Journal of the Nigerian Association of Mathematical Physics
Volume 10 (November 2006), **137-142**
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Active control versus recursive backstepping control of a chaotic system

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Abstract

In this paper active controllers and recursive backstepping controllers are designed for a third order chaotic system. The performances of these controllers in the control of the dynamics of the chaotic system are investigated numerically and are found to be effective. Comparison of their transient performances show that the rate of convergence of error is faster for the active controllers than for the recursive backstepping controllers. However, the flexibility in the choice of the control laws for recursive backstepping design gives room for further improvement in its performance and enables it to achieve the goals of stabilization and tracking.

PACS: 05.45.-a, 05.45.Pq, 05.45.Ac
***Keywords*:** Active Control; Recursive Backstepping control chaotic system

1.0 Introduction

Chaos control is directly related to the observer problem in control theory [1]. It has been show that the control of chaotic system has many potential applications in chemical reactors, biomedical systems, ecology, laser physics, secure communication [2-4], to mention but a few.

The origin of chaos control is traceable to the pioneering work of Pecora and Carroll [5], in which replacement method was employed to achieve synchronization, in 1990. Thereafter several methods were developed, notable among which are linear feedback [6-9], adaptive synchronization [10-11], sliding mode control [12-13] active control [14-18], backtepping design [19-21].

Chaos control using active control was proposed, in 1997, by Bai and Lonngren to control Lorenz system [17-18]; and has been used to control other system [16-18]. The method was later extended to non-identical systems by Ho and Hung [17] using a scheme called generalized active control (GAC), thereby demonstrating the advantage of the active control technique over other controls schemes. Active control has been used to control many chaotic systems [22-26].

In the last decade, backstepping based designs emerged as powerful tools for stabilizing chaotic systems for tracking and regulation purposes [27]. Based on recursive application of Lyapunov's direct trajectory. A major advantage of the backstepping controller is the flexibility in the choice of control law so that the goals of both stabilization and tracking are achieved. This flexibility is possible through the construction of a Lyapunov function whose derivative can be made negative definite by a variety of control laws rather than by a specific control law [27-28]. In this paper active controllers and recursive backstepping controllers are designed for a typical third order chaotic system and their performance, investigated by numerical computation, are compared. The rest of the paper is organized as follows. In section 2 the active controllers are designed; in section 3 the recursive backstepping controllers are designed; section 4 present and discusses the results while section 5 concludes the paper.

2.0 Design of active controllers

As a simple yet practical nonlinear system example one chooses the following third order state representation

$$\dot{x}_1 = -x_3, \ \ \dot{x}_2 = x_1 - x_2$$
$$\dot{x}_3 = 3.1x_1 + x_2^2 + F_L x_1 \tag{2.1}$$

Suppose that system (1) is the drive system, then the response system is

$$\begin{aligned}\dot{y}_1 &= -y_3 + u_1 \\ \dot{y}_2 &= y_1 - y_2 + u_2 \\ \dot{y}_3 &= 3.1y_1 + y_2^2 + F_L y_1 + u_3 \end{aligned} \qquad (2.2)$$

where $u_i(t), i = 1,2,3$ are control functions to be determined. Subtracting (2.1) form (2.2) we obtain the error dynamics as

$$\begin{aligned}\dot{e}_1 &= -e_3 + u_1 \\ \dot{e}_2 &= e_2 - e_2 + \\ \dot{e}_2 &= 3.1e_1 + \left(y_2^2 - x_2^2\right) + F_L e_3 + u_3 \end{aligned} \qquad (2.3)$$

where $e_i = y_i - x_i, i = 1,2,3$. We now re-define the control functions, to eliminate terms in (2.3) which cannot be expressed as linear terms in e_1, e_2 and e_3, as follows:

$$\begin{aligned}u_1 &= v_1(t) \\ u_2 &= v_2(t) \\ u_3 &= v_3(t) - \left(y_2^2 - x_2^2\right) \end{aligned} \qquad (2.4)$$

Substituting (2.4) into (2.3) we have

$$\begin{aligned}\dot{e}_1 &= -e_3 + v_1 \\ \dot{e}_2 &= e_1 - e_2 + v_2 \\ \dot{e}_3 &= 3.1e_1 + F_L e_3 + v_3 \end{aligned} \qquad (2.5)$$

Using the active control method, we choose a constant matrix **A** which will control the error dynamics (2.5) such that

$$\begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{pmatrix} = \mathbf{A} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} \qquad (2.6)$$

with

$$\mathbf{A} = \begin{pmatrix} \lambda_1 & 0 & 1 \\ -1 & \lambda_2 + 1 & 0 \\ -3.1 & 0 & \lambda_3 - F_L \end{pmatrix} \qquad (2.7)$$

In (2.7) the three eigenvalues λ_1, λ_2 and λ_3 have been chosen as -1,-1 and -1 in order that a table synchronization of systems (2.1) and (2.2) in achieved.

3.0 Design of recursive backstepping controllers

The system (2.1) is now written as, [29]

$$\begin{aligned}\dot{x}_1 &= -x_3 \\ \dot{x}_2 &= x_1 - x_2 + u_1 \\ \dot{x}_3 &= 3.1x_1 + x_2^2 + F_L x_1 + u_2 \end{aligned} \qquad (3.1)$$

where $u_1(t)$ and $u_2(t)$ are the control function to be determined. In recursive backstepping control the values of x_1, x_2 and x_3 are controlled to take desired values x_{1d}, x_{2d} and x_{3d}, respectively, and the error signals are, therefore defined as

$$\begin{aligned}e_1 &= -x_1 - x_{1d} \\ e_2 &= x_2 - yx_{2d}\end{aligned}$$

$$e_3 = +x_3 - x_{3d} \quad (3.2)$$

In the design method we let $x_{1d} = 0$ (or any desired function) $x_{2d} = c_1 e_1$

$$x_{3d} = c_2 e_1 + c_3 e_2 \quad (3.3)$$

Substituting system (3.3) in system (3.2) and substituting the resulting system of equations in system (3.1) gives

$$\dot{e}_1 = -e_3 + c_2 e_1$$
$$\dot{e}_2 = (1 - c_1 + c_1 c_2) e_1 - e_2 + c_2 e_3 + u_1$$
$$\dot{e}_3 = c_2 (e_3 + c_2 e_1) + 3.1 e_1 + (e_2 + c_1 e_1)^2 F_L (e_3 + c_2 e_1) + u_2 \quad (3.4)$$

Where c_3 has been set to zero $(c_3 = 0)$ to eliminate the term containing u_1 in the expression for $\dot{e}_3$.

$$V = \frac{1}{2} \sum_{i=1}^{3} k_i e_i \quad (3.5)$$

The time derivative of equation (3.5) is

$$\dot{V} = \sum_{i=1}^{3} k_i e_i \dot{e}_i = k_1 e_1 \dot{e}_1 + k_2 e_2 \dot{e}_2 + k_3 e_3 \dot{e}_3 \quad (3.6)$$

Substituting (3.4) in (3.6) and making the choices $c_1 = 1$ and $c_2 = -1 - F_L$, which reduce the number of terms and eliminate F_L, we have $u_1 = +e_2 - c_2 e_1 - e_3$

$$u_2 = e_3 - (3.1 - c_2) e_1 + (e_1 + e_2)^2 \quad (3.7)$$

where $k_i, i = 1,2,3$ and $u_i, i = 1,2$ have been chosen such that $\dot{V} = 0$; in which case $k_1 = 0, k_2 = k_3 \neq 0$ since k_2 and k_3 are coefficient of u_1 and u_2 respectively

4.0 Numerical result

The 4th order Runge-Kutta algorithm was employed to solve systems (2.1) and (2.2) simultaneously with a time grid of $0.05, F_L = 0.5$ and the initial conditions $x_1(0) = 0, x_2(0) = 0, x_3(0) = 0$ .01, $y_1(0) = 0.01$, $y_2(0) = 0.01\, y_3(0) = 0.06$. Figure (1) shows the attractor in phase space of the drive system (1). When the active controllers $u_i, i = 1,2,3$ are switched off the error states $(e_1, e_2$ and $e_3)$ oscillator chaotically as shown in Figure (2), for e_1. When the active controllers are switched on at $t = 60$, the error states e_1, e_2 and e_3 diminish to zero as shown in Figure (3), for e_1 showing that the two systems are synchronized.

The 4th order Rung-Kutta algorithm was also employed to solve the backstepping control system (8) with the same conditions as in the active control above. When the controllers are switched off one obtains the same chaotically oscillating error states as in Figure (2). Figure (4) Shows the error states when the controllers switched on at $t = 60$. Again the errors diminish to zero showing that the state variables are controlled to take the desired values. Figure (5). Compares the transient error states of the two controllers. It can be observed that the active controllers reduce the errors to zero much earlier than the recursive backstepping controllers. However, the flexibility in the choice of control laws for the recursive backstepping controllers gives room for further improvement in its performance.

5.0 Conclusion

Active controllers and recursive backstepping controllers were designed for a third order chaotic system. The performance of these controllers in the control of the dynamics of the chaotic system were investigated numerically and found to be effective. Comparison of their transient performances shows that the rate of convergence of error is faster for the active controller than for the recursive backstepping controller. However, the flexibility in the choice of the control laws for recursive backstepping design gives room for further improvement in its performance and enables it to achieve the goals of stabilization and tracking.

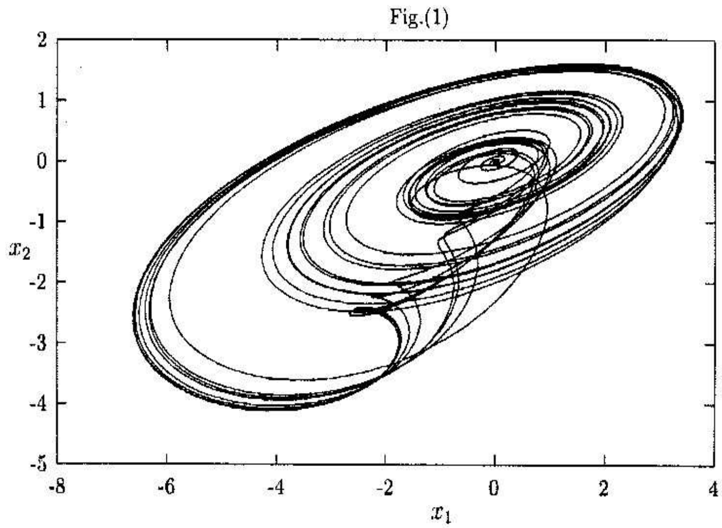

Figure 1: Chaotic attractor in phase space of the drive system (1)

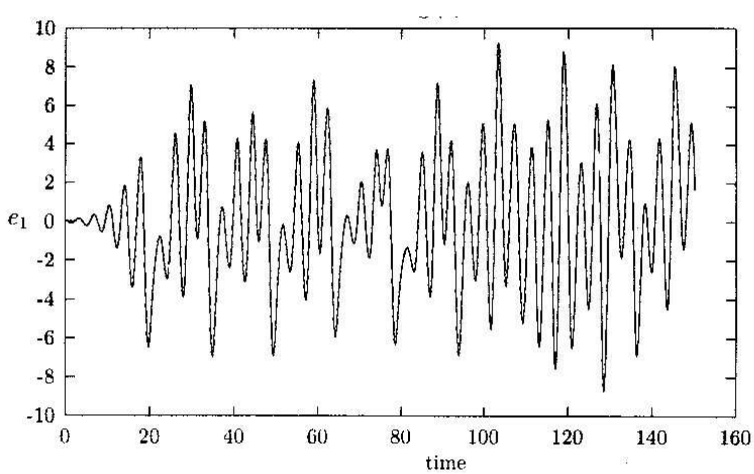

Figure 2: Time evolution of the error state (e_1) in the absence of control.

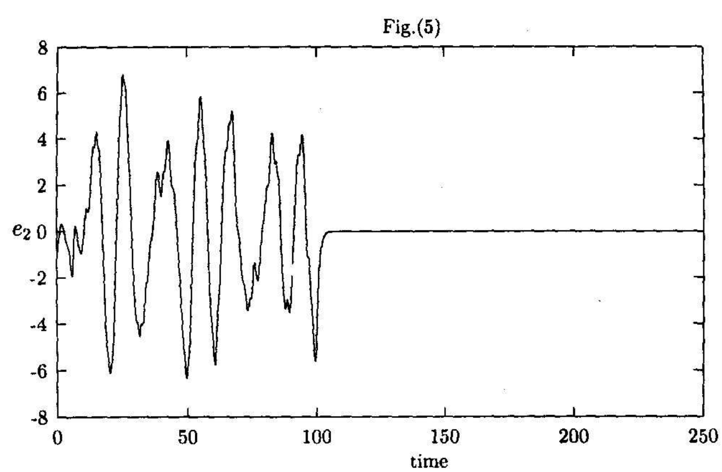

Figure 3: Time evolution of the error stare (e_1) with the active controllers $(u_i, i = 1,2,3)$ activated at $t = 60$

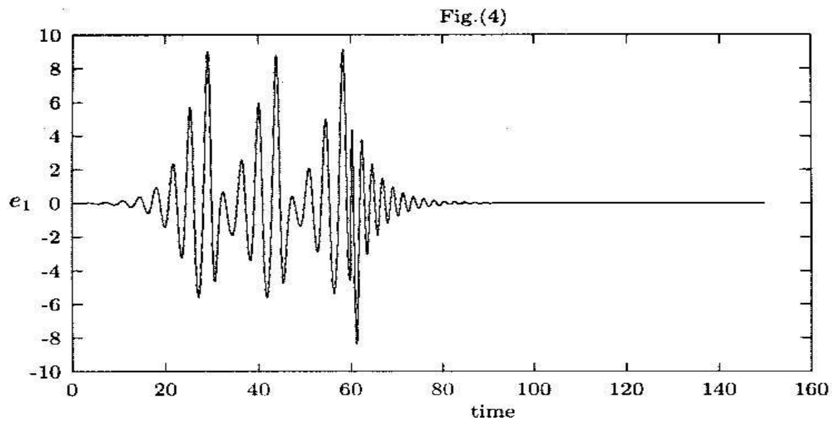

Figure 4: Time evolution of the error state (e_1) with the backstopping controllers $(u_i, i = 1,2)$ activated at $t = 60$.

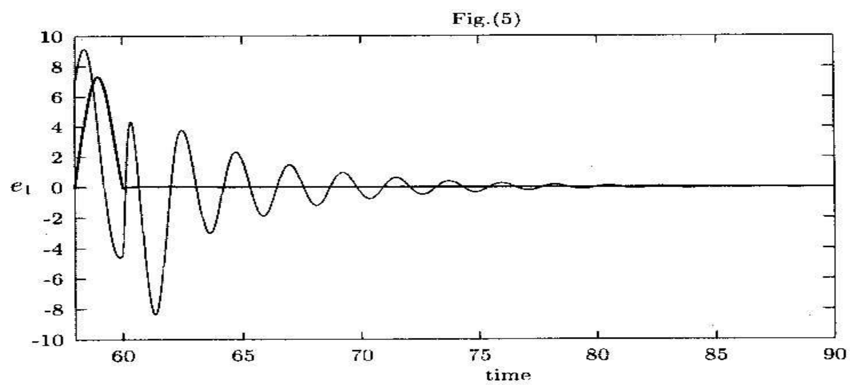

Figure 5: Time Evolution Of The Transient Error State (e_1) With Active Controllers (Thick Line) And Backstopping Controllers (Thin Line) Activated at $t = 60$

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