Journal of the Nigerian Association of Mathematical Physics
Volume 10 (November 2006), 197 - 202

Asymptotic solution on the dynamic buckling of a column stressed by a dynamically slowly varying load

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Abstract

This paper analysis the dynamic stability of a dynamically oscillatory system with slowly varying time dependent parameters. It utilizes the concept of multiple times scaling in an asymptotic evaluation of the dynamic buckling load of the imperfect elastic structure under investigation. Unlike most similar investigations to date , the time dependence is explicit in the formulation and this creates a situation of non-autonomous differential equation that accurately models the dynamic stability of the structure .The dynamic buckling load is obtained nontrivially and the results are found to generalize earlier results obtained for step loading situation. It is established that the results depend strongly on the first derivative of the load function evaluated at the initial time.

1.0 Introduction

This paper is concerned with asymptotic solution of a purely nonlinear oscillatory system where the structure involved is suddenly trapped by an explicitly time dependent loading history that is dynamically slowly varying over a natural period of vibration of the structure. Over the years, most of the investigations that have received attention have been those in which the loading histories prescribed do not prescribed the time variable explicitly. Such loading histories include step loading, impulsive loading and rectangular loading among others. Exceptions however exist in the sense that within the same period specified, there were few other investigations involving loading histories in which the time variable was explicitly expressed. These include such loading histories as periodic loading [1] and triangular loading [2]. Others include investigations by Aksogan and Sofiyev [3] , who analyzed a case where cylindrical shells were subjected to a time dependent pressure varying as a power function of time, and Svalbonas and Kalnins [1] , who developed new computer programs for the dynamic buckling of shells under a general time dependent loading. Except for such few cases, there has been a dearth of analytical studies of dynamic buckling investigations for cases where the loading history is explicitly time dependent.

We remark here that we are concerned with a nonlinear dynamical system, with small perturbations, where the ensuing formulation contains dynamically slowly varying parameters .The first study of a strictly nonlinear oscillators with slowly varying parameters was initiated by Kuzmak [4] , whose formulations were essentially restricted to systems yielding second order differential equations. Later, Luke [5], in his study on nonlinear nearly periodic waves, extended Kuzmak's study to include higher orders of differential equations. Since this noble beginning, the concept of slowly varying parameters in nonlinear dynamical systems has been steadily and severally studied by other investigators including Bourland and Haberman [6], Kevorkian [7] and Li and Kevorkian [8] among others.

2.0 Formulation

The dimensional differential equation satisfied by the lateral displacement W(X,Y,T) of a finite simply-supported imperfect column resting on a nonlinear (cubic) elastic foundation [9] , trapped by (for now) an arbitrary time dependent load P(T) , where is X and T are the spatial and time dependent variables respectively , is given by

$$m_0 W,_{TT} + EIW,_{XXXX} + P(T)W,_{XX} + k_1 W - \alpha k_3 W^3 = -P(T)\frac{d^2\overline{W}}{dT^2}, \; T > 0 \qquad (2.1)$$

where EI is the bending stiffness while E is the Young's modulus and I is the moment of inertia. Similarly, $\overline{W}$ is a twice differentiable stress-free function of space variable which serves as the initial imperfection and m_0 is the mass per unit length of the column while k_1 and k_3 are constants and are such that $k_1 > 0$, $k_3 > 0$. α is the imperfection-sensitivity parameter and is such that for a non-linear elastic foundation that behaves like a "softening" spring, α takes the values α=1 where as for a nonlinear elastic foundation that behaves like a "hardening" spring, α takes the value α=1. In this study, the nonlinear elastic foundation is deemed to behave like a "softening" spring and so, we shall henceforth set α=1. The column however rests on a nonlinear elastic foundation that produces a restoring force per unit length of $k_1W - \alpha K_3 W^3$. Here, a subscript following a comma indicates partial differentiation and we have neglected all nonlinear geometric effects as well as axial inertia and shall assume homogeneous initial conditions. We now introduce the following quantities

$$X = \left(\frac{k_1}{E}\right)^{\frac{1}{4}} x = w = \left(\frac{k_3}{k_1}\right)^{\frac{1}{2}} W, \in \overline{w} = \left(\frac{k_3}{k_1}\right)^{\frac{1}{2}} \overline{W}, \; \lambda f(\in^2 \tilde{t}) = \frac{P(T)}{2(EIk_1)^{\frac{1}{2}}}, \tilde{t} = \left(\frac{k_1}{m_0}\right)^{\frac{1}{2}} T \qquad (2.2)$$

On substitution these quantities into (2.1) and simplifying, we get

$$w,_{\tilde{t}\tilde{t}} + w,_{xxxx} + 2\lambda f(\in^2 \tilde{t})w,_{xx} + w - w^3 = -2\lambda \in f(\in^2 \tilde{t})\frac{d^2\overline{w}}{dx^2}, \tilde{t} > 0 \qquad (2.3a)$$

$$w = w,_{xx} = 0 \;\; at \;\; x = 0, \pi; t \geq 0 \qquad (2.3b)$$

$$w(x,0) = w,_t(x,0) = 0, \; 0 < x < \pi \qquad (2.3c)$$

Here $\in$ is a small parameter satisfying the condition $0 < \in << 1$ and representing the amplitude of the imperfection. Similarly λ is a nondimensional load amplitude satisfying the condition $0 < \lambda < 1$, while $f = f(\in^2 \tilde{t})$ is a slowly varying continuous load function of time $\tilde{t}$ and satisfies the conditions

$$f(0) = 1, \; \left|f(\in^2 \tilde{t})\right| \leq 1, \quad \tilde{t} > 0 \qquad (2.4)$$

and has right hand derivatives of all orders at $\tilde{t} = 0$.

We have carried out the non-dimensionalization in such a way that the classical buckling load λ_c takes the value λ_c=1. In our guest for solution, we are to determine a particular value of λ, namely λ_D, called the dynamic buckling load satisfying the inequality $0<\lambda_D<\lambda_S<\lambda_c$ where λ_s is the associated static buckling load. The dynamic buckling λ_D is defined as the largest load parameter for which the solution of the problem remains bounded for all time $\tilde{t} > 0$. For solution, we are first to determine, using multiple scaling regular perturbations in asymptotic approximations, a uniformly valid asymptotic expression of the lateral displacement, $w(x,\tilde{t})$. We shall next determine the maximum value of $w(x,\tilde{t})$ and lastly determine the dynamic buckling load λ_D using a suitable maximization.

Analysis reported here, is similar to those in Wang and Tian [10-12], Popov [13], Zhu et al [14] and Schenk and Schueller [15].

3.0 Solution of the problem

Based on (2.3b) we assume

$$\overline{w} = \overline{a}_m \sin mx, \; m = 1,2,3..., \; |a_m| < 1 \;\; \forall m \qquad (3.1)$$

We set $\tau = \in^2 \tilde{t}$, and for each m in (3.1), we let $\frac{dt}{d\tilde{t}} = \Omega^{\frac{1}{2}}, \; \Omega = \left(m^4 - 2m^2\lambda f(\tau) + 1\right)$ (3.2)

We further let $w(x,\tilde{t}) = U(x,t,\tau)$, so that $w,_{\tilde{t}} = \Omega^{\frac{1}{2}} U,_t + \in^2 U,\tau$ (3.3a)

$$w,_{\tilde{t}\tilde{t}} = \Omega U,_{tt} + 2\in^2 U,_{t\tau} + \in^4 U,_{\tau\tau} - \left(\frac{\in^2 m^2 \lambda \in f'}{\Omega^{\frac{1}{2}}}\right) U,_t \qquad (3.3b)$$

We now let $$U(x,t,\tau,\in)=\sum_{i=1}^{\infty}U^{(i)}(x,t,\tau)\in^{i} \qquad (3.4)$$

On substituting (3.1), (3.3a,b) and (3.4) into (2.3a-c) and equating the coefficients of powers of $\in$, we get

$$MU^{1}\equiv U,_{tt}^{(1)}+\frac{1}{\Omega}\left(U,_{xxxx}^{(1)}+2\lambda fU,_{xx}^{(1)}+U^{(1)}\right)=\frac{2\overline{a}_{m}\lambda m^{2}f(\tau)}{\Omega}\sin mx \qquad (3.5)$$

$$MU^{2}=0 \qquad (3.6)$$

$$MU^{(3)}=\frac{\left(U^{(1)}\right)^{3}}{\Omega}-2\Omega^{-1/2}U,_{t\tau}^{(1)}+\frac{\left(m^{2}\lambda f'\right)U,_{t}^{(1)}}{\Omega^{3/2}} \qquad (3.7)$$

The initial conditions are evaluated at (t,τ) = (0, 0) and are given by

$$U^{(i)}=0,\quad i=1,2,3..., \qquad (3.8a)$$

$$U_{,t}^{(1)}=U,_{t}^{(2)}=0 \qquad (3.8b)$$

$$U,_{t}^{(r)}+\Omega^{-1/2}(0)U,_{\tau}^{(r-2)}=0 \qquad r=3,4,5 \qquad (3.8c)$$

We shall next let $$U^{(i)}(x,t,\tau)=\sum_{n=1}^{\infty}U_{n}^{(i)}(t,\tau)\sin nx \qquad (3.9)$$

On substituting (3.9) into (3.5) we observe that, when $n = m$, we have

$$U_{m,tt}^{(1)}+U_{m}^{(1)}=B,\quad \left\{B(\tau)=\frac{2\overline{a}_{m}\lambda m^{2}f(\tau)}{\Omega(\tau)}\right\} \qquad (3.10a)$$

$$U_{m}^{(1)}(0,0)=U_{m,t}^{(1)}(0,0)=0 \qquad (3.10b)$$

The solution of (3.10a,b) is

$$U_{m}^{(1)}(t,\tau)=a,(\tau)\cos t+b_{1}(\tau)\sin t+B \qquad (3.11a)$$

$$a_{1}(0)=-B_{0};\ B_{0}=B(0)=\frac{2\overline{a}_{m}\lambda m^{2}}{\Omega(0)}=\frac{2\overline{a}_{m}\lambda m^{2}}{\left(m^{4}-2m^{2}\lambda+1\right)}, \qquad (3.11b)$$

$$(0)=\frac{B_{0}\left(m^{4}+1\right)f'(0)}{\Omega(0)}\ ,\quad b_{1}(0)=0 \qquad (3.11c)$$

Since (3.6) is homogeneous with homogeneous initial conditions, we expect

$$U^{(2)}(t,\tau)\equiv 0 \qquad (3.12)$$

We substitute (3.9) into (3.7) for i =3 and get

$$\begin{aligned}MU^{(3)}&=\frac{(3\sin mx-\sin 3mx)}{4\Omega}\left[r_{0}+r_{1}\cos 2t+r_{2}\cos 3t+r_{3}\cos t\right]\\&+2\Omega^{-1/2}\left(a_{1}'\sin t-b'\cos t\right)\sin mx+\frac{m^{2}\lambda' f}{\Omega^{3/2}}\left(-a_{1}\sin t+b_{1}\cos t\right)\end{aligned} \qquad (3.13a)$$

where $\frac{d(\ )}{d\tau}=(\ )'$ and where

$$r_{0}=B^{3}+\frac{3a_{1}^{2}B}{2},\ \mathrm{r}_{1}(0)=\frac{5}{2}B_{0}^{3};\ \mathrm{r}_{0}'(0)=\frac{3B_{0}^{3}f'(0)R_{1}}{2\Omega_{0}},\ \mathrm{R}_{1}=3\left(m^{4}+1\right)+m^{2}\lambda \qquad (3.13b)$$

$$r_{1}=\frac{3a_{1}^{2}B}{2},\ \mathrm{r}_{1}(0)=\frac{3}{2}B_{0}^{3};\ \mathrm{r}_{1}'(0)=\frac{3B_{0}^{3}f'(0)R_{2}}{2\Omega_{0}},\ \mathrm{R}_{2}=\left(m^{4}+1\right)+m^{2}\lambda \qquad (3.13c)$$

$$r_{2}=\frac{\alpha_{1}^{3}}{4},\ \mathrm{r}_{2}(0)=-\frac{B_{0}^{3}}{4};\ \mathrm{r}_{2}'(0)=-\frac{3B_{0}^{2}m^{2}\lambda f'(0)}{8\Omega_{0}}\ ;\ \mathrm{r}_{3}=\frac{3\alpha_{1}^{3}}{4}+3\alpha_{1}B^{2}\mathrm{r}_{3}(0)=-\frac{15B_{0}^{3}}{4} \qquad (3.13d$$

$$r_{3}'(0)=-\frac{B_{0}^{3}f'(0)R_{4}}{4\Omega_{0}},\ \mathrm{R}_{4}=9m^{2}\lambda+24\left(m^{4}+1\right) \qquad (3.13e)$$

$$\Omega_{0}=\Omega(0)=\left(m^{4}-2m^{2}\lambda+1\right) \qquad (3.13f)$$

For n=m in (3.13a), we get

$$U^{(3)}_{m,tt}+U^{(3)}_{m}=\frac{3}{4\Omega}\left[r_0+r_1\cos 2t+r_2\cos 3t+r_3\cos t\right]+2\Omega^{-1/2}\left(a_1'\sin t-b_1'\cos t\right)$$
$$+\frac{m^2\lambda f'}{\Omega^{3/2}}\left(-a_1\sin t+b_1\cos t\right) \qquad (3.14a)$$
$$U^{(3)}_{m}(0,0)=0\,,U^{(3)}_{m,t}(0,0)+\Omega^{1/2}(0)U^{(1)}_{m,\tau}(0,0)=0 \qquad (3.14b)$$

However for n =3m in (3.13a), we get

$$U^{(3)}_{3m,tt}+\Theta U^{(3)}_{3m}=-\frac{1}{4\Omega}\left[r_0+r_1\cos 2t+r_2\cos 3t+r_3\cos t\right] \qquad (3.14c)$$
$$U^{(3)}_{3m}(0,0)=0\ ;\ U^{(3)}_{3m,t}(0,0)=0 \qquad (3.14d)$$
$$\Theta=\left(\frac{81m^2-18m^2\lambda f+1}{m^4-2m^2\lambda f+1}\right) \qquad (3.14e)$$

To ensure a uniformly valid solution in the time scale t, we equate to zero in (3.14a) the coefficients of cost and sint and get respectively $\quad b_1'-\frac{m^2\lambda f'\,b_1}{2\Omega}=\frac{3r_3}{8\Omega^{1/2}}\ ;\ a_1'-\frac{m^2\lambda f'\,a_1}{2\Omega}=0 \qquad$ (3.15)

The solutions of (3.15) are $\quad b_1(\tau)=\frac{3}{8}\Omega^{-1/4}\int_0^{\tau}r_3(s)\Omega^{-1/4}(s)ds\ \ ;\ \ a_1(\tau)=a_1(0)\left(\frac{\Omega(0)}{\Omega(\tau)}\right)^{1/4}$ (3.16)

The remaining equation in (3.14a,b) is now solved to get

$$U^{(3)}_{m}(t,\tau)=a_3(\tau)\cos t+b_3(\tau)\sin t+\frac{3}{4\Omega}\left[r_0-\frac{r_1\cos 2t}{3}-\frac{r_2\cos 3t}{8}\right] \qquad (3.17a)$$
$$a_3(0)=-\frac{195B_0^3}{128\,\Omega_0},\ b_3(0)=0 \qquad (3.17b)$$

To solve (3.14c), we note that

$$\Theta=\omega^2+\tau\,\Theta'(0)+\frac{\tau^2\Theta''(0)}{2}+\cdots;\ \omega^2=\left(\frac{81m^4-18m^2\lambda+1}{m^4-2m^2+1}\right)>0\,\forall m \qquad (3.18a)$$

The terms $\tau\,\Theta'(0)+\frac{\tau^2\Theta''(0)}{2}+\cdots$ will definitely contribute to accuracy outside that retained in this investigation. Thus, within the degree of accuracy retained here, we let $\qquad \Theta\cong\omega^2$ (3.18b)
On substituting (3.18b) into (3.14c) and solving, we get

$$U^{(3)}_{3m}(t,\tau)=a_4(\tau)\cos\omega t+b_4(\tau)\sin\omega t-\frac{1}{4\Omega}\left[\frac{r_0}{\omega^2}+\frac{r_1\cos 2t}{\omega^2-4}+\frac{r_2\cos 3t}{\omega^2-9}+\frac{r_3\cos t}{\omega^2-1}\right] \qquad (3.19a)$$

where

$$a_4(0)=\frac{B_0^3\,Q_0}{8\Omega_0};\ Q_0=\left[\frac{5}{\omega^2}-\frac{15}{2(\omega^2-1)}+\frac{3}{\omega^2-4}-\frac{1}{2(\omega^2-9)}\right],\ b_4(0)=0 \qquad (3.19b)$$

Thus we have $\quad U(x,t,\tau)=\in U^{(1)}_m\sin mx+\in^3\left[U^{(3)}_m\sin mx+U^{(3)}_{3m}\sin 3mx\right]+O(\in^4) \qquad$ (3.20)

We shall now determine the maximum lateral displacement $U_a=U(x_a,t_a,\tau_a)$ Where x_a,t_a and τ_a are the values of the associated variables at maximum displacement. As a function of space and time variables, the conditions for maximum displacement are

$$U_{,x}=U_{,t}+\in^2\Omega^{-1/2}(\tau_a)U_{,\tau}=0 \qquad (3.21)$$

By substituting (3.20) into the first of (3.21), we get $\qquad x_a=\frac{\pi}{2} \qquad$ (3.22)

where we have taken the least positive value of x_a in (3.22). We now let

$$t_a=t_0+\in^2 t_1+\cdots;\ \tilde{t}_a=\tilde{t}_0+\in^2\tilde{t}_1+\cdots;\tau_a\in^2\tilde{t}_a=\in^2\left(\tilde{t}_0+\in^2\tilde{t}_1+\cdots\right) \qquad (3.23a)$$

By substituting (3.20) into the second of (3.21) and equating the coefficient of $\in$, we get

$$\sin t_0 = 0 \qquad (3.23b)$$

This gives

$$t_0 = \pi \qquad (3.23c)$$

where we have taken the least nontrivial positive value of t_0 in (3.23c).To determine the maximum displacement $U_a = U(x_a, t_a, \tau_a)$, we evaluate (3.20) at the critical values of the variables , using (3.22) and (3.23c) and obtain

$$U_a = \in (B(0) - a_1(0)) + \in^3 \left[U_m^{(3)}(t_0, 0) - U_{3m}^{(3)}(t_0, 0) + (B'(0) - a_1'(0))\tilde{t}_0\right] + O(\in^4) \qquad (3.24)$$

To determine $\tilde{t}_0$, as it appears in (3.24), we note, from (3.2) evaluated at the critical values of the variables, that we can easily get

$$t_a = \int_0^{\tilde{t}_a} (m^4 - 2m^2\lambda f(s) + 1)^{1/2} ds = \Omega_0^{1/2}\left[\tilde{t}_a - \frac{m^2\lambda}{2\Omega_0}\left\{\in^2 \tilde{t}_a^2 f'(0) + \frac{2\in^4 f''(0)\tilde{t}_a^3}{3} + \cdots\right\}\right] + \cdots \qquad (3.25a)$$

On substituting into (3.25a) for t_a and $\tilde{t}_a$ from (3.25a), we get $t_0 = \pi = \Omega_0^{1/2}\tilde{t}_0$ (3.25b)

This gives

$$\tilde{t}_0 = \frac{\pi}{\Omega_0^{1/2}} \qquad (3.25c)$$

On substituting into (3.24c), using all the already evaluated terms, we have

$$U_a = \in C_1 + \in^3 C_3 + \cdots \qquad (3.26a)$$

$$C_1 = 2B_0 \ , C_3 = \frac{3B_0^3 Q_1}{\Omega_0}; \qquad (3.26b)$$

$$Q_1 = \left[1 - \left(\frac{1-\cos\omega t_0}{24}\right)\left\{\frac{15}{2(\omega^2 -)} + \frac{1}{2(\omega^2 - 9)} - \frac{5}{\omega^2} - \frac{3}{\omega^2 - 4}\right\} + \frac{\pi f'(0)\{m^2\lambda + 2(m^4+1)\}}{6B_0^2\Omega_0^{3/2}}\right] \qquad (3.26c)$$

We shall now determine the dynamic buckling load λ_D, which, according to Ette [16] follows from the maximization

$$\frac{d\lambda}{dU_a} = 0 \qquad (3.27)$$

As in [16], the usual procedure is to first reverse the series (3.26a) and so obtain

$$\in = d_1 U_a + d_3 U_a^3 + \cdots \qquad (3.28a)$$

By substituting into (3.28a) for U_a from (3.26a) and equating the coefficients of equation of powers of $\in$, we get

$$d_1 = \frac{1}{C_1},\ d_3 = -\frac{C_3}{C_1^4} \qquad (3.28b)$$

The maximization (3.27) now easily follows from (3.28a) to yield, after some simplification

$$\in = \frac{2}{3}\sqrt{\frac{C_1}{3C_3}} \qquad (3.29)$$

which is evaluated at $\lambda = \lambda_D$ .On substituting into (3.29) for C_1 and C_3 from (3.26b) and simplifying, we get

$$(m^4 - 2m^2\lambda_D + 1)^{3/2} = \frac{9|\in \bar{a}_m| m^2\lambda_D}{\sqrt{2}}\left[1 - \left(\frac{1-\cos\omega t_0}{24}\right)\left\{\frac{15}{2(\omega^2 -)} + \frac{1}{2(\omega^2 - 9)} - \frac{5}{\omega^2} - \frac{3}{\omega^2 - 4}\right\} + \frac{\pi f'(0)\{m^2\lambda + 2(m^4+1)\}}{6B_0^2\Omega_0^{3/2}}\right]^{1/2} \qquad (3.30)$$

where (3.30), is evaluated at $\lambda = \lambda_D$.

4.0 Analysis of result and conclusion

The result (3.30) is implicit in the load parameter λ_D and it is valid provided

$$\left\|\left[\left(\frac{1-\cos\omega t_0}{24}\right)\left\{\frac{15}{2(\omega^2-)}+\frac{1}{2(\omega^2-9)}-\frac{5}{\omega^2}-\frac{3}{\omega^2-4}\right\}+\frac{\pi\ f'(0)\ \{m^2\lambda+2(m^4+1)\}}{6B_0^2\Omega_0^{3/2}}\right]\right\|<1$$ We observe that (3.30) is independent of any form of the load function $f(\in^2 \tilde{t})$ provided that equation (2.4) is satisfied. However, the result depends strongly on $f'(0)$ which is the first derivative of $f(\in^2 \tilde{t})$ evaluated at the initial time .The dominant result is for the case where m=1 and this gives the following result derivable from (3.30)

$$(1-\lambda_D)^{3/2} = \frac{9|\bar{a}_1\in|}{4}\left[1-\left(\frac{1-\cos\omega t_0}{24}\right)\left\{\frac{15}{2(\omega^2-)}+\frac{1}{2(\omega^2-9)}-\frac{5}{\omega^2}-\frac{3}{\omega^2-4}\right\}+\frac{\sqrt{2}\pi f'(0)(1-\lambda_D)(\lambda_D+4)}{24(\lambda_D\bar{a}_1)^2}\right]^{\frac{1}{2}} \quad (4.1)$$

Where (4.1) is evaluated at m = 1.For step loading situation we readily observe that $f'(0)=0$ so that

(3.27) and (4.1) become respectively

$$(m^4+2m^2\lambda_D+1)^{\frac{3}{2}} = \frac{9|\in\bar{a}_m|\,m^2\lambda_D}{\sqrt{2}}\left[1-\left(\frac{1-\cos\omega t_0}{24}\right)\left\{\frac{15}{2(\omega^2-)}+\frac{1}{2(\omega^2-9)}-\frac{5}{\omega^2}-\frac{3}{\omega^2-4}\right\}\right]^{\frac{1}{2}} \quad (4.2a)$$

$$(1-\lambda_D)^{\frac{3}{2}} = \frac{9|\in\bar{a}_1|\,\lambda_D}{4}\left[1-\left(\frac{1-\cos\omega t_0}{24}\right)\left\{\frac{15}{2(\omega^2-)}+\frac{1}{2(\omega^2-9)}-\frac{5}{\omega^2}-\frac{3}{\omega^2-4}\right\}\right]^{\frac{1}{2}} \quad (4.2b)$$

where (4.2b) is evaluated at m=1 and $\bar{a}_1$ is the value of $\bar{a}_m$ at m=1. In all the results so far obtained (that is ,equations (3.27) - (4.2b)), there is a tacit assumption that the buckling mode be partly in the shape of the imperfection and partly in some higher eigen modes of the imperfection. If we are however demanding for the result of the case in which the buckling mode is strictly in the shape of the imperfection , then we must neglect the term $U_{3m}^{(3)}(t,\tau)$. This is equivalent to neglecting the term Q_0 in the results as in (3.26b). This boils down to taking $Q_1=1$ (30c) and the corresponding result for (4.26b) is

$$(1-\lambda_D)^{\frac{3}{2}} = \frac{9|\in\bar{a}_1|\,\lambda_D}{4} \quad (4.3)$$

The result (4.3) is exactly the same as that obtained by Amazigo and Frank [9] for step loading consideration.

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