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Remarks on thermal explosions in the early evolution of the earth.

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1.0 Introduction

Earth's origin and the formation of its shells are fundamental problems of natural sciences. Owing to the joint efforts of space physicists and space chemists, planetologists and geophysicists the main physicochemical processes have been studied, computer models of planet formation from smaller bodies of asteroid dimensions have been developed and the times of planet formation supported by isotope data have been calculated. It is evident that during the formation of the main structural units of the Earth – its core and mantle – there was a considerable energy generation due to gravitational differentiation (equivalent heat by 2500 ^{0}C) [3].

The presence of fluid core of an electrically conducting fluid permits the interaction of the fluid flow and the magnetic lines of force to produce an electromotive force (e.m.f) which helps the magnetic field to regenerate itself. The subject of the study of the processes of regeneration of a magnetic field is known as the dynamo theory [2].

During the gravitational differentiation (GD) in the large material volume in the Earth's gravitational field the generated potential energy becomes heat due to viscous dissipation [3].

In this paper we study the time evolution of the Earth. Of course, the planetary scales and characteristic geologic times of the thermal processes in the interior differ from the corresponding characteristics of the classical thermal explosion, but, and in essence and form, they are analogous to the thermally activated processes [3].

2.0 The mathematical model

We start with the usual Navier – Stokes energy equation

$$\rho\, c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \mu\left(\frac{\Delta u}{\Delta h}\right)^2 + \in_r , \qquad (2.1)$$

where

T	*is temperature*	μ	*is viscosity*
u	*is velocity*	c_p	*is specific heat*
$2h$	*is the thickness of the flat layer*	ρ	*is density*
z	*is distance*	$\in_r$	*is energy due to radioactive sources.*
k	*is thermal conductivity*		

Following Vityazev [3], we consider a certain volume V of a matrix of viscosity μ and density ρ_2 which contains inclusions of radius a and density $\rho_1 = \rho_2 + \Delta\rho$ with

$$\mu = \mu_0\, e^{E/RT} \qquad (2.2)$$

$$k = \lambda + \lambda d \qquad (2.3)$$

$$\lambda_d = \lambda\, pe^{\frac{1}{n}}\, e^{\frac{E}{nRT}},\ \ 1 \le n \le 4 \qquad (2.4)$$

$$u = (1-2.5c)\, 2\,\Delta p\, ga^2 \big/ q\mu \qquad (2.5)$$

$$\rho = \rho_0\left(1-\alpha(T-T_d)-\beta(c_1-c_0)\right) \qquad (2.6)$$

where

P_e	*is the Pe'clet number*	c	*is the fraction of the volume accupied by inclusions.*

E	*is the activation energy*	g	*is the acceleration due to gravity*
R	*is the universal gas constant*	c_1	*is the concentration of the light material*
λ_d	*is the thermal conductivity due convective flows*	α	*is the coefficient of volume expansion*
λ	*is the ordinary thermal conductivity*	β	*is the coefficient of compositional expansion*

3.0 Method of solution

Writing (and assuming $\rho = \rho_0$)

$$\theta = \frac{E}{RT_d^2}\left(T - T_d\right), \quad x = \frac{z}{h}, \quad \tau = \frac{\lambda t}{\rho c_p h^2},$$

we obtain

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial x}\left(\left(1 + p_e^{\frac{1}{n}} e^{\frac{\theta}{n(1+\in\theta)}}\right)\frac{\partial \theta}{\partial x}\right) + \Gamma_d\, e^{\frac{\theta}{1+\in\theta}} + \Gamma_r, \tag{3.1}$$

where $\Gamma_d = \frac{E}{RT_d^2}\frac{(1-2.5c)^2\, 4(\Delta\rho)^2\, g^2\, a^4}{81\lambda\, \mu_0}$, $\Gamma_r = \frac{h^2 E}{\lambda\, RT_{d^2}}\in_r$, $\in = \frac{RT_d}{E}$

In the limit $\varepsilon \to 0$, $$\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial x}\left(1 + p_e^{\frac{1}{n}} e^{\frac{\theta}{n}}\right)\frac{\partial \theta}{\partial x} + \Gamma_d\, e^{\theta} + \Gamma_r \tag{3.2}$$

We assume as initial and boundary conditions

$$\theta(-1,) = \theta(1,\tau) = \theta(x,0) = 0 \tag{3.3}$$

4.0 Thermal runaway time

We consider two cases

(i) $\quad p_e\, e^{\frac{\theta}{n}} >> 1 \qquad (ii) \quad p_e\, e^{\frac{\theta}{n}} << 1$

Let $w = 1 - e^{-\theta}$, then for .(i) $\quad \frac{\partial w}{\partial \tau} - p_e^{\frac{1}{n}}\frac{\partial^2 w}{\partial x^2} \geq \Gamma_d$, $\quad (ii) \quad \frac{\partial w}{\partial \tau} - \frac{\partial^2 w}{\partial x^2} \geq \Gamma_d$

Then by [1], theorem 4.5, there exists a t_0 such that

(i) $\quad w(0, t_0) \to 1 \;\; and\; \theta \to \infty\infty \qquad (ii) \quad w\left(0, \; t_0 / p_e^{1/n}\right) \to 1 \;\; and \;\; \theta \to \infty$

In particular, for some $\alpha > 0$, $\frac{\Gamma_d\, t_0}{2\sqrt{\pi\,(t_0 + \alpha)}} = 1$ for case (i) and $\frac{\Gamma_d\, t_0}{2\sqrt{\pi\,(p_e\, t_0 + \alpha)}} = 1$ for case (ii) when $n = 1$.

Hence $\frac{d\, t_0}{d\; p_e} > 0$ and time of explosion increases as p_e increases.

5.0 Conclusion

We have ignored the dependence of density on temperature in preexplosion heating. In reality as $\theta \to \infty$, $\rho \to 0$ and equation (7) is no longer valid.
Thus in reality θ never becomes too large.

References

[1] R. O. Ayeni (1978): Thermal Runaway Ph.D Thesis Cornell University U.S.A

[2] R.O. Ayeni (1999): Earth's magnetic field and compositional theory National Mathematical Centre Lecture Series

[3] A.V. Vityasev (2004): Thermal explosions in the early evolution of the Earth, Combustion, explosion and shock waves vol 40 No 6 pp 720 – 723.