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On the steady state temperature profiles of biological tissues during microwave heating.

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Abstract

The Maxwell equations are solved together with the Pennes Bio-heat equation analytically. The procedure of solution is provoked by the solution to the Maxwell equation. The result revealed the effect of the model parameters such as: the thermal conductivity, blood perfusion coefficient, and the thickness of the tissues and the location of the effect of the electric field. Our result agrees with the results obtained by El-dabe et al (2003) the results are significant to medical experts and engineers.

Key words: Maxwell equation microwave heating, Pennes Bio-heat Biological tissue equation steady state. Author to which correspondence should be addressed

1.0 Introduction

Research works have continued to grow wide and broad in the use of heat to destroy or control the growth of cancer in biological tissues. In the literature various forms of heating and heat deposition methods have been advanced and practiced. Such heating modalities are magnetic heating, microwave heating, ultrasound heating and so on.

Heating with the aid of microwave has been found useful for smelting, sintering, drying and in fact it has it has a lot of application in joining mathematical and medical field in areas such as clinical cancer therapy. (See Hill and Pincombe (1992). El-dabe et al (2003)).

Some of the works done are the investigation of the effects of thermal properties and geometrical dimension in skin burn injuries by Jiang et al. (2002). Ng and Chua (2002) proposed a comparism of one and two-dimensional programmes for predicting the state of skin burn. Liu (2000) discussed the preliminary survey on the mechanism of the wave-like behaviour of heat transfer in living tissues. He introduced a new concept of multi-mode energy coupling, a phenomenon logical thermal wave mode by Bio-heat transfer. Merchant and Liu (2001) on the other hand considered the steady state of microwave heating of a finite one dimensional slab. They put into use the temperature dependence of the electrical conductivity and thermal absorptivity. These are taken to be of the Arrhenius law, while both the electrical permitivity and permeability were assumed constant. In a recent study Liu and Marchant (2002) went further to study the microwave heating of three dimensional block with transverse magnetic wave-guide in a long rectangular wave guide Adebile (1997) has investigated the temperature rise in tumor and surrounding normal tissues. Conditions for preferential treatment were given for physically reasonable situations. Possibility of hot spot and approach to hinder the occurrence were given. Very recently Adebile (2004) discussed the effect of variable thermal conductivity on temperature rise in biological tissues and the 'possibility of multiple solution is revealed. In a very interesting paper El-dabe et al (2003), studied the effect of microwave heating in the thermal state of biological tissues. They solved their one-dimensional multi-layer model using a numerical method to analyses the transient temperature profile.

The aim of the present work is to investigate the problem of El-dabe et al. (2003) analytically. We consider in the first instance a steady state situation. The transient situation is the subject of another work.

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The introduction is presented in the first section while the mathematical formulation is in section two. The method of solution is displayed in section three, the result and discussion is in section four and the conclusion is highlighted in section six.

Nomenclature

ρ	Density of tissue
c_p	Specific heat of tissue
t	time
c_b	Specific heat of blood
E	electric field
x	space coordinate
H	magnetic field
T	tissue temperature
T_a	artery temperature
T_b	blood temperature
T_c	core temperature
T_w	wall temperature
L	distance from skin surface to body core
m	positive integer number
ρ_b	density of the blood
ω_b	blood perfusion rate
k	thermal conductivity of tissue
Q	body heating coefficient
μ_e	magnetic permeability
ε	electric permittivity
σ	electrical conductivity
H_0	is the magnetic field in the free space upon the tissue
E_0	is the magnetic field in the free space upon the tissue
E_0	is the electric field in the free space upon the tissue
P_r	Prandtl number, $P_r = \frac{\mu c_p}{k}$
v	kinematics viscosity, $v = \frac{\mu}{\rho}$
μ	viscosity of the tissue

2.0 Mathematical formulation

The heat source arising from the microwave irradiation is proportional to the square of the modulus of the electric field intensity (Hill and Pincombe, 1992) . We therefore solve the Maxwell equation together with the Bio-heat equation. These equations are:

$$\frac{\partial H}{\partial x}+\varepsilon\frac{\partial E}{\partial t}+\alpha E=0 \quad (2.1)$$

$$\frac{\partial E}{\partial x}+\mu\frac{\partial H}{\partial t}=0 \quad (2.2)$$

$$\alpha c_p\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)-\omega_b p_b c_b\left(T-T_b\right)+Q(T)\left|E\right|^2 \quad (2.3)$$

with the initial and boundary conditions $T(x,0)=\frac{T_c}{L}x;\quad T(0,t)=0,\ T(L,t)=T_c$

$$E(x,0)=\frac{E_0}{L}x; \quad E(0,t)=0,\ E(L,t)=E_0,\ H(x,0)=\frac{H_0}{L}x; \quad H(0,\tau)=0, \quad H(L,t)=H_0 \qquad (2.4)$$

El-dabe et al (2003) solved the above mathematical model numerically for m=1, assuming that the body-heating coefficient has he form $Q(T)=T^m, m\geq 1$ (2.5)
as it has been advanced through experimental work of Marchant and Liu (2001) that the physical properties of material have power law dependence on temperature.

In this paper we solve the mathematical model in (2.4) analytically for the steady case when m = 1. The solution of the case when $m\geq 1$ will be for another paper. Using the following dimensionless variables:

$$T = \frac{tv}{L^2}\eta = \frac{x}{L},\ \theta = \frac{T}{T_b},\ C_1 = \frac{C_b}{C_p},\ \rho_1 = \frac{\rho_b}{\rho},\ \bar{E} = \frac{E}{E_0},\bar{H} = \frac{H}{H_0}, \lambda_1 = \frac{v \in E_0}{LH_0}$$

$$\lambda_2 = \frac{L\sigma E_0}{H_0}, \lambda_3 = \frac{\mu H_0 v}{LE_0}; \quad \lambda = \frac{L^2|E_0|^2}{v_p c_p} \qquad (2.6)$$

The dimensionless equation corresponding to equation (2.1)-(2.4) after dropping the bar;

$$\frac{\partial H}{\partial \eta}+\lambda_1\frac{\partial E}{\partial \tau}+\lambda_2 E=0 \qquad (2.7)$$

$$\frac{\partial E}{\partial \eta}+\lambda_3\frac{\partial H}{\partial \tau}=0 \qquad (2.8)$$

$$\frac{\partial \theta}{\partial \tau}=\frac{1}{p_r}\frac{\partial^2\theta}{\partial \eta^2}-\omega_1 p_1 c_1(\theta-1)+\lambda Q|E|^2 \qquad (2.9)$$

subject to the following dimensionless initial and boundary conditions

$$\theta(\eta.0)=\frac{T_c}{T_b}\eta, \theta(0,\tau)=0;\ \theta(1,\tau)=\frac{T_c}{T_b},\ E(\eta,0)=\eta',\ E(0,\tau)=0;\ \ E(1,\tau)=1$$

$$H(\eta,0)=\eta;\ H(0,\tau)=0,\ H(1,\tau)=1 \qquad (2.10)$$

3.0 Method of solution.

The equations for the steady state are:

$$\frac{\partial H}{\partial \eta}+\lambda_2 E=0 \qquad (3.1)$$

$$\frac{\partial E}{\partial \eta}=0 \qquad (3.2)$$

$$\Omega\frac{\partial^2\theta}{\partial \eta^2}-\alpha(\theta-1)+\lambda|E|^2T^m=0 \qquad (3.3)$$

With the boundary conditions:, $\theta(0)=0 \ \ ;\theta(1)=\Omega_1$

$$E(0)=0 \ \ ;E(1)=1 \qquad (3.4)$$

$H(0)=0 \ \ ;H(1)=1$. The solution to (3.1) and (3.2) using (3.4) $_{b,c}$ *are* :

$$E=H(\eta-a)=\begin{cases}0;\eta\prec a\\1,\eta\geq a\end{cases} \qquad (3.5)$$

$$H=\eta H(\eta-a)=\begin{cases}0\ ;\eta<a\\\eta;\eta\geq a\end{cases} \qquad (3.6)$$

The energy equation in (3.3) now becomes:

$$\Omega\frac{\partial^2\theta}{\partial \eta^2}-\alpha(\theta-1)+\lambda|H(\eta-a)|^2\theta=0 \qquad (3.7)$$

$$\theta(0) = 0\ ,\ \theta(1) = \Omega_1 = 0 \qquad (3.8)$$

where $\Omega = (P_r)^{-1}$, $\Omega_1 = T_c T_b^{-1}, \alpha = \omega_1 p_1 c_1$. $H(\eta - a)$ is the Heaviside function. We seek for solution to equation (3.7) and (3.8) by splitting the tissue volume into two regions (Region I and II) in view of the behavior of E(η) in (3.5). The relevant equation for Region I and II are:

$$\Omega_1 \frac{\partial^2 \theta_{11}}{\partial \eta^2} - \alpha(\theta_{11} - 1) = 0 \qquad (3.9)$$

$$\theta_{11}(0) = 0,\ \theta_{11}(a) \ = \ \Theta \qquad (3.10)$$

for Region I and

$$\Omega \frac{\partial^2 \theta_{12}}{\partial \eta^2} - \alpha(\theta_{12} - 1) + \lambda \theta_{12} = 0 \qquad (3.11)$$

$$\theta_{12}(a) = \ \Theta \ \ ; \theta_{12}(1) = \Omega_1 \qquad (3.12)$$

for Region II . The solution to equation (3.9) and (3.10) is:

$$\theta_{11}(\eta) = 1 - \cosh m_1 \eta + \frac{suhm_1\eta}{suhm_1 a}\{(\Theta - 1) + \cosh m_1 a\},\ 0 \le \eta \le a \qquad (3.13)$$

and the solution to (3.11) and (3.12) is

$$\theta_{12}(\eta) = A\cosh m_2\eta + B\, suh\, m_2\eta + \frac{\alpha}{\alpha - \lambda}; \quad a \le v \le 1 \qquad (3.14)$$

where $m_1 = \sqrt{\frac{\alpha}{\Omega}}$, $m_2 = \sqrt{\frac{(\alpha - \lambda)}{\Omega}}$

$$A = \frac{1}{suh(1-a)m_2}\ \left(\ \Theta\ \ suh\, m_2 - \Omega_1\, suh\, m_2\, a\right) - \left(\left\{\frac{\alpha}{\alpha - \lambda}\right\} suhm_2 a - \frac{\alpha}{(\alpha - \lambda)} suhm_2 a\right) \qquad (3.15)$$

$$B = \frac{1}{suh\, m_2 a}\ \left\{\Theta - A\cosh m_2\, a - \frac{\alpha}{(\alpha - \lambda)}\right\} \qquad (3.16)$$

4.0 Results and discussion

The analytical solution to a one-dimensional multi-layer model for predicting the temperature profiles in living tissues under going microwave heating in discussed.

The results are displayed in the graphs and interpretations in relation to tissues are given.

In figure 1, the temperature distribution for different tissue thickness is shown, the temperature profiles increases with increases in tissue thickness in the first region from the surface to the point before the electric field acts, in this region (i.e. a > 0.75) the temperature decreases as the tissue thickness increases.

Figure 2 illustrates the effect of variation of blood perfusion on temperature distribution. The temperature increases with increase in blood perfusion I region one (1) but decrease with increase in blood perfusion region two (2)

Figure 3 shows the relation between the temperature distribution and the different thermal conductivity of tissue. The temperature decreases with increases in the thermal conductivity either in region 1 or 2

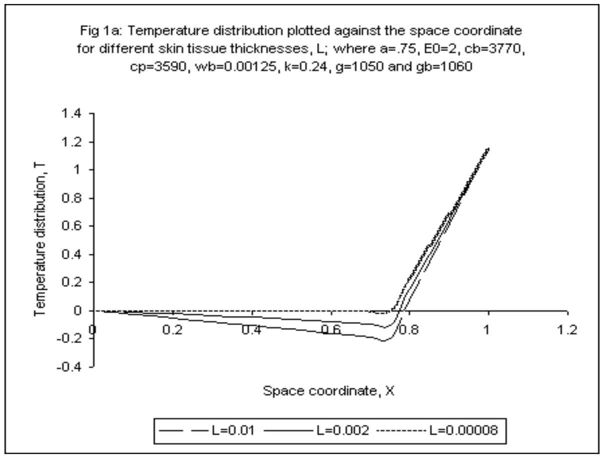

Fig 1a: Temperature distribution plotted against the space coordinate for different skin tissue thicknesses, L; where a=.75, E0=2, cb=3770, cp=3590, wb=0.00125, k=0.24, g=1050 and gb=1060

In figure 4, the temperature distribution of the tissue varies directly with the electric field in the free space, E_o.

Figure 5 reflect the variation of temperature distribution in tissue location as the location of electric field (a) affect changes. As a increases towards the core the temperature distribution decreases.

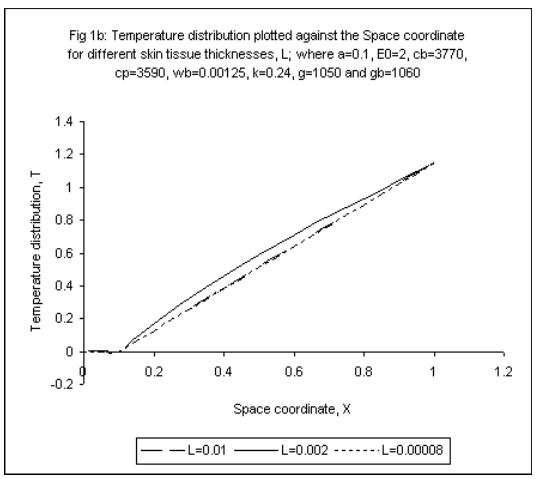

Fig 1b: Temperature distribution plotted against the Space coordinate for different skin tissue thicknesses, L; where a=0.1, E0=2, cb=3770, cp=3590, wb=0.00125, k=0.24, g=1050 and gb=1060

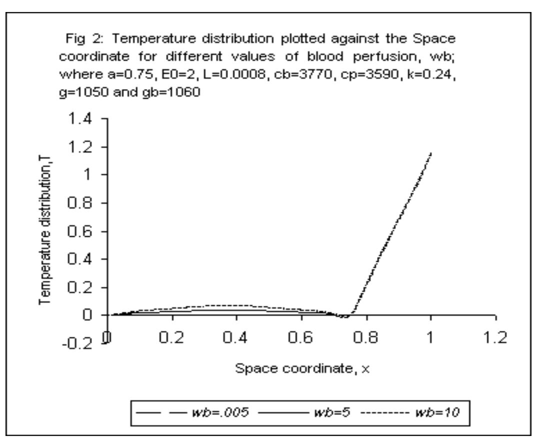

Fig 2: Temperature distribution plotted against the Space coordinate for different values of blood perfusion, wb; where a=0.75, E0=2, L=0.0008, cb=3770, cp=3590, k=0.24, g=1050 and gb=1060

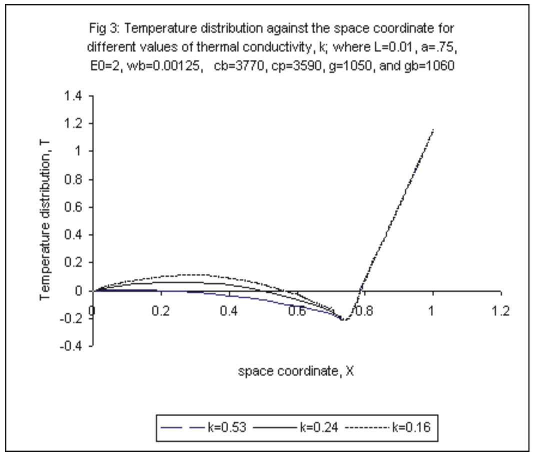

Fig 3: Temperature distribution against the space coordinate for different values of thermal conductivity, k; where L=0.01, a=.75, E0=2, wb=0.00125, cb=3770, cp=3590, g=1050, and gb=1060

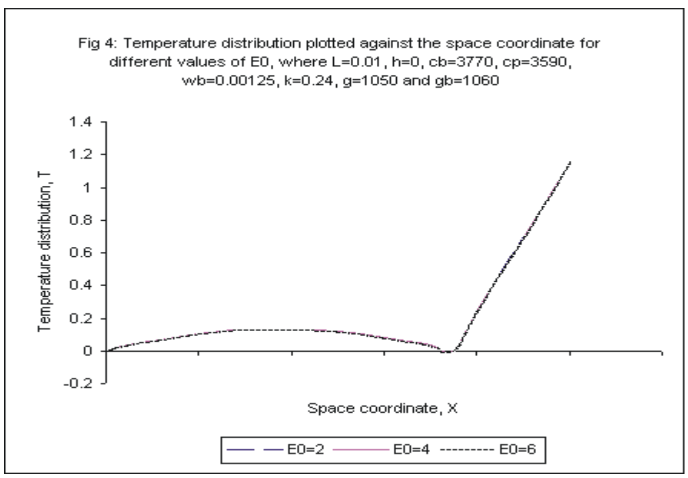

Fig 4: Temperature distribution plotted against the space coordinate for different values of E0, where L=0.01, h=0, cb=3770, cp=3590, wb=0.00125, k=0.24, g=1050 and gb=1060

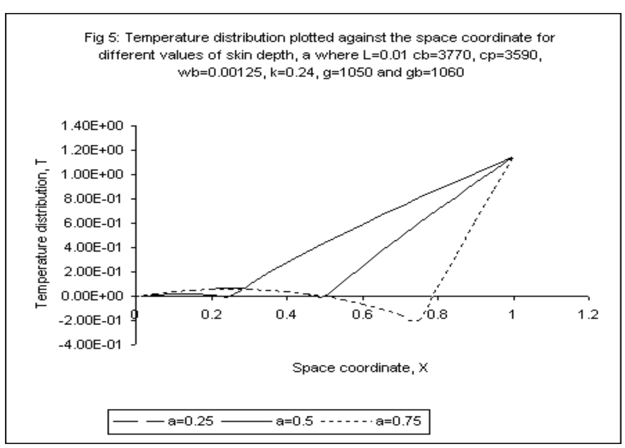

Fig 5: Temperature distribution plotted against the space coordinate for different values of skin depth, a where L=0.01 cb=3770, cp=3590, wb=0.00125, k=0.24, g=1050 and gb=1060

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