Journal of the Nigerian Association of Mathematical Physics
Volume 10 (November 2006), **59 - 68**
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Analysis of Stokes waves theory as a diffusion problem

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Abstract

This mathematical model concerns the theory of Stokes waves. These wave types belong to the class of ocean surface waves found in deep and intermediate waters. In this consideration, the fifth order expansion was obtained using Korteweg de Vries equation with diffusion term. This study suggests that the phase velocity grows with increase in wave steepness whilst the group velocity shows the opposite tendency .The effect of diffusion introduced through depth distribution is obvious as the solutions apparently depend strongly on the water depth in inverse form. Interestingly, this analysis strongly suggests that the peak for potential energy lies between second and third order solutions while that of kinetic energy attains the peak at second and then becomes fairly stable. High seismic response associated with sea-bed motion corresponding to second order solution strongly support the result. However, the effect of additional terms on the wave profile appears somewhat insignificant. The wave profile of first order to fifth order in this consideration remains unchanged as expected.

1.0 Introduction

Stokes expansion of wave profile usually grows numerically with increasing order. However, Longuet-Higgins & Fenton (1974) considered the theory using viscous fluid model. Instead, the diffusing term is introduced into the equation governing the wave profile η(x,t) (Whitham,1974) with rather more generalization (Okeke,1997). The consideration will apply to the sea waves in the intermediate water range $\left(0.0015 \le {}^{h_0}\!/_{gT^2} \le 0.055\right)$, the problem being transformed to time – domain; g is the acceleration due to gravity and T is the period of dominant wave mode, h_o is a measure of water depth. In this range, both finite amplitude short and long waves interact freely. One of the main objective of this investigation is thus, to determine to what extent viscous theory of Longuet-Higgins compares with diffusion effects being proposed in terms of wave energy. In this consideration and following Whitham (1974), η(*x, t*) satisfies the nonlinear equation.

$$\eta_t + c_0\left(1 + \frac{3}{2}\frac{\eta}{h_0}\right)\eta_x + \gamma\eta_{xxx} = 0 \qquad (1.1)$$

γ is the dispersion coefficient defined by $\gamma = \frac{c_0 h_0^2}{6}, c_0 = \sqrt{\frac{g}{k}}$ in deep water and $c_0 = \sqrt{gh}$ g = acceleration due to gravity h_0 = water depth measured when the water is undisturbed. If $\eta = \xi(\theta)h$, $\theta = kx - \omega\tau$, $\eta = \xi(kx - \omega t)h$, $\frac{\partial}{\partial x} = \frac{\partial}{\partial\theta}\frac{\partial\theta}{\partial x} = k\frac{\partial}{\partial\theta}, \frac{\partial}{\partial t} = \frac{\partial}{\partial\theta}\frac{\partial\theta}{\partial t} = -\omega\frac{\partial}{\partial\theta}$.

Thus (1.1) gives

$$(kc_0 - w)\xi^1 + \frac{3}{2}kc_0\xi\xi^1 + \gamma k^3\xi^{111} = 0 \qquad (1.2)$$

where $k = \frac{2\pi}{L}, L =$ wavelength. Take $\varepsilon = ka$ as the wave steepness, a being the wave amplitude, and expand in Fourier series of the form $\eta(x,t)$

$$\eta(x,t) = \varepsilon\xi_1(\theta) + \varepsilon^2\xi_2(\theta) + \varepsilon^3\xi_3(\theta) + \varepsilon^4\xi_4(\theta) + \varepsilon^5\xi_5(\theta) + o(\varepsilon^6) \quad (1.3)$$

(1, 2) is if the form,

$$\sum_{i=1}^{n}(kc_0 - w)\xi_i^1\varepsilon^i + \frac{3}{2}kc_0\sum_{i=1}^{n}\varepsilon^i\xi_i\sum_{s=1}^{n}\varepsilon^s\xi_s^1 + \gamma k^3\sum_{i=1}^{n}\varepsilon^i\xi_i^{111} = 0 \quad (1.4)$$

$\xi = \xi(\theta)$ only, $\xi^1 = \dfrac{d\xi}{d\theta}$

2.0 Derivation of equation with diffusion term

From (1.1)

$$\eta_t + c_0\left(1 + \frac{3}{2}\frac{\eta}{h_0}\right)\eta_x + \gamma\eta_{xxx} = 0, \quad \frac{\eta}{h_0} = \xi(\theta) \quad (2.1)$$

$\theta = kx - wt$

$$(h_0\xi)_t + c_0\left(1 + \frac{3}{2}\frac{\eta}{h_0}\right)(h_0\xi)_x + \gamma(h_0\xi_{xxx}) = 0 \quad (2.2)$$

$$\xi_t + c_0\left(1 + \frac{3}{2}\xi\right)\xi_x + \gamma\xi_{xxx} = 0 \quad (2.3)$$

Let $\xi = \varepsilon\xi_1(\theta) + \varepsilon^2\xi_2(\theta) + \varepsilon^3\xi_3(\theta) + \varepsilon^4\xi_4(\theta) + \varepsilon^5\xi_5(\theta)$ (2.4)

to fifth order. From (2.4)

$$\left(\varepsilon\xi_1(\theta)+\varepsilon^2\xi_2(\theta)+\varepsilon^3\xi_3(\theta)+\varepsilon^4\xi_4(\theta)+\varepsilon^5\xi_5(\theta)\right)_t + c_0\left(1+\frac{3}{2}\left[\begin{array}{r}\varepsilon\xi_1(\theta)+\varepsilon^2\xi_2(\theta)+\varepsilon^3\xi_3(\theta)\\ +\varepsilon^4\xi_4(\theta)+\varepsilon^5\xi_5(\theta)\end{array}\right]\right)\left(\begin{array}{l}\varepsilon\xi_1(\theta)+\varepsilon^2\xi_2(\theta)+\varepsilon^3\xi_3(\theta)\\ +\varepsilon^4\xi_4(\theta)+\varepsilon^5\xi_5(\theta)\end{array}\right)_x$$
$$+\gamma\left(\varepsilon\xi_1(\theta)+\varepsilon^2\xi_2(\theta)+\varepsilon^3\xi_3(\theta)+\varepsilon^4\xi_4(\theta)+\varepsilon^5\xi_5(\theta)\right)_{xxx} = 0 \quad (2.5)$$

$$\left(-\varepsilon\omega\xi_1^1 - \varepsilon^2\omega\xi_2^1 - \varepsilon^3\omega\xi_3^1 - \varepsilon^4\omega\xi_4^1 - \varepsilon^5\omega\xi_5^1\right) + c_0\left(1+\frac{3}{2}\varepsilon\xi_1+\frac{3}{2}\varepsilon^2\xi_2+\frac{3}{2}\varepsilon^3\xi_3+\frac{3}{2}\varepsilon^4\xi_4+\frac{3}{2}\varepsilon^5\xi_5\right)\left(\varepsilon k\xi_1^1+\varepsilon^2 k\xi_2^1+\varepsilon^3 k\xi_3^1+\varepsilon^4 k\xi_4^1+\varepsilon^5 k\xi_5^1\right)$$
$$+\gamma\left(\varepsilon k^3\xi_1^{111}+\varepsilon^2k^3\xi_2^{111}+\varepsilon^3k^3\xi_3^{111}+\varepsilon^4k^3\xi_4^{111}+\varepsilon^5k^3\xi_5^{111}\right) = 0 \quad (2.6)$$

$$\left(-\varepsilon\varepsilon\omega_1^1 - \varepsilon^2\omega\xi_2^1 - \varepsilon^3\omega\xi_3^1 - \varepsilon^4\omega\xi_4^1 - \varepsilon^5\omega\xi_5^1\right) + c_\circ\Big(k\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)+\tfrac{3}{2}\varepsilon\xi_1k\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)$$
$$+\tfrac{3}{2}\varepsilon^2\xi_2k\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)+\tfrac{3}{2}\varepsilon^3\xi_3k\Big(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1$$
$$+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\Big)+\tfrac{3}{2}\varepsilon^4\xi_4k\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)+\tfrac{3}{2}\varepsilon^5\xi_5k\Big(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1$$
$$+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\Big)\Big)+\gamma\left(\varepsilon k^3\xi_1^{111}+\varepsilon^2k^3\xi_2^{111}+\varepsilon^3k^3\xi_3^{111}+\varepsilon^4k^3\xi_4^{111}+\varepsilon^5k^3\xi_5^{111}\right) = 0 \quad (2.7)$$

Equating coefficients, we have

$$\varepsilon : (\omega - c_0k)\xi_1^1 - \gamma k^3\xi_1^{111} = 0 \quad (2.8)$$

$$\varepsilon^2 : (\omega - c_0k)\xi_2^1 - \gamma k^3\xi_2^{111} - \frac{3}{2}c_0k\xi_1\xi_1^1 = 0 \quad (2.9)$$

$$\varepsilon^3 : (\omega - c_0k)\xi_3^1 - \gamma k^3\xi_3^{111} - \frac{3}{2}c_0k\left(\xi_1\xi_2^1 + \xi_2\xi_1^1\right) = 0 \quad (2.10)$$

$$\varepsilon^4 : (\omega - c_0k)\xi_4^1 - \gamma k^3\xi_4^{111} - \frac{3}{2}c_0k\left(\xi_1\xi_3^1 + \xi_2\xi_2^1 + \xi_3\xi_1^1\right) = 0 \quad (2.11)$$

$$\varepsilon^5 : (\omega - c_0k)\xi_5^1 - \gamma k^3\xi_5^{111} - \frac{3}{2}c_0k\left(\xi_2\xi_3^1 + \xi_3\xi_2^1 + \xi_1\xi_4^1 + \xi_4\xi_1^1\right) = 0 \quad (2.12)$$

that is,.

$$(\omega - c_0k)\xi_1^1 - \gamma k^3\xi_1^{111} = 0$$

$$(\omega - c_0k)\xi_2^1 - \gamma k^3\xi_2^{111} = \frac{3}{2}c_0k\xi_1\xi_1^1 \quad (2.13)$$

$$(\omega - c_0k)\xi_3^1 - \gamma k^3\xi_3^{111} = \frac{3}{2}c_0k\left(\xi_1\xi_2\right)^1 \quad (2.14)$$

$$(\omega - c_0k)\xi_4^1 - \gamma k^3\xi_4^{111} = \frac{3}{2}c_0k\left[\left(\xi_1\xi_3\right)^1 + \xi_2\xi_2^1\right] \quad (2.15)$$

$$(\omega - c_0k)\xi_5^1 - \gamma k^3\xi_5^{111} = \frac{3}{2}c_0k\left[\left(\xi_1\xi_4\right)^1 + \left(\xi_2\xi_3\right)^1\right] \quad (2.16)$$

From (2.7) $\left(\varepsilon\omega\xi_1^1+\varepsilon^2\omega\xi_2^1-\varepsilon^3\omega\xi_3^1-\varepsilon^4\omega\xi_4^1-\varepsilon^5\omega\xi_5^1\right)-c_0k\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)$

$$-\gamma k^3\left(\varepsilon\xi_1^{111}+\varepsilon^2\xi_2^{111}+\varepsilon^3\xi_3^{111}+\varepsilon^4\xi_4^{111}+\varepsilon^5\xi_5^{111}\right)-\frac{3}{2}c_0\xi_1k\varepsilon\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)$$
$$-\frac{3}{2}c_0\xi_2k\varepsilon^2\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)-\frac{3}{2}c_0\xi_3k\varepsilon^3\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right) \quad (2.17)$$
$$-\frac{3}{2}c_0\xi_4k\varepsilon^4\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)-\frac{3}{2}c_0\xi_5k\varepsilon^5\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)=0$$

Putting $\omega(k)=\sum_{s=0}^{6}\omega_s\varepsilon^s$. Thus,

$$\omega(k)=\omega_0+\omega_1\varepsilon^1+\omega_2\varepsilon^2+\omega_3\varepsilon^3+\omega_4\varepsilon^4+\omega_5\varepsilon^5+\cdots \quad (2.18)$$

Using (2.17) above, we have

$$\varepsilon\left(\omega_0+\omega_1\varepsilon^1+\omega_2\varepsilon^2+\omega_3\varepsilon^3+\omega_4\varepsilon^4+\omega_5\varepsilon^5+\ldots\right)\xi_1^1+\varepsilon^2\left(\omega_0+\omega_1\varepsilon^1+\omega_2\varepsilon^2+\omega_3\varepsilon^3+\omega_4\varepsilon^4+\omega_5\varepsilon^5+\ldots\right)\xi_2^1$$
$$+\varepsilon^3\left(\omega_0+\omega_1\varepsilon^1+\omega_2\varepsilon^2+\omega_3\varepsilon^3+\omega_4\varepsilon^4+\omega_5\varepsilon^5+\ldots\right)\xi_3^1+\varepsilon^4\left(\omega_0+\omega_1\varepsilon^1+\omega_2\varepsilon^2+\omega_3\varepsilon^3+\omega_4\varepsilon^4+\omega_5\varepsilon^5+\ldots\right)\xi_4^1$$
$$+\varepsilon^5\left(\omega_0+\omega_1\varepsilon^1+\omega_2\varepsilon^2+\omega_3\varepsilon^3+\omega_4\varepsilon^4+\omega_5\varepsilon^5+\ldots\right)\xi_5^1-c_0k\left(\varepsilon\xi_1^1+\varepsilon^2\xi_2^1+\varepsilon^3\xi_3^1+\varepsilon^4\xi_4^1+\varepsilon^5\xi_5^1\right)$$
$$-\gamma k^3\left(\varepsilon\xi_1^{111}+\varepsilon^2\xi_2^{111}+\varepsilon^3\xi_3^{111}+\varepsilon^4\xi_4^{111}+\varepsilon^5\xi_5^{111}\right)-\frac{3}{2}c_0\xi_1k\left(\varepsilon^2\xi_1^1+\varepsilon^3\xi_2^1+\varepsilon^4\xi_3^1+\varepsilon^5\xi_4^1+\varepsilon^6\xi_5^1\right)$$
$$-\frac{3}{2}c_0\xi_2k\left(\varepsilon^3\xi_1^1+\varepsilon^4\xi_2^1+\varepsilon^5\xi_3^1+\varepsilon^6\xi_4^1+\varepsilon^7\xi_5^1\right)-\frac{3}{2}c_0\xi_3k\left(\varepsilon^4\xi_1^1+\varepsilon^5\xi_2^1+\varepsilon^6\xi_3^1+\varepsilon^7\xi_4^1+\varepsilon^8\xi_5^1\right) \quad (2.19)$$
$$-\frac{3}{2}c_0\xi_4k\left(\varepsilon^5\xi_1^1+\varepsilon^6\xi_2^1+\varepsilon^7\xi_3^1+\varepsilon^8\xi_4^1+\varepsilon^9\xi_5^1\right)-\frac{3}{2}c_0\xi_5k\left(\varepsilon^6\xi_1^1+\varepsilon^7\xi_2^1+\varepsilon^8\xi_3^1+\varepsilon^9\xi_4^1+\varepsilon^{10}\xi_5^1\right)=0$$

Equating coefficients of ε, we have $\xi:\quad(\omega_0\text{-}c_0k)\xi_1^1-\gamma k^3\xi_1^{111}=0$ (2.20)

$$\xi^2:\omega_1\xi_1^1+\omega_0\xi_2^1-c_0k\xi_2^1-\gamma k^3\xi_2^{111}-\frac{3}{2}c_0k\xi_1\xi_1^1=0$$
$$\Rightarrow(\omega_0\text{-}c_0k)\xi_2^1-\gamma k^3\xi_2^{111}=\frac{3}{2}c_0k\xi_1\xi_1^1 \quad (2.21)$$

$$\xi^3:\omega_2\xi_1^1+\omega_1\xi_2^1+\omega_0\xi_3^1-c_0k\xi_3^1-\gamma k^3\xi_3^{111}-\frac{3}{2}c_0k\left(\xi_1\xi_2^1+\xi_2\xi_1^1\right)=0$$
$$\Rightarrow\quad(\omega_0-c_0k)\xi_3^1-\gamma k^3\xi_3^{111}=\frac{3}{2}c_0k(\xi_1\xi_2)^1-\omega_2\xi_1^1-\omega_1\xi_2^1 \quad (2.22)$$

$$\xi^4:\omega_3\xi_1^1+\omega_2\xi_2^1+\omega_1\xi_3^1+\omega_0\xi_4^1-c_0k\xi_4^1-\gamma k^3\xi_4^{111}-\frac{3}{2}c_0k\left(\xi_2\xi_2^1+\xi_1\xi_3^1+\xi_3\xi_1^1\right)=0$$
$$\Rightarrow(\omega_0-c_0k)\xi_4^1-\gamma k^3\xi_4^{111}=\frac{3}{2}c_0k\left[(\xi_1\xi_3)^1+\xi_2\xi_2^1\right]-\omega_3\xi_1^1-\omega_2\xi_2^1-\omega_1\xi_3^1 \quad (2.23)$$

$$\xi^5:\omega_4\xi_1^1+\omega_3\xi_2^1+\omega_2\xi_3^1+\omega_1\xi_4^1+\omega_0\xi_5^1-c_0k\xi_5^1-\gamma k^3\xi_5^{111}-\frac{3}{2}c_0k\left(\xi_2\xi_3^1+\xi_3\xi_2^1+\xi_1\xi_4^1+\xi_4\xi_1^1\right)=0$$
$$\Rightarrow(\omega_0\text{-}c_0k)\xi_5^1-\gamma k^3\xi_5^{111}=\frac{3}{2}c_0k\left[(\xi_1\xi_4)^1+(\xi_2\xi_3)^1\right]-\omega_4\xi_1^1-\omega_3\xi_2^1-\omega_2\xi_3^1-\omega_1\xi_4^1 \quad (2.24)$$

Let $\xi_1=\cos\theta,\ \xi_1^1=-\sin\theta,\ \xi_1^{11}=-\cos\theta,\ \xi_1^{111}=\sin\theta$. Substituting in (2.20)

$$-(\omega_0\text{-}c_0k)\sin\theta=\gamma k^3\sin\theta,\ \ c_0k\text{-}\omega_0=\gamma k^3,\ \omega_0=c_0k-\gamma k^3 \quad (2.25)$$

(2.25) is the dispersion equation at the lowest order. From (2.21)

$$-\gamma k^3\xi_2^1-\gamma k^3\xi_2^{111}=-\frac{3}{2}c_0k\cos\theta\sin\theta+\omega_1\sin\theta \quad (2.26)$$

$$\gamma k^3\left(D\xi_2+D^3\xi_2\right)=\frac{3}{4}c_0k\sin2\theta+\omega_1\sin\theta,\ D=\frac{d}{d\theta},\ \left(D+D^3\right)\xi_2=\frac{3c_0k}{4\gamma k^3}\sin2\theta+\frac{\omega_1\sin\theta}{\gamma k^3}$$

$$\xi_2=\frac{3c_0k}{4\gamma k^3}\frac{\sin2\theta}{(D+D^3)}+\frac{\omega_1\sin\theta}{(D+D^3)\gamma k^3}=\frac{3c_0k}{4\gamma k^3}\frac{D\sin2\theta}{(D^2+D^4)}$$
$$\omega_1\sin\theta=0 \quad (2.27)$$

as the secular term is unbounded when operated on by D.

$$\omega_1 = 0,\ \xi_2 = \frac{3c_0 k}{2\gamma k^3}\frac{\cos 2\theta}{(-2)^2 + (-2^2)^2} = \frac{3c_0 k}{24\gamma k^3}\cos 2\theta,\ \xi_2 = \frac{c_0}{8\gamma k^2}\cos 2\theta$$

From (2.22)

$$(\omega_0 - c_0 k)\xi_3^1 - \gamma k^3 \xi_3^{111} = \frac{3}{2}c_0 k(\xi_1\xi_2)^1 - \omega_2\xi_1^1 - \omega_1\xi_2^1 - \gamma k^3(D + D^3)\xi_3 = \frac{3c_0 k}{2}\left[(\cos\theta)\left(\frac{c_0}{8\gamma k^2}\cos 2\theta\right)\right]^1$$

$$-\omega_2(-\sin\theta) = \frac{3c_0^2}{16\gamma k}(\cos\theta\cos 2\theta)^1 + \omega_2\sin\theta \qquad (2.29)$$

$$= \frac{3c_0^2}{16\gamma k}[\cos\theta(-2\sin 2\theta) + \cos 2\theta(-\sin\theta)] + \omega_2\sin\theta = -\frac{3c_0^2}{16\gamma k}[2\sin 2\theta\cos\theta + \cos 2\theta\sin\theta] + \omega_2\sin\theta$$

But $2\sin 2\theta\cos\theta = \sin 3\theta + \sin\theta$, $\cos 2\theta\sin\theta = \frac{1}{2}\sin 3\theta - \frac{1}{2}\sin\theta$. Substituting, we have

$$= -\frac{3c_0^2}{16\gamma k}\left[\sin 3\theta + \sin\theta + \frac{1}{2}\sin 3\theta - \frac{1}{2}\sin\theta\right] + \omega_2\sin\theta = -\frac{3c_0^2}{16\gamma k}\left[\frac{3}{2}\sin 3\theta + \frac{1}{2}\sin\theta\right] + \omega_2\sin\theta$$

$$-\gamma k^3(D + D^3)\xi_3 = -\frac{3c_0^2}{32\gamma k}[3\sin 3\theta + \sin\theta] + \omega_2\sin\theta$$

$$\gamma k^3(D + D^3)\xi_3 = \frac{3c_0^2}{32\gamma k}[3\sin 3\theta + \sin\theta] - \omega_2\sin\theta$$

$$\xi_3 = \frac{3c_0^2}{32\gamma^2 k^4}\frac{(3\sin 3\theta + \sin\theta)}{(D + D^3)} - \frac{1}{\gamma k^3}\omega_2\sin\theta$$

$$\xi_3 = \frac{3c_0^2}{32\gamma^2 k^4}\frac{D(3\sin 3\theta)}{(D^2 + D^4)} + \frac{3c_0^2\sin\theta}{32\gamma^2 k^4} - \frac{1}{\gamma k^3}\omega_2\sin\theta \qquad (2.30)$$

To eliminate the resonant term as $\sin\theta$ terms become unbounded when operated upon, and let

$$\frac{3c_0^2\sin\theta}{32\gamma^2 k^4} - \frac{1}{\gamma k^3}\omega_2\sin\theta = 0 \qquad (2.31)$$

$$\xi_3 = \frac{3c_0^2}{32\gamma^2 k^4}\frac{D(3\sin 3\theta)}{(D^2 + D^4)}$$

$$\xi_3 = \frac{3c_0^2}{256\gamma^2 k^4}\cos 3\theta \qquad (2.32)$$

From 2.31, $\frac{\omega_2}{\gamma k^3} = \frac{3c_0^2}{32\gamma^2 k^4}$

$$\omega_2 = \frac{3c_0^2}{32\gamma k} \approx 0.09\ \frac{c_0^2}{\gamma k} \qquad (2.33)$$

From (2.23)

$$(\omega_0 - c_0 k)\xi_4^1 - \gamma k^3\xi_4^{111} = \frac{3}{2}c_0 k\left[(\xi_1\xi_3)^1 + \xi_2\xi_2^1\right] - \omega_3\xi_1^1 - \omega_2\xi_2^1 - \omega_1\xi_3^1 - \gamma k^3(D + D^3)\xi_4 =$$

$$\frac{3c_0 k}{2}\left[\left((\cos\theta)\frac{3c_0^2}{256\gamma^2 k^4}\cos 3\theta\right)^1 + \left(\frac{c_0}{8\gamma k^2}\cos 2\theta\right)\left(\frac{c_0}{8\gamma k^2}\cos 2\theta\right)^1\right] - \omega_2\left(\frac{c_0}{8\gamma k^2}\cos 2\theta\right)^1 - \omega_3(\cos\theta)^1$$

$$= \frac{3c_0 k}{2}\left[\frac{3c_0^2}{256\gamma^2 k^4}(\cos\theta\cos 3\theta)^1 + \frac{c_0^2}{64\gamma^2 k^4}(\cos 2\theta)(\cos 2\theta)^1\right] + \frac{\omega_2 c_0}{4\gamma k^2}\sin 2\theta + \omega_3\sin\theta \qquad (2.34)$$

$$= \frac{3c_0^3}{128\gamma^2 k^4}\left[\frac{3}{4}(\cos\theta(-3\sin 3\theta) + \cos 3\theta(-\sin\theta)) + (\cos 2\theta)(-2\sin 2\theta)\right] + \frac{\omega_2 c_0}{4\gamma k^2}\sin 2\theta + \omega_3\sin\theta$$

$$= -\frac{3c_0^3}{128\gamma^2 k^4}\left[\frac{3}{4}(3\sin 3\theta\cos\theta + \cos 3\theta\sin\theta) + 2\sin 2\theta\cos 2\theta\right] + \frac{\omega_2 c_0}{4\gamma k^2}\sin 2\theta + \omega_3\sin\theta \qquad (2.35)$$

But $2\sin 2\theta\cos 2\theta = \sin 4\theta$, $3\sin 3\theta\cos\theta + \cos 3\theta\sin\theta = 2\sin 3\theta\cos\theta + \sin 3\theta\cos\theta + \cos 3\theta\sin\theta$

$= \sin 4\theta + \sin 2\theta + \sin 4\theta = 2\sin 4\theta + \sin 2\theta$, substituting in (2.35),

$$= -\frac{3c_0^{\;3}}{128\gamma^2 k^4}\left[\frac{3}{4}\left(2\sin 4\theta + \sin 2\theta\right) + \sin 4\theta\right] + \frac{\omega_2 c_0}{4\gamma k^2}\sin 2\theta + \omega_3 \sin\theta \qquad (2.36)$$

$$= -\frac{3c_0^{\;3}}{128\gamma^2 k^4}\left[\frac{3}{2}\sin 4\theta + \sin 4\theta + \frac{3}{4}\sin 2\theta\right] + \frac{\omega_2 c_0}{4\gamma k^2}\sin 2\theta + \omega_3 \sin\theta$$

$$-\gamma k^3\left(D + D^3\right)\xi_4 = -\frac{3c_0^{\;3}}{128\gamma^2 k^4}\left[\frac{5}{2}\sin 4\theta + \frac{3}{4}\sin 2\theta\right] + \frac{\omega_2 c_0}{4\gamma k^2}\sin 2\theta + \omega_3 \sin\theta$$

$$\gamma k^3\left(D + D^3\right)\xi_4 = \frac{3c_0^{\;3}}{128\gamma^2 k^4}\left[\frac{5}{2}\sin 4\theta + \frac{3}{4}\sin 2\theta\right] - \frac{\omega_2 c_0}{4\gamma k^2}\sin 2\theta - \omega_3 \sin\theta$$

$$\xi_4 = \frac{3c_0^{\;3}}{128\gamma^3 k^6 (D + D^3)}\left[\frac{5}{2}\sin 4\theta + \frac{3}{4}\sin 2\theta\right] - \frac{\omega_2 c_0}{4\gamma^2 k^5 (D + D^3)}\sin 2\theta - \frac{1}{\gamma k^3}\omega_3 \sin\theta$$

The term containing ω_3 is eliminated by setting $\omega_3 = 0$

$$\xi_4 = \frac{3c_0^{\;3}D}{128\gamma^3 k^6 (D^2 + D^4)}\left[\frac{5}{2}\sin 4\theta\right] + \frac{3c_0^{\;3}D}{128\gamma^3 k^6 (D^2 + D^4)}\left[\frac{3}{4}\sin 2\theta\right] - \frac{\omega_2 c_0 D[\sin 2\theta]}{4\gamma^2 k^5 (D^2 + D^4)} \qquad (2.37)$$

$$= \frac{1}{1024}\frac{c_0^{\;3}}{\gamma^3 k^6}\cos 4\theta + \frac{9}{3072}\frac{c_0^{\;3}}{\gamma^3 k^6}\cos 2\theta - \frac{1}{24}\frac{c_0}{\gamma^2 k^5}\omega_2\cos 2\theta$$

Putting $\frac{9}{3072}\frac{c_0^{\;3}}{\gamma^3 k^6}\cos 2\theta - \frac{1}{24}\frac{c_0}{\gamma^2 k^5}\omega_2\cos 2\theta = 0$ (2.38)

$$\xi_4 = \frac{1}{1024}\frac{c_0^{\;3}}{\gamma^3 k^6}\cos 4\theta \qquad (2.39)$$

From (2.24) $(\omega_0 - c_0 k)\xi_5^1 - \gamma k^3 \xi_5^{111} = \frac{3}{2}c_0 k\left[(\xi_1\xi_4)^1 + (\xi_2\xi_3)^1\right] - \omega_4\xi_1^1 - \omega_3\xi_2^1 - \omega_2\xi_3^1 - \omega_1\xi_4^1$

$$-\gamma k^3\left(D + D^3\right)\xi_5 = \frac{3c_0 k}{2}\left[\left((\cos\theta)\left(\frac{c_0^{\;3}}{1024\gamma^3 k^6}\cos 4\theta\right)\right)^1 + \left(\left(\frac{c_0}{8\gamma k^2}\cos 2\theta\right)\left(\frac{3c_0^{\;2}}{256\gamma^2 k^4}\cos 3\theta\right)\right)^1\right] \qquad (2.40)$$

$$-\omega_4(\cos\theta)^1 - \omega_3\left(\frac{c_0}{8\gamma k^2}\cos 2\theta\right)^1 - \omega_2\left(\frac{3c_0^{\;2}}{256\gamma^2 k^4}\cos 3\theta\right)^1$$

$$= \frac{3c_0 k}{2}\left(\frac{c_0^{\;3}}{1024\gamma^3 k^6}\right)\left[\left(\cos\theta\cos 4\theta\right)^1 + \frac{3}{2}\left(\cos 2\theta\cos 3\theta\right)^1\right] - \omega_4(\cos\theta)^1 - \frac{c_0}{8\gamma k^2}\omega_3\left(\cos 2\theta\right)^1$$

$$-\frac{3c_0^{\;2}}{256\gamma^2 k^4}\omega_2\left(\cos 3\theta\right)^1 = \frac{3c_0^{\;4}}{2048\gamma^3 k^5}\left[\cos\theta(-4\sin 4\theta) + \cos 4\theta(-\sin\theta) + \frac{3}{2}\left[\cos 2\theta(-3\sin 3\theta) + \cos 3\theta(-2\sin 2\theta)\right]\right]$$

$$+\omega_4\sin\theta + \frac{c_0}{4\gamma k^2}\omega_3\sin 2\theta + \frac{9c_0^{\;2}}{256\gamma^2 k^4}\omega_2\sin 3\theta$$

$$= -\frac{3c_0^{\;4}}{2048\gamma^3 k^5}\left[\left(4\sin 4\theta\cos\theta + \cos 4\theta\sin\theta\right) + \frac{3}{2}\left[\left(3\sin 3\theta\cos 2\theta + 2\cos 3\theta\sin 2\theta\right)\right]\right]$$

$$+\omega_4\sin\theta + \frac{c_0}{4\gamma k^2}\omega_3\sin 2\theta + \frac{9c_0^{\;2}}{256\gamma^2 k^4}\omega_2\sin 3\theta \qquad (2.41)$$

But $4\sin 4\theta\cos\theta + \cos 4\theta\sin\theta = 3\sin 4\theta\cos\theta + \sin 4\theta\cos\theta + \cos 4\theta\sin\theta = \frac{3}{2}(\sin 5\theta + \sin 3\theta) + \sin 5\theta = \frac{5}{2}\sin 5\theta + \frac{3}{2}\sin 3\theta$. Also, $\frac{3}{2}(3\sin 3\theta\cos 2\theta + 2\cos 3\theta\sin 2\theta)$

$$= \frac{3}{2}(\sin 3\theta\cos 2\theta + 2\sin 3\theta\cos 2\theta + 2\cos 3\theta\sin 2\theta)$$

$$= \frac{3}{2}(\frac{1}{2}\sin 5\theta + \frac{1}{2}\sin\theta + 2\sin\theta) = \frac{3}{2}\left(\frac{5}{2}\sin 5\theta + \frac{1}{2}\sin\theta\right) = \frac{15}{4}\sin 5\theta + \frac{3}{4}\sin\theta$$

Substituting in (2.41)

$$= -\frac{3}{2048}\frac{c_0^{\ 4}}{\gamma^3 k^5}\left[\frac{5}{2}\sin 5\theta + \frac{3}{2}\sin 3\theta + \frac{15}{4}\sin 5\theta + \frac{3}{4}\sin\theta\right] + \omega_4\sin\theta + \frac{c_0}{4\gamma k^2}\omega_3\sin 2\theta$$

$$+ \frac{9c_0^{\ 2}}{256\gamma^2 k^4}\omega_2\sin 3\theta = -\frac{3}{2048}\frac{c_0^{\ 4}}{\gamma^3 k^5}\left[\frac{25}{4}\sin 5\theta + \frac{3}{2}\sin 3\theta + \frac{3}{4}\sin\theta\right]$$

$$+ \omega_4\sin\theta + \frac{c_0}{4\gamma k^2}\omega_3\sin 2\theta + \frac{9c_0^{\ 2}}{256\gamma^2 k^4}\omega_2\sin 3\theta \qquad (2.42)$$

$$-\gamma k^3(D + D^3)\xi_5 = -\frac{3}{4096}\frac{c_0^{\ 4}}{\gamma^3 k^5}\left[\frac{25}{2}\sin 5\theta + 3\sin 3\theta + \frac{3}{2}\sin\theta\right] + \omega_4\sin\theta + \frac{c_0}{4\gamma k^2}\omega_3\sin 2\theta + \frac{9c_0^{\ 2}}{256\gamma^2 k^4}\omega_2\sin 3\theta$$

$$\xi_5 = \frac{3}{4096}\frac{c_0^{\ 4}}{\gamma^4 k^8(D + D^3)}\left[\frac{25}{2}\sin 5\theta + 3\sin 3\theta + \frac{3}{2}\sin\theta\right] - \frac{1}{\gamma k^3(D + D^3)}\omega_4\sin\theta$$
$$- \frac{c_0}{4\gamma^2 k^5(D + D^3)}\omega_3\sin 2\theta - \frac{9c_0^{\ 2}}{256\gamma^3 k^7(D + D^3)}\omega_2\sin 3\theta \qquad (2.43)$$

Putting $$\frac{3}{4096}\frac{c_0^{\ 4}}{\gamma^4 k^8(D + D^3)}\left[\frac{3}{2}\sin\theta\right] - \frac{1}{\gamma k^3(D + D^3)}\omega_4\sin\theta = 0, \qquad (2.44)$$

(the two terms become unbounded) $$\omega_3 = 0 \qquad (2.45)$$

$$\xi_5 = \frac{3}{4096}\frac{25}{2}\frac{c_0^{\ 4}}{\gamma^4 k^8}\frac{D}{(D^2 + D^4)}\sin 5\theta = \left(\frac{3}{4096}\right)\left(\frac{25}{2}\right)\left(\frac{5}{600}\right)\frac{c_0^{\ 4}}{\gamma^4 k^8}\cos 5\theta \qquad (2.46)$$

$$\xi_5 = \frac{1}{65536}\frac{c_0^{\ 4}}{\gamma^4 k^8}\cos 5\theta \qquad (2.47)$$

$$\frac{3}{4096}\frac{c_0^{\ 4}}{\gamma^4 k^8(D + D^3)}[3\sin 3\theta] - \frac{9c_0^{\ 2}}{256\gamma^3 k^7(D + D^3)}\omega_2\sin 3\theta = 0$$

To obtain ω_4

$$(\omega_0 - c_0 k)\xi_6^1 - \gamma k^3\xi_6^{111} = \frac{3}{2}c_0 k\left[\xi_3\xi_3^1 + (\xi_5\xi_1)^1 + (\xi_2\xi_4)^1\right] - \omega_5\xi_1^1 - \omega_4\xi_2^1 - \omega_3\xi_3^1 - \omega_2\xi_4^1 = 0 \qquad (2.48)$$

$$-\gamma k^3(D + D^3)\xi_6^1 = \frac{3}{2}c_0 k\left[\left(\frac{3c_0^{\ 2}}{256\gamma^2 k^4}\right)^2(\cos 3\theta)(\cos 3\theta)^1 + \frac{5c_0^{\ 4}}{65536\gamma^4 k^8}(\cos 5\theta\cos\theta)\right.$$
$$\left. + \frac{c_0}{8\gamma k^2}\left(\frac{c_0^{\ 3}}{1024\gamma^3 k^6}\right)(\cos 4\theta\cos 2\theta)^1\right] + \omega_5\sin\theta + \omega_4\left(\frac{c_0}{4\gamma k^2}\right)\sin 2\theta \qquad (2.49)$$

$$\frac{3}{2}c_0 k\left[\begin{matrix}\frac{3^3 c_0^{\ 4}}{(256)^2\gamma^4 k^8}(-\cos 3\theta\sin 3\theta) + \frac{5c_0^{\ 4}}{65536\gamma^4 k^8}(-\cos 5\theta\sin\theta - 5\sin 5\theta\cos\theta) \\ + \left(\frac{c_0^{\ 4}}{8192\gamma^4 k^8}\right)(-4\sin 4\theta\cos 2\theta - 2\cos 4\theta\sin 2\theta)\end{matrix}\right] \qquad (2.50)$$

$$+ \omega_5\sin\theta + \omega_4\left(\frac{c_0}{4\gamma k^2}\right)\sin 2\theta + \omega_3\frac{9c_0^{\ 2}}{256\gamma^2 k^4}\sin 3\theta + \omega_2\frac{c_0^{\ 3}}{256\gamma^3 k^6}\sin 4\theta = 0$$

$$=-\frac{3}{2}\frac{c_0^5 k}{\gamma^4 k^8}\left(\frac{1}{8192}\right)\left[\frac{27}{8}(\cos 3\theta \sin 3\theta)+\frac{5}{8}(\sin\theta\cos 5\theta+5\cos\theta\sin 5\theta)+(4\sin 4\theta\cos 2\theta+2\cos 4\theta\sin 2\theta\right]$$

$$+\omega_4\frac{c_0}{4\gamma k}\sin 2\theta (\text{since}\, \omega_3=0 \text{ and } \omega_5=0) \quad (2.51)$$

Again,

$\cos 3\theta \sin 3\theta = \frac{1}{2}\sin 6\theta,\ \sin\theta\cos 5\theta + 5\cos\theta\sin 5\theta = (\sin\theta\cos 5\theta + \cos\theta\sin 5\theta) + \cos\theta\sin 5\theta$

$= \sin 6\theta + 2\sin 6\theta + 2\sin 4\theta = 3\sin 6\theta + 2\sin 4\theta, 4\sin 4\theta\cos 2\theta + 2\cos 4\theta\sin 2\theta = 2(\sin 4\theta\cos 2\theta$

$+ \cos 4\theta\sin 2\theta\} + 2\sin 4\theta\cos 2\theta = 2\sin 6\theta + \sin 6\theta + \sin 2\theta 1 = 3\sin 6\theta + \sin 2\theta$

substituting, we have,

$$-\gamma k^3(D+D^3)\xi_6 = -\frac{3}{2}\frac{c_0^5}{\gamma^4 k^7}\left(\frac{1}{8192}\right)\left[\frac{27}{8}(\frac{1}{2}\sin 6\theta+\frac{5}{8}(3\sin 6\theta+2\sin 4\theta)+(3\sin 6\theta+\sin 2\theta\right] + \omega_4\frac{c_0}{4\gamma k}\sin 2\theta = 0 \quad (2.52)$$

since we are interested in ω_4

$$\frac{3}{2}\frac{c_0^5}{\gamma^4 k^7}\left(\frac{1}{8192}\right)\frac{1}{\gamma k^3}\frac{D(\sin 2\theta)}{(D^2+D^4)} = \omega_4\frac{c_0}{4\gamma^2 k^5}\frac{D(\sin 2\theta)}{(D^2+D^4)} \quad (2.53)$$

$$\frac{3}{2}\frac{c_0^5}{\gamma^5 k^{10}}\left(\frac{1}{8192}\right)\left(\frac{2}{12}\right)\cos 2\theta = \omega_4\frac{c_0}{4\gamma^2 k^5}\left(\frac{2}{12}\right)\cos 2\theta$$

$$\omega_4 = \frac{3}{4096}\frac{c_0}{\gamma^3 k^5} \quad (2.54)$$

Thus, $\omega(k) = \omega_0 + \varepsilon\omega_1 + \varepsilon^2\omega_2 + \varepsilon^3\omega_3 + \varepsilon^4\omega_4 + \varepsilon^5\omega_5 + \cdots$. But $\omega_1 = \omega_2 = \omega_3 = 0$

$$\therefore \qquad \omega(k) = \omega_0 + \varepsilon^2\omega_2 + \varepsilon^4\omega_4\cdots \quad (2.55)$$

But, $\omega_0 = c_0 k - \gamma k^3$, from equation,

$$\therefore \qquad \omega(k) = c_0 k - \gamma k^3 + \frac{3}{32}\frac{c_0^2}{\gamma k}\varepsilon^2 + \frac{3}{4096}\frac{c_0^4}{\gamma^3 k^5}\varepsilon^4 + \cdots \quad (2.56)$$

Recall from equation (2.4),

$$\xi = \varepsilon\xi_1(\theta) + \varepsilon^2\xi_2(\theta) + \varepsilon^3\xi_3(\theta) + \varepsilon^4\xi_4(\theta) + \varepsilon^5\xi_5(\theta)$$

to fifth order.

Substituting for $\xi_1, \xi_2, \xi_3, \xi_4, \xi_5$ respectively, we have-

$$\xi = \frac{\eta}{h_0} = \varepsilon\cos\theta + \frac{\varepsilon^2 c_0}{8\gamma k^2}\cos 2\theta + \frac{3\varepsilon^3 c_0^2}{256\gamma^2 k^4}\cos 3\theta + \frac{\varepsilon^4 c_0^3}{1024\gamma^3 k^6}\cos 4\theta + \frac{5\varepsilon^5 c_0^4}{65536\gamma^4 k^8}\cos 5\theta \quad (2.57)$$

where $\theta = (kx - \omega t)$ but $\gamma = \frac{c_0 h_0^2}{6}$

$$\omega(k) = c_0 k - \gamma k^3 + \frac{3}{32}\frac{c_0^2}{\gamma k}\varepsilon^2 + \frac{3}{4096}\frac{c_0^4}{\gamma^3 k^5}\varepsilon^4 + \cdots \quad (2.58)$$

Take $h_0 = 15\text{m}$ (diivide through by $c_0 k$ and substitute for $\gamma = \frac{c_0 h_0^2}{6}$)

$$\frac{\omega}{c_0 k} = 1 - \frac{1}{6}k^2 h_0^2 + \frac{9}{16}\frac{\varepsilon^2}{k^2 h_0^2} + \frac{81}{512}\frac{\varepsilon^4}{k^6 h_0^6} + ... \quad (2.59)$$

Also,

$$\frac{\eta}{h_0} = \varepsilon\cos\theta + \frac{3\varepsilon^2}{4k^2 h_0^2}\cos 2\theta + \frac{27\varepsilon^3}{64k^4 h_0^2}\cos 3\theta + \frac{27\varepsilon^4}{128k^6 h_0^6}\cos 4\theta + \frac{81\varepsilon^5}{4096k^8 h_0^8}\cos 5\theta \quad (2.60)$$

But $\frac{\omega}{k} = c$ (phase velocity)

$$\frac{c}{c_0} = 1 - \frac{1}{6}k^2 h_0^2 + \frac{9}{16}\frac{\varepsilon^2}{k^2 h_0^2} + \frac{81}{512}\frac{\varepsilon^4}{k^6 h_0^6} + \cdots \quad (2.61)$$

Also, $\frac{d\omega}{dk} = c_g$ (group velocity.). Differentiating (2.58), with respect to *k*, we have,

$$\frac{d\omega}{dk} = c_0 - 3\gamma k^2 - \frac{3}{32}\frac{c_0^{\ 2}}{\gamma k^2}\varepsilon^2 - \frac{15}{4096}\frac{c_0^{\ 4}}{\gamma^3 k^6}\varepsilon^4 + ... \qquad (2.62)$$

$$c_g \ = \ c_0 - 3\gamma k^2 - \frac{3}{32}\frac{c_0^{\ 2}}{\gamma k^2}\varepsilon^2 - \frac{15}{4096}\frac{c_0^{\ 4}}{\gamma^3 k^6}\varepsilon^4 + ... \qquad (2.63)$$

$$\frac{c_g}{c_0} = 1 - \frac{3\gamma k^2}{c_0} \ - \frac{3}{32}\frac{c_0}{\gamma k^2}\varepsilon^2 - \frac{15}{4096}\frac{c_0^{\ 3}}{\gamma^3 k^6}\varepsilon^4 + ... \qquad (2.64)$$

substituting for $\gamma = \frac{c_0 h_0^{\ 2}}{6}$, we have $\frac{c_g}{c_0} = 1 - \frac{1}{2}h_0^{\ 2}k_0^{\ 2} \ - \frac{9}{16}\frac{\varepsilon^2}{h_0^{\ 2}k_0^{\ 2}} \ - \ \frac{3240}{4096}\frac{\varepsilon^2}{h_0^{\ 6}k_0^{\ 6}} \ + ...$ (2.65)

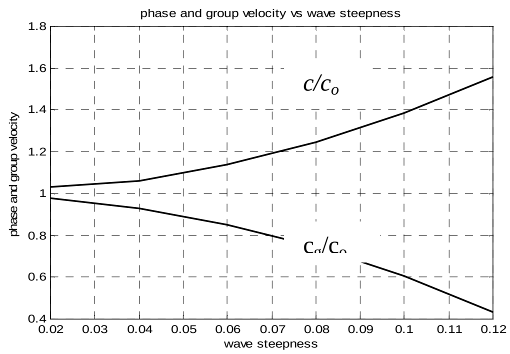

Figure 1: The phase and group velocities as functions of wave steepness ε .

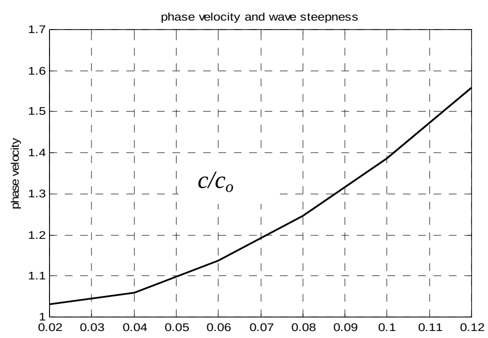

Figure 2:The phase velocity as a function of wave steepness ε .

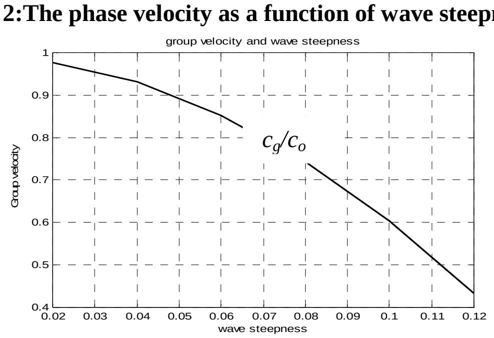

Figure 3: The group velocity as function of wave steepness $\mathcal{E}$.

The origin is shitted to $c_\circ - \gamma k^2$ along the vertical axis. The data so depicted isnearer to the observed than that of Johnson and Aneborg (1995).

Figure 4 is computed from equation (2.57) expressing wave energy density as function of solution order. This is based on the theory that wave energy is proportional to the square of wave amplitude. The effect of diffusion introduced through depth distribution is apparent. This is so because the solutions apparently depend strongly on the water depth in inverse form. Interestingly, this analysis strongly suggests that the peak energy lies between second and third order solutions. High seismic response associated with sea-bed motion corresponding to second order solution strongly support the result (Darbyshire, 1990).

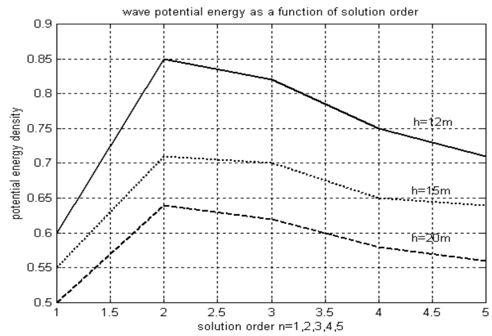

Figure 4: Wave potential density energy as a function of solution order for various depth of intermediate water.

It is also noted that the average in this consideration is over a wavelength (Okeke, 1999) of the fundamental mode. The peak at n =2 is due to the numerical increase of diffusion coefficient $\gamma = \frac{c_0 h_0^2}{6}$ with wave speed c (order of solution). But this coefficient is at the denominator of each term of the solution. Hence, the diminishing effects increase with increasing n. This is the case because in computing (for example) fifth order wave energy density, fifth order wave velocity is used; for fourth order solution, fourth order wave velocity is used and so on. The increase in wave velocity with increasing n is however calculated to be of order 1.2%.

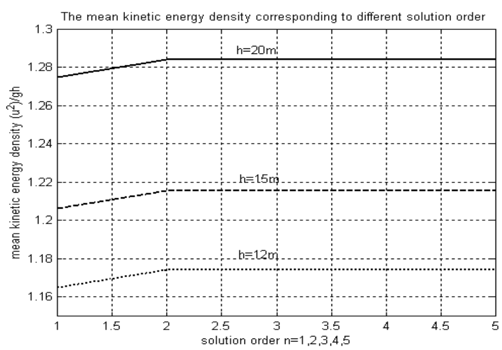

Figure 5: Average kinetic energy density corresponding to different solution order (n =1, 2, 3, 4, and 5)

The mean kinetic energy (ensemble mean) can also be calculated from the equation of the wave profile η(x,t) using the expansion in eqn (2.61) and (2.65) .

Thus, we obtain in non-dimensional form $\frac{\overline{u}}{gh} = 1 + \overline{\xi}, \ \overline{\xi} h = \overline{\eta}$. The total energy density (ensemble mean) $\overline{E}$ is calculated from

$$\overline{E}_T = 1 + \overline{\xi} + \overline{\xi}^2 . \qquad (2.66)$$

The energy flow F beneath the wave crest per unit horizontal width perpendicular to the direction of wave advance is calculated from (non-dimensional) $F = \frac{\overline{u}}{\sqrt{gh}} \overline{E}r$, $\overline{u}$ is now regarded as the mean depth current velocity.

3.0 Mean Water Surface (Mws)

Equation (2.66) may compare with that of Jonsson et al (1995). The later contain term associated with radiation stress.

Tsec	L(m)	$\bar{u}^2$ (m/s)	$F\rho\sqrt{gh}$	F_{MWS}
2	6.24	3.24	3.65	3.69
3	14.00	4.23	4.12	4.22

Tsec	L(m)	$\bar{u}^2$ (m/s)	$F\rho\sqrt{gh}$	F_{MWS}
4	20.33	5.21	6.33	6.35
5	39.12	7.80	8.21	8.45
8	97.21	11.20	9.10	9.33
10	156.10	15.60	9.98	10.10

Table (3.1): *h* =18.2m

With ρ = 1.026gm/cm^3, the energy density flux is measured *kwsec*/*m*^{2} in columns four and five respectively in table (3.1)

Similarity concerning the two (latter) is interesting. column five is the sum total of the following:

(i) kinetic energy due to the current motion.
(ii) transport of wave energy
(iii) the work on the flow due to radiation stress.

The latter component may act as an opposing component. Column four depicts only the effect of diffusion term which in the series expansion carried out here is quite dominant.

The similarity pattern between the two columns seems to suggest that radiation stress in wave motion can be adequately taken care of by diffusion. The effect of energy dissipation as it affects wave motion appears negligible. This is confirmed by the calculations involving wave equations with energy dissipation terms.

4.0 Conclusion

Diffusion term of the form $\frac{c_0 h_0^2}{6}$ was introduced for the fifth order expansion in the Stokes wave theory. Introduced further is the wave celerity expanded to fifth order. The aim is to consider further the convergence of the Stokes wave series. Since practitioners are more interested in wave energy and its evolutions, expressions for kinetic and potential energies were computed from the model.

This study strongly suggests that the peak energy is associated with the second order solution of the Stokes waves after which the energy becomes stable. Seismic energy density spectrum generated by the sea wave interactions (Darbyshire, 1990) agreed totally with this result. Thus, after the second term, extra terms appear rapidly diffused. Even though in this consideration, the model is vorticity free, various calculation thus obtained were quite close to those of viscous model of Jonsson et al (1995).

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