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Synchronization of Forced damped Pendulum via Active Control

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Abstract

In this paper active controllers are designed to synchronize two identical forced damped pendula. The performance of the controllers in the synchronization of the chaotic dynamics of the two pendula, resulting from different initial conditions, is investigated numerically and found to be effective. Transition from nonsynchronous state via both temporary phase lock (TPL) and intermittent synchronous states to complete synchronous state was observed.

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Keywords: Active Control; synchronization; chaotic pendulum

1.0 Introduction

In 1990, Pecora and Carroll [1] introduced a method to synchronize two identical chaotic systems evolving from different initial conditions. Due to their pioneering works, chaos synchronization has been intensively investigated in the contexts of many specific problems in Physical systems [2-4], chemical systems, ecological systems [5;6], biomedical systems, secure communications [7-9], to mention but a few. Extensive research has been made in developing various types of synchronization schemes, such as adaptive control [10], backstepping design [11], active control [12-15], nonlinear control [16-18], sliding mode control [19,20] and projective synchronization [21].

The method of active control was proposed, in 1997, by Bai and Lonngren [12,13] The method was later extended to non-identical systems by Ho and Hung [14] using scheme called generalized active control (GAC). GAC was recently employed to synchronize non-identical systems consisting of Lorenz, Chen and Lu systems with a new chaotic system of Chen and Lee [22]. Due to the inherent advantage of the active control technique in synchronizing chaotic systems, the method has over the years received wide applications in a variety of dynamical systems, including Geophysical model [23], spatiotemporal dynamical system [24] and he so-called unified chaotic system [25]. Using the active control, phenomena such as phase and anti-phase synchronizations were reported in [26]; while temporary phase locking states (TPL) as precursors to global synchronization were reported [24].

This paper studies the synchronization behaviour of identical forced damped pendula using active control approach. In section 2 the model is discussed; in section 3 the active controllers that synchronize between identical forced damped pendula are designed; numerical simulations are given in section 4 to validate the synchronization approach and section 5 concludes the paper.

2.0 Description of the model

The equation describing the forced damped pendulum is of the form

$$\ddot{\theta} = -c\dot{\theta} - \sin\theta + \rho\sin t \qquad (2.1)$$

This differential equation includes a damping term with damping constant c and the force of gravity, which pulls the pendulum bob down, as well as the sinusoidal force $\rho \sin t$, which accelerates the bob first clockwise and then counterclockwise continuing in a periodic way. This periodic forcing guarantees that the pendulum will keep swinging, provided ρ is nonzero. The term $\rho \sin t$ has a period of 2π. It follows that if $\theta(t)$ is a solution of (1) then $\theta(t+2\pi)$ and $\theta(t+2\pi N)$ for any integer N, are solution of (2.1). Let $x(t)=\theta(t+2\pi)$ then $x(t)$ also satisfies (2.1) as follows [28]

$$\ddot{x} = -c\ddot{x} - \sin x + \rho\sin t \qquad (2.2)$$

3.0 Formulation of the active controllers

Equation (2.2) can be written as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -cx_1 - \sin\ x_1 + \rho\ \sin\ t \quad (3.1)$$

Suppose that system (3.1) is the drive system, then the response system is

$$\dot{y}_1 = y_2 + u_1$$
$$\dot{y}_2 = -cy_1 - \sin\ y_1 + \rho\ \sin\ t\ + u_2 \quad (3.2)$$

where $u_i(t), i = 1,2$ are control functions to be determined. Subtracting (3.1) from (3.2) we obtain the error dynamics as

$$\dot{e}_1 = e_2 + u_1$$
$$\dot{e}_2 = -ce_1 - (\sin\ y_1 - \sin\ x_1)\ u_2 \quad (3.3)$$

where $e_i = y_i - x_i,\ i = 1,2$.

We now re-define the control functions, to eliminate terms in (3.3) which cannot be expressed as linear terms in e_1 and e_2, as follows:

$$u_1 = v_1(t)$$
$$u_2 = +v_2(t) + (\sin\ y_1 - \sin\ x_1) \quad (3.4)$$

Substituting (3.4) into (3.3) we have

$$\dot{e}_1 = e_2 + v_1$$
$$\dot{e}_2 = -ce_1 + v_2 \quad (3.5)$$

Using the active control method, we choose a constant matrix **A** which will control the error dynamics (3.5) such that

$$\begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix} = \mathbf{A} \begin{pmatrix} e_1 \\ 2_2 \end{pmatrix} \quad (3.6)$$

with

$$\mathbf{A} = \begin{pmatrix} \lambda_1 & -1 \\ x & \lambda_2 \end{pmatrix} \quad (3.7)$$

In (3.7) the two eigenvalues λ_1 and λ_2 have been chosen as-1and -1 in order that a stable synchronization of systems (3.1) and (3.2) is achieved.

4.0 Numerical results

The 4th order Runge-Kutta algorithm was used to solve systems (3.1) and (3.2) simultaneously with a time step of $0.01, c = 0.2, \rho = 1.66$ and the initial conditions $x_1 = 0, x_2 = 0, y_1 = 0.4, y_2 = 1.0$. The results of the simulation are presented in Figures (1-6). When the active controllers $u_i,\ i = 1,2$ are deactivated the error states e_1 and e_2 oscillate chaotically as shown in Figure (1), for e_1. When u_1 and u_2 are simultaneously activated at $t = 100$ the error states e_1 and e_2 diminish to zero as shown in Figure (2), for e_1, showing that systems (3.1) and (3.2) have synchronized. However, when the controllers are activated sequentially (as proposed by Bai and Lonngren [13] to check the weakness inherent in the simultaneous activation of active controllers) with u_1 activated first at $t = 100$ followed by u_2 at $t = 150$ e_1 diminishes to zero after the activation of u_1 just like in the case of simultaneous activation, Figure (2), while e_2 is temporarily phase locked (TPL) for the period $100 < t \le 150$ and after the activation of u_2 at $t = 150$ e_2 diminishes to zero thus achieving complete synchronization (CS), Figure (3). However, if the sequence of activation of u_1 and u_2 is reversed with u_2 activated first at $t = 100$ followed by u_1 at $t = 150$, e_1 undergoes TPL for the period $100 < t \le 150$, Figure (4), and e_2 undergoes intermittent synchronization for the same period Figure (5) and after the activation of u_1 at

$t = 150$ both e_1 and e_2 diminish to zero thereby indicating that the systems have achieved complete synchronization. Figure (6) magnifies the region $100 < t \leq 150$ for a better observation of the intermittent synchronization. TPL resulting from sequential application of active controllers was first observed by U. E. Vincent [23]. It has been observed in this paper that sequential activation of the controllers results in both TPL and intermittent synchronization.

4.0 Conclusion

In summary, active controllers have been designed to synchronize two identical forced damped pendula. The performance of the controllers in the synchronization of the chaotic dynamics of the two pendula, resulting from different initial conditions was investigated numerically and found to be effective. Transition from nonsynchronous state via both temporary phase (TPL) and intermittent systemsynchronous state was observed.

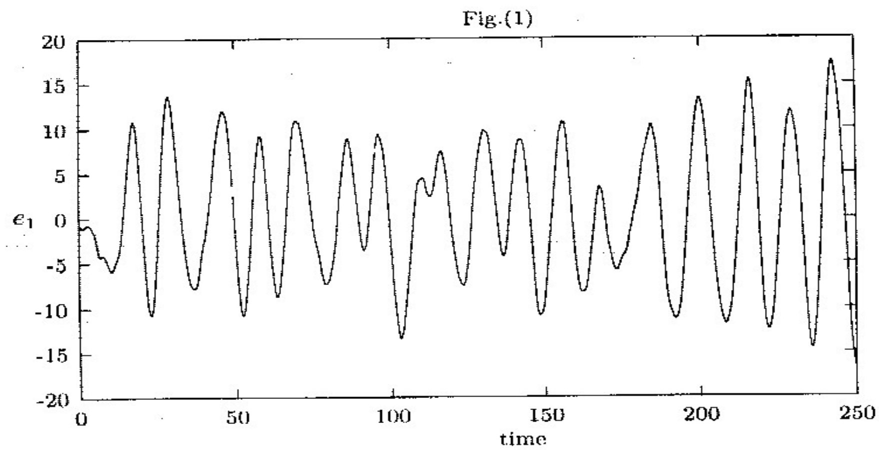

Figure 1: Time evolution of the error state (e_1) without any control.

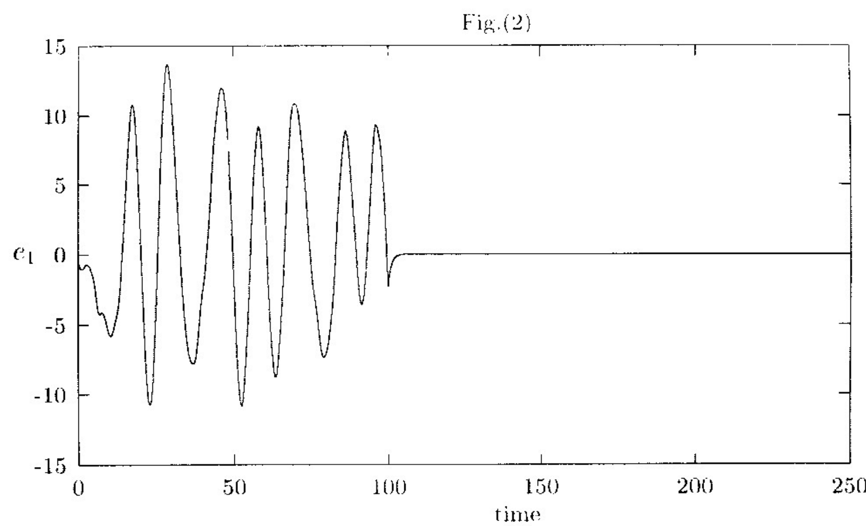

Figure 2: Time evolution of the error stare (e_1) with the active controllers $(u_i, i = 1,2)$ activated at $t = 100$.

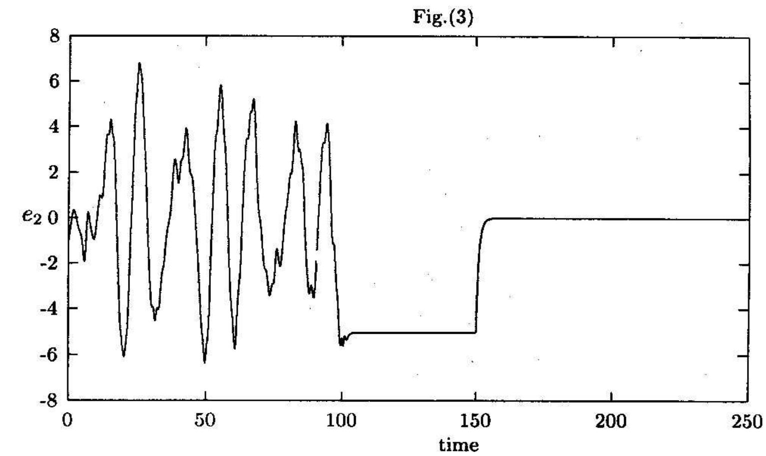

Figure 3: Time evolution of the error states e_2 for sequential activation of u_1 at t =100 and u_2 at 150

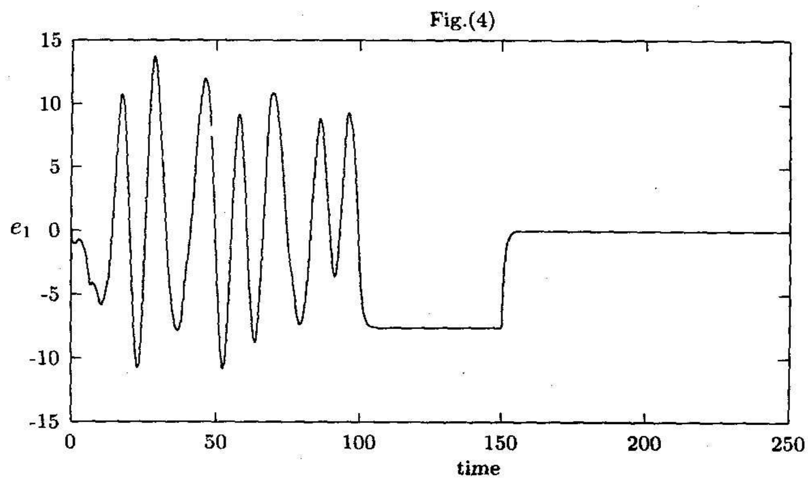

Figure 4: Time evolution of the error state e_1 for sequential activation of u_1 at $t = 150$ and u_2 at $t = 100$.

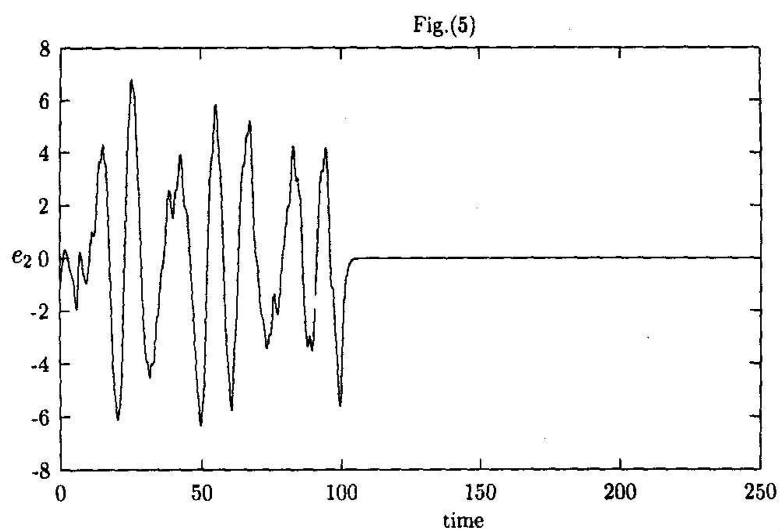

Figure 5: Time evolution of the error state e_2 for sequential activation of u_1 at $t = 150$ and u_2 at $t = 100$.

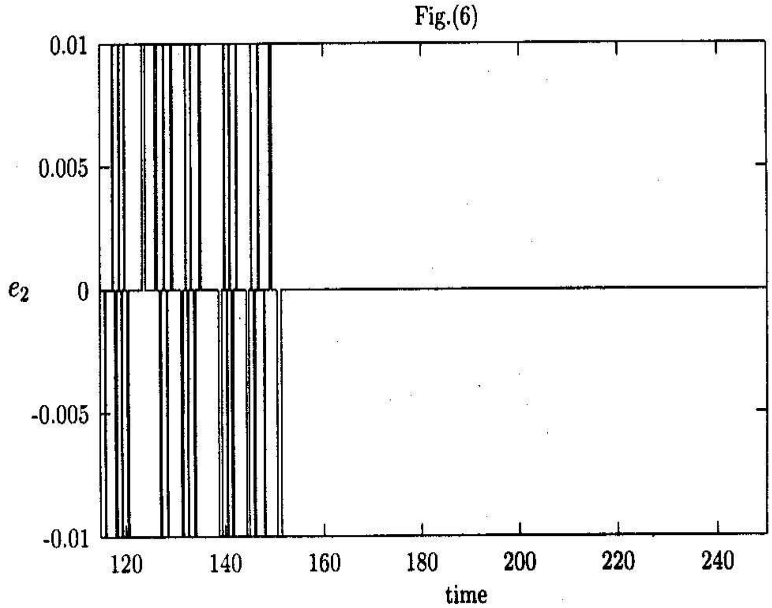

Figure 6: Time evolution of the error state (e_2) in the region $100 < t \leq 150$ and u_2 is on and u_1 is off showing intermittent synchronization.

References

[1] Pecora LM, Carroll TL. Phys. Rev. Lett. 1990; 64:821.
[2] Lakshmanan M, Murali K. Chaos in Nonlinear Oscillators: Controlling and Synchronization. Singapore: World Scientific; 1996.
[3] Vincent UE, Njah AN, Akinlade O, Solarin ART. Chaos 2004; 14: 1018.
[4] Vincent UE, Njah AN, Akinlade O, Solarin ART. Physica A 2006; 360: 180.
[5] Han SK, Kerrer C, Kuramoto Y. Phys. Rev. Lett 1995; 75: 3190.
[6] Blasius B, Huppert A, Stone L. Nature 1999; 399: 354.
[7] Cuomo KM, Oppenheim AV. Phys. Rev. Lett. 1993; 71:65.
[8] Kocarev L, Parlitz U, Phys. Rev. Lett. 1995; 74: 5028.
[9] Kwok HS, Wallace K, Tang S, Man KF. Int. J. Bifurcation and Chaos 2004; 14: 285
[10] Chen SH, Lu J. Chaos, Solitons & Fractals 2002; 14: 643.
[11] Tan X, Zhang J, Yang Y. Chaos Solitons & Fractals 2003; 16: 37.
[12] Bai EW, Linngren KE. Chaos Solitons & Fractals 1997; 8: 51.
[13] Bar EW, Lonngren KE, Chaos Solitons & Fractals 2000; 11: 1041.
[14] Ho MC, Hung YC. Phys. Lett A 2002; 301: 424.
[15] Chen HK. Chaos, Solitons & Fractals 2005; 25: 1049.
[16] Huang L, Feng R, Wang M. Phys. Lett A 2004; 320: 271.
[17] Chen HK. Chaos Solitons & Fractals 2005; 23: 1245.
[18] Moukam Kakmeni FM, Bowong S, Tchawoua C. Phys. Lett A 2006; 355: 47.
[19] Konishi K, Hirari M. Kokame H. Phys. Lett. A 1998; 245: 571.
[20] Jang M. Int. J. Bifurcation and Chaos 2002; 12: 1437.
[21] Yang J, Li C. Chaos Solitons & Fractals 2005; 26: 1119.
[22] Chen HK and Lee CI. Chaos Solitons & Fractals 2004; 21: 957
[23] Vincent UE. Pyhs. Lett. A 2005: 343: 133.
[24] Codreanu S. Chaos Solitons & Fractals 2003; 15: 507.
[25] Ucar A, Lonngren KE, Bai EW. Chaos Solitons & Fractals 2006; 27: 1292
[26] Ho MC, Hung YC, Chou Ch, Phys. Lett. A 2002; 296: 43
[27] Stoker JJ. Nonlinear Vibration in Mechanical and Electrical Systems. New York: Interscience Publisher; 1950.

[28] Alligood K. T., Sauer T. D. and Yorke J. A., CHAOS: An introduction to dynamical system, 1996, Springer-Verlag New York.