Journal of the Nigerian Association of Mathematical Physics
Volume 10 (November 2006), 35 - -40

On temperature control of buildings by adobe wall design: Duffin and Knowles' exponential transmission line model revisited

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Abstract

Duffin and Knowles (Solar Energy, Vol. 27(3), 1981) developed an equation for attenuation factor of an Adobe wall modelled as 4-terminal electrical transmission line network. The modelled electrical system and the derived formula for the real attenuation factor of the wall have been critically examined and then modified by taking into cognisance the true conceptualisation of a physical filter network as analogue of the thermal wall. By comparing results from the two versions of the exponential transmission line network models, it is shown that the effect of the correction on the attenuation factor is significant.

1.0 Introduction

The absence of building codes and standards for thermal design of buildings adaptable to the harsh external climates of Nigeria and other tropical locations have often led to the design of buildings totally unfit for comfortable habitation.

The Adobe (mud) house developed for the hot arid climate by the Indians of the American Southwest is an example of the practical utilisation of design strategies to achieve natural air conditioning effect (Conklin, 1958; Bahadori, 1978; Duffin and Knowles, 1981).

In recent times, the principles and design strategies used in the construction of the Adobe buildings have attracted worldwide attention. In Nigeria, the Nigerian Building and Road Research Institute has been in the forefront of projecting and promoting the use of stabilised laterite bricks for the construction of low-cost houses mainly for the rural populace. Amongst the rich diversity of Chinese vernacular buildings, adobe buildings have been classified as a special kind of buildings using rammed earth for its external walls (Yan et al., 2005). Adobe walls have good properties of thermal insulation, fireproof and a certain load bearing capacity.

Nomenclature

ω	$2\pi / 24$ angular frequency (daily)
t	time, hr
L	wall width
x	distance to the outside of the wall
T	$T(x)e^{i\omega t}$ temperature i n the wall
Y	$y(x)e^{i\omega t}$ current density in the wall
r	thermal resistivity in the wall
c	heat capacitivity in the wall
R_0	outside skin resistance of the wall per unit area
R	inside skin resistance of the wall per unit area
ψ	complex attenuation factor
f	real attenuation factor

z_0 characteristic impedance of a uniform layer
α $\mu + i\nu$ forward solution exponent
β $\varepsilon + \mu + i\nu$ reverse solution exponent
ε characterising exponent of exponential wall

In the analysis of the exponential transmission line model of the Adobe wall (see Duffin and Knowles, 1981; section 4), the authors derived the characteristic impedance of the wall model with reference to the interior parameters and the "load," represented by the building interior – a resistance in series with a capacitance and other input parameters without the "actual and real representative parameters of the building material matrix. In other words, the "thermal wall" was taken inevitably as a "black box" but made references only to coefficients associated with the major wall parameters.

In this work, modification of the modelled exponential thermal wall (adobe) has been carried out. The correction is expected to introduce remarkable changes in the expressions representing the characteristic impedance and attenuation factor of the wall component. The building models to be discussed assume that heat is transmitted into the interior only through walls and that all walls are of the same construction.

2.0 Four-terminal electrical network as a thermal analogue of an adobe wall

A four-terminal network is a network having a pair of input terminals and another pair of output terminals. The thermal wall may be modelled as a two-port or 4-terminal network by assuming a symmetrical structure. In this case, the electrical properties of the network are unaffected by interchanging the input and output terminals. The ratio of the input and output current densities are also equal.

The symmetrical network is normally defined by a fundamental electrical property called the characteristic impedance. The characteristic impedance is often described in terms of the iterative impedance of the network. This impedance may be taken as the value of the resistance measured at a pair of terminals of the network when the other pair of terminals is closed with an impedance of the same value. Iterative impedances are thus equal in symmetrical networks and their common value is also called the characteristic impedance.

The exponential electrical transmission line model of an adobe wall gives almost optimum performance. From the perspective of characteristic impedance in the work of Duffin and Knowles (1981), an expression for the attenuation factor is derived. Details may be found in the cited reference but briefly summarised here. Figure 1 shows the thermal wall (adobe) modelled as an electrical transmission line system. The equations governing the transmission line S are expressed in the form:

$$\frac{\partial T}{\partial x} = -rY\,;\ \frac{\partial Y}{\partial x} = -c\frac{\partial T}{\partial t} \qquad (2.1)$$

It is assumed in the network of Duffin and Knowles (1981), that the capacitivity and the resistivity of the wall vary as continuous exponential functions of the form:

$$\left.\begin{aligned} c(x) &= c_0 \exp(-\varepsilon t) \\ r(x) &= r_0 \exp(\varepsilon t) \end{aligned}\right\} \qquad (2.2)$$

where c_0, r_0 and ε are real constants.

Let

$$\left.\begin{aligned} T(x,t) &= T(x)e^{i\omega t} \\ Y(x,t) &= y(x)e^{i\omega t} \end{aligned}\right\} \qquad (2.3)$$

Then, the heat equation (2.1) becomes:

$$\left.\begin{aligned} &\frac{dT}{dx} = -ry\,; \frac{dY}{dx} = -i\omega cT \\ &\frac{d^2T}{dx^2} = -ry\,\frac{dT}{dx} \end{aligned}\right\} \qquad (2.4)$$

By making appropriate substitutions in equation (2.4), it is shown that the temperature, T, satisfies the equation,

$$\frac{d^2T}{dx^2} - \varepsilon\frac{dT}{dt} - ir_0c_0\omega T = 0 \qquad (2.5)$$

The general solution of (2.5) has the form $\alpha^2 + \varepsilon\alpha - ir_0c_0\,\omega = 0$ so that

$$\mu^2 + 2i\mu\nu - \nu^2 + \varepsilon\ \mu + i\varepsilon\nu - ir_0 c_0\ \omega = 0 \qquad (2.6)$$

Separating real and imaginary components yields, $\mu^2 - \nu^2 + \varepsilon\,\mu = 0$

$$\varepsilon = \left(\frac{\nu^2 - \mu^2}{\mu}\right) \qquad (2.7)$$

Again, $2i\mu\nu + i\varepsilon\nu - ir_0 c_0\ \omega = 0$. Hence, $r_0 c_0\ \omega = \dfrac{(\nu^2 + \mu^2)\nu}{\mu}$ (2.8)

Suppose we seek solutions of the form: $T = Ae^{-\alpha x}$ (2.9)

and recalling from (2.4), that $\dfrac{dT}{dx} = -ry$, then, we have: $y = -\dfrac{1}{r}\dfrac{dT}{dx}$, $y = -\dfrac{1}{r}\dfrac{dT}{dx} = \dfrac{A\alpha e^{-\alpha x}}{r_0 e^{\varepsilon x}}$

$$\frac{A\alpha}{r_0} e^{-\beta x} \qquad (2.10)$$

where $\varepsilon + \alpha = \beta$. Simulating the interior of the building as a resistor R in series with a capacitor C as shown, the circuit output impedance Z_0 is: $Z_0 = R + \dfrac{1}{i\omega C} = \dfrac{u_1}{y_1}$ (2.11

where u_1 and y_1 denote voltage and current at $x = L$. Hence, $Z_0 = R + \dfrac{1}{i\omega C} = \dfrac{Ae^{-\alpha L}}{\left(\dfrac{A\alpha e^{-\beta L}}{r_0}\right)}$,

$$Z_0 = \frac{r_0}{\alpha} e^{-\alpha L} \cdot e^{\beta L} = \frac{r_0}{\alpha} e^{-L(\alpha - \beta)} = \frac{r_0}{\alpha} e^{\varepsilon L} \qquad (2.12)$$

Duffin and Knowles (1981) termed equation (2.12 the characteristic impedance of the transmission line at $x = L$.

Assuming the transmission line is uniform, which indeed is the case in practice, then when terminated in an impedance equal to its characteristic impedance, there is no reflected heat wave and the impedance at any point of the line (including the input terminals) is also equal to the line's characteristic impedance. The load impedance thus matches the characteristic impedance. Therefore, $R + \dfrac{1}{i\omega C} = \dfrac{r_0 e^{\varepsilon L}}{\mu + i\nu}$. Equating real and imaginary parts, yields,

$$R = \frac{r_0 e^{\varepsilon L}\mu}{\mu^2 + \nu^2} \qquad (2.13a)$$

$$\frac{1}{\omega C} = \frac{r_0 e^{\varepsilon L}\nu}{\mu^2 + \nu^2} \qquad (2.13b)$$

$$\omega CR = \frac{\mu}{\nu} \qquad (2.13)$$

Ordinarily, the fact that the characteristic impedance matches the load impedance at $x = L$ simply imply (in electrical circuit principles) that the maximum power transfer theory should hold. The concept of impedance matching is a way to increase the inside wave amplitude. Impedance matching is extensi vely used in electrical engineering to maximise power transfer from source to load. The relationship given by equation (2.13) is particularly elegant, not because of the simplicity with which it is derived, but its major contribution towards the formulation of the attenuation factor of the wall. The beauty of this relationship will become clearer in due course. Let V_2 be the voltage at the output across the capacitor C.

$$\frac{V_2}{U_1} = \frac{(1/i\omega C)}{(R + 1/i\omega C)} = \left[\frac{1}{1 + i(\mu/\nu)}\right] \qquad (2.14)$$

The complex attenuation factor ψ is defined as the ratio of input voltage to the output voltage. Thus,

$$\psi = \frac{U_0}{V_2} = fe^{i\alpha} \qquad (2.15)$$

But $\psi = \frac{U_0}{V_2} = (U_0/V_1)(U_1/V_2)$, $\psi = e^{\mu L} \cdot e^{i\nu L}\left(1+\frac{i\mu}{\nu}\right)$. The final expression for the real attenuation factor f of the adobe wall as represented by the electrical transmission line analogue is:

$$f = \frac{\psi}{e^{i\alpha}} = e^{\mu L}\left[1+\left(\frac{\mu}{\nu}\right)^2\right]^{1/2} . \qquad (2.16)$$

The adobe wall has the unique property of acting as a heat wave or temperature filter (Duffin and Knowles, 1981, Yan et al., 2005). In electrical parlance, a filter network may be defined fundamentally as a 4-terminal network designed to attenuate certain ranges of frequency and to pass others without loss. The range of interest in this study is the range subject to attenuation and is called the attenuation band. The daily angular frequency of periodic heat transfer is assumed to meet this criterion.

One major problem identified in the analysis carried out in the preceding section is that the actual thermal resistance of the wall layer $R(= rd)$ and the thermal capacitance $C(= cd)$ per unit area (where d is the thickness of the wall layer) were made insignificant. The analogy carried out involved a situation in which the wall capacitivity c decreases exponentially with the thickness of the wall, while the wall resistivity, r increases with the wall thickness.

Secondly, since the thermal wall analogue of the transmission line system is modelled as a filter, it is appropriate to represent the wall as a true, physical filter if the concept of the transmission line network model is to be considered flawless.

2.0 Modification of the transmission line network

In Figure 2, the true representation of a simple filter is shown, consisting of pure reactive components, namely the inductance, L and the capacitance, C. Let the filter be closed or terminated in its characteristic impedance, Z_0. By definition, $Z_{in} = Z_0$. Generally, the characteristic impedance Z_0 may be defined for 4-terminal network systems by the circuit elements as:

$$Z_0 = (Z_1^2 + 2Z_1Z_2)^{1/2} \qquad (3.1)$$

Let $Z_1 = \frac{i\omega L}{2}$ and $Z_2 = \frac{1}{i\omega C}$. The total impedance Z_{0T} of the network becomes:

$$Z_{0T} = \left\{\left(\frac{i\omega L}{2}\right)^2 + 2\left(\frac{i\omega L}{2}\cdot\frac{1}{i\omega L}\right)\right\}^{1/2}$$

$$Z_{0T} = \left\{\frac{L}{C} - \frac{\omega^2 L^2}{4}\right\}^{1/2} \qquad (3.2)$$

$$Z_{0T} = R\left(1-\frac{\omega^2 C^2 R^2}{4}\right)^{1/2} \qquad (3.3)$$

where $R(=\sqrt{L/C})$ may be taken as the design impedance or nominal impedance of wall section. From equation (2.13), i.e., $\omega CR = \left(\frac{\mu}{\nu}\right)$, it is observed that,

$$Z_{0T} = R\left[1-\frac{(\mu/\nu)^2}{4}\right]^{1/2} \qquad (3.4)$$

Put $\frac{(\mu/\nu)^2}{4} < 1$, compulsorily. Hence, $\quad Z_{0T} = R \qquad (3.5)$

Equation (3.5) indicates again that the total impedance of the entire system equals the design impedance of the network. If the filter is appropriately terminated and works in such a manner that Z_0 becomes resistive as in equation (3.5), then all the power (heat wave and/or temperature) delivered to the input (source) must be transferred to the output (i.e., the interior of the building), in accordance with the maximum power transfer theory; and there will be no attenuation. In practice, however, attenuation in an adobe wall must have some finite value.

4.0 Derivation

Attenuation commences only if in equation (3.2),

$$\frac{L}{C} = \frac{\omega^2 L^2}{4} \qquad (4.1)$$

$$\frac{\omega^2 C^2 R^2}{4} = 1$$

Since $\omega CR = \frac{\mu}{\nu}$ (= 2, from above), substituting this value in equation (2.16) gives:

$$f = e^{\mu L}\left[1+(2)^2\right]^{1/2} = 2.24e^{\mu L} \qquad (4.2)$$

The new attenuation factor f is now expressed as a function of only μ and L. It would be appropriate here to introduce some modifications in Duffin and Knowles' work in line with the procedure above. As a first approximation, let $(\mu/\nu) < 1$. The expression for f given by equation (2.16) reduces to:

$$f = e^{\mu L} \qquad (4.3)$$

Following similar deduction, put $(\mu/\nu) = 1$. The attenuation factor becomes:

$$f = 1.41e^{\mu L} \qquad (4.4)$$

5.0 Results and discussion

Any section of a uniform transmission line is a two-port network. More specifically, all uniform transmission line sections are reciprocal symmetric two-port (4-terminal) networks. In the work of Duffin and Knowles (1981), the characteristic impedance was deduced primarily from parameters associated with the "building interior" components and the modelled transmission line network contributing to the derived attenuation factor through the coefficients - resistivity and capacitivity of the wall layers. It was concluded then that the characteristic impedance matches the load impedance at $x = L$ (by equation (2.12)).

With the introduction of a physical filter network to replace the original model network, it was again shown that the total impedance of the network also equals the characteristic impedance of the 4-terminal network (equation (3.5)). The two equations simply satisfy the conditions of maximum power (heat wave / temperature) transfer from the input terminals (source) to the output (load), which for an adobe wall negates the unique property of the building component, that it acts as a heat wave or temperature filter.In furtherance of the analysis and as a matter of notation, let μ = 2. The plots of f versus L using equations 4.1, 4.2 and 4.3 shown in Figure 3, give indications of remarkable differences in the results due to the effected modifications / corrections carried out in the present study (results from equation 4.2) compared to that originally proposed by Duffin and Knowles (1981), assuming $(\mu/\nu) \leq 1$ (results from equations 4.3 and 4.5). Another significant observation from the plots is that attenuation generally increases with the width of the wall.

The exponential wall model according to Duffin and Knowles (1981), gives optimum performance, but is not practically possible with available building materials. However, the corrections and / or the modifications done in this study, tend to erase that impression. Specifically, if μ is assumed to represent thermal diffusivity of the building material, it is then possible to show that for adobe (earth bricks, rammed earth or puddled mud) bricks, the determination of the attenuation factor is feasible, provided the property values of the building materials, such as thermal conductivity, density and specific heat capacity are known.

6.0 Conclusion

The network model and the equation for the real attenuation factor proposed by Duffin and Knowles (1981) in the form: $f = e^{\mu L}\left[1+(\mu/\nu)^2\right]^{1/2}$ has been critically examined in this study. By setting $(\mu/\nu) < 1$ and $(\mu/\nu)^2 = 1$, it is shown that the expression reduces to $f = e^{\mu L}$ and $f = 1.41e^{\mu L}$, respectively.

Modification carried out on the model by introducing a physical filter network as analogue of the thermal screen has led to the modification of the formula for attenuation factor as: $f = 2.24 e^{\mu L}$. Defining μ as the thermal diffusivity with a specific value and comparing results from the two versions of the network, it is observed that the correction made on the original network through the incorporation of a simple low-pass filter network has significantly impacted on the value of the attenuation factor as a function of the thickness of the wall. This is crucial to studies on the control of thermal fluxes entering adobe buildings in composite climates.

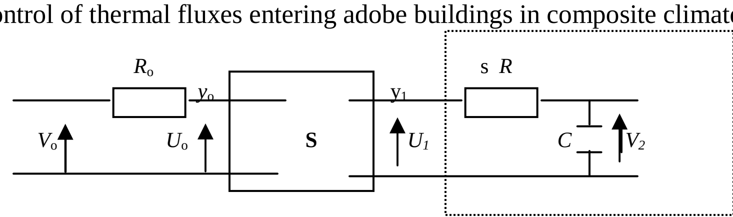

Figure 1: Transmission line with load (After Duffin and Knowles, 1981)s

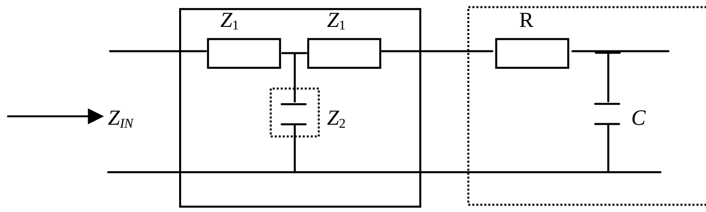

Figure 2: Modified transmission line system (incorporating a physical filter network)

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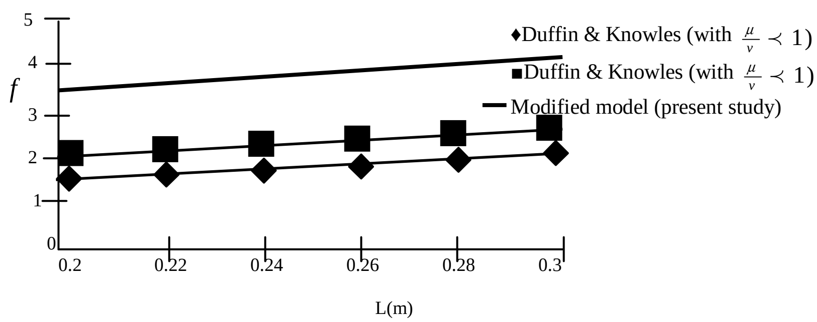

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Figure 3: Variation of attenuation with width of wall

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