Ayeni [3], gave some remarks on the projectile motion of a particle in a resistant medium under the influence of a magnetic field. They showed that the velocity and path angle increase as magnetic field increases and vice versa.

In this paper, we shall compare the impact of electric and magnetic fields in a resistant medium on the velocity of a particle subject to varying path angles by using numerical integration of finite difference method. We shall also consider a special case of this comparison.

2.0 Mathematical formulation

Let the Cartesian coordinates be x, y, z. We shall assume that the velocity field $\underline{V} = (V_x, V_y, V_z)$ and its magnitude is $V = V(t)$. Also the magnetic field $\underline{B} = (B_x, B_y, B_z)$ and its magnitude $B = \beta =$ constant. Similarly, the elective field $\underline{E} = (E_x, E_y, E_z)$ and its magnitude is $E = \alpha =$ constant. In this work, the set of coupled non linear equations are:

$$\frac{dV}{dt} = -Sin\phi + \beta V^m - \alpha V^n,\ V(0) = 1 \tag{2.1}$$

$$V^m \frac{d\phi}{dt} = -Sin\phi,\ \phi(0) = \phi_0 \tag{2.2}$$

where $V(t)$ is the velocity in ms^{-1}, $\phi(t)$, is the local path angle in degrees, α, is the electric field, β is the imposed magnetic field, m and n are real constants with $m,\ n \geq 1$ and t (time) is an independent variable.

3.0 Method of solution

With $m = 1$ and $n = 2$, equations (2.1) and (2.2) are equivalent to

$$\frac{dV}{d\phi} = \frac{V\left(-Sin\phi + \beta V - \alpha V^2\right)}{-Cos\phi} \tag{3.1}$$

The equivalent finite difference scheme for (3.1) is

$$V_{i+1} = V_i + \frac{h\left[V_i\left(-Sin\phi_i + \beta V_i - \alpha V_i^2\right)\right]}{-Cos\phi_i} \tag{3.2}$$

Similarly, consider the special case when $m = n = 2$, equations (2.1) and (2.2) are equivalent to

$$\frac{dV}{d\phi} = \frac{V\left(-Sin\phi + (\beta - \alpha)V^2\right)}{-Cos\phi} \tag{3.3}$$

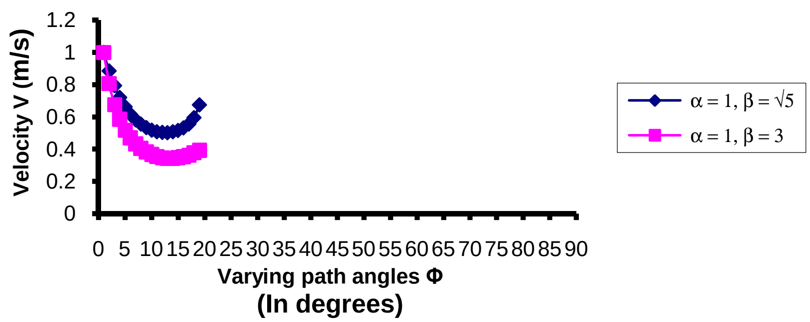
The equivalent finite difference scheme for (3.3) is

$$V_{i+1} = V_i + \frac{h\left[V_i\left(-Sin\phi_i + (\beta - \alpha)V_i^2\right)\right]}{-Cos\phi_i} \tag{3.4}$$

where $i = 0, 1, 2, 3, \ldots,$ $h = 0.1,$ $V_0 = 1$ with $\phi_0 = 0^o$, $\phi_1 = 5^o$, $\phi_2 = 10^o$, . . . , $\phi_{17} = 85^o$, $\phi_{18} = 89^o$

Then, the graphs of V are then plotted against ϕ for different α and β values as in figures 1 and 2 below.

Equation 3.2 gives the figure below



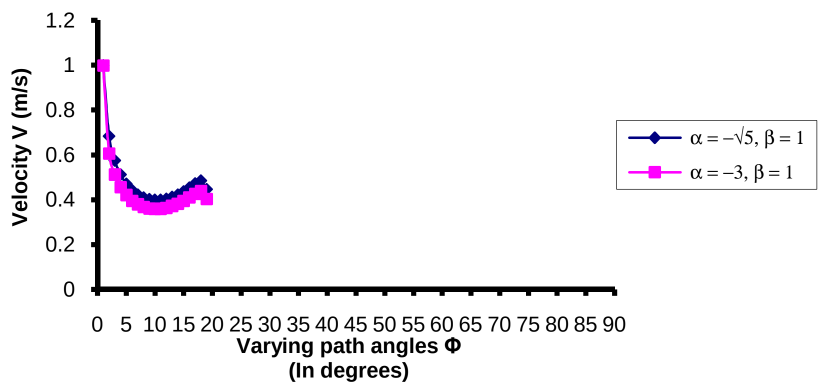
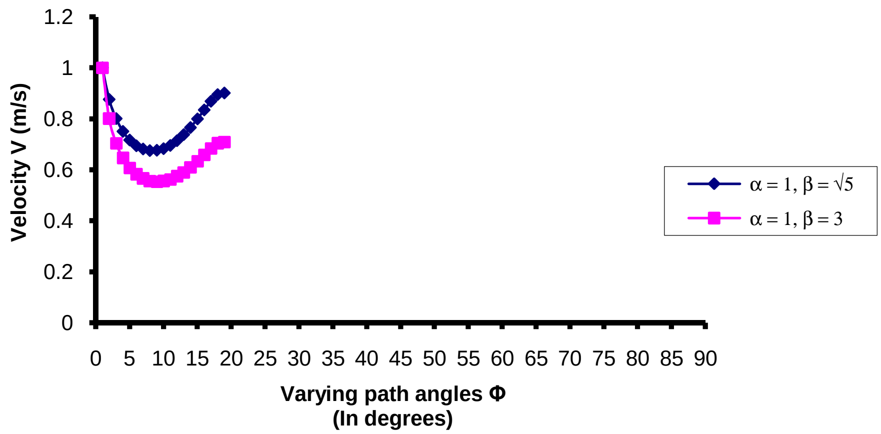


Figure 1: The graph of Velocity V against Path angle ϕ for equation (3.2)

Equation 3.4 gives the figure below



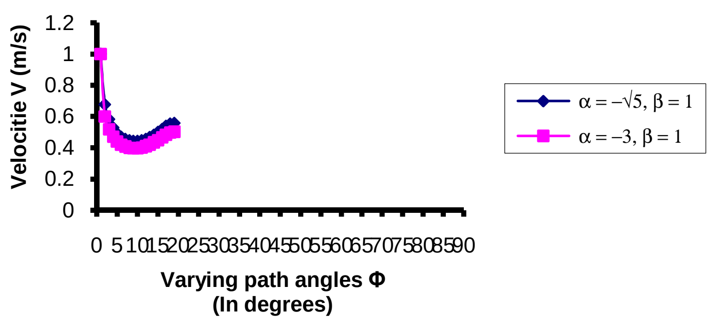


Figure 2: The graph of velocity V against path angle ϕ for equation (3.4)

4.0 Results and discussions

In the graphs of velocity V against the varying path angles ϕ with the two cases as in equations (3.2) and (3.4), it was confirmed as in [3] that the magnetic field β has much impact on the velocity than the electric field. This is because, the change in velocities as shown in figures 1 and 2 is more pronounced when there is a change in magnetic field compared to when there is a change in electric field. Moreover, if $\frac{dV}{d\phi} = 0$ in equations (3.1) and (3.3) respectively, then

$$V = 0 \text{ or } V = \frac{\beta \pm \sqrt{\beta^2 - 4\alpha}\sin\phi}{2\alpha} \tag{4.1}$$

and
$$V = 0 \text{ or } V = \pm\sqrt{\frac{\sin\phi}{\beta - \alpha}} = \frac{\pm\sqrt{(\beta-\alpha)\sin\phi}}{\beta - \alpha} \tag{4.2}$$

Hence, (4.1) and (4.2) are respectively true iff $\beta^2 \geq 4\alpha\sin\phi$ and $\beta > \alpha$ as shown in figures 1 and 2.

Generally, in (4.1) and (4.2) respectively, the velocity of the particle increases only when $\beta^2 > 4\alpha\sin\phi$ and $\beta > \alpha$, the particle cannot take off when $\beta^2 = 4\alpha\sin\phi$ and $\beta = \alpha$ and the electric field is an hindrance to the flight of the particle when $\beta^2 \leq 4\alpha\sin\phi$ and $\beta < \alpha$.

4.0 Conclusion

The equation $V = \sqrt{\dfrac{\sin\phi}{(\beta - \alpha)}}$ shows that motion is only possible if $\dfrac{\sin\phi}{\beta - \alpha} \geq 0$. This is possible if $\beta > \alpha$ and $0^0 \leq \phi \leq 180^0$. However, when $\beta < \alpha$ and $180^0 < \phi < 360^0$, motion is also possible.

This paper extends other paper by highlighting the ways of manipulating the velocity through adjustment of electric and magnetic fields and the direction of motion.

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