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Contribution of oblateness of the sun to radar sounding according to Newtonian mechanics

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Abstract

The Newtonian theory of radar sounding in the gravitational field of a spherical sun is well known [1]. It is now well established that most of the astronomical bodies including the sun are spheroidal (proplate or oblate) in shape [5,11,12]. The Newtonian mechanics has been used to resolve satisfactorily the radar sounding phenomenon to the order of c^{-5} within the gravitational field established by the homogenous spherical massive sun. In this paper the Newtonian mechanics shall be used to resolve satisfactorily the radar sounding phenomenon within the gravitational field established by the homogenous spheroidal oblate massive sun.

1.0 Introduction

Since 1915 when Einstein came out with his gravitational field equation, which is a second order non-linear partial differential equation, only two exact solutions have been provided. One by K. Shwartchild and the other by Robertson-Walker [1]. The Schwarzschild's metric has resolved satisfactorily the problems of [2] (i) Orbital perihelion precession. (ii) Gravitational redshift (iii) Gravitational deflection of starlight and (iv) Radar sounding, while the Robertson – Walker metric has given a lot of insight into cosmological studies [1].

Newtonian Mechanics has also been applied to the radar sounding phenomenon [1] to the order of c^{-3} and recently it has been extended to c^{-5}.

Both General Relativity (GR) and Newtonian mechanics have developed the field equations for solving the above mentioned physical phenomenon by considering the massive sun, planetary bodies and other stars as homogenous spherical bodies. But it is well known that the only reason for these restriction is mathematical convenience and simplicity [3]. The fact of nature is that the sun, which is a G2, star in the milky way galaxy is spheroidal [3, 11, 12] in shape. Several studies and observations have been undertaken since 1966 to evaluate the solar oblateness. Measurements began with the Princeton solar Distortion Telescope and Dicke and Goldenberg found a value for the oblatenness of the sun as $\sum = (4.51 \pm 0.4) \cdot 10^{-5}$, (Dicke and Goldenberg, 1967) [12] Goldrich and Schubert in 1968 showed that the theoretical maximum solar oblateness, consistent with the stability of the sun is $\left[(3.5) \cdot 10^{-\frac{1}{2}}\right]^{13}$. Maier, Twiyg and sofia gave in 1992, their preliminary results of the solar diameter from a ballon flight of the solar Disk sextant (SDS) experiment and found $\sum = (5.6 \pm 0.35) \cdot 10^{-5}$, for the solar oblateness [13]. The equatiorial bulging (ie oblateness) of the sun was also advanced by Steve Carlip in 1996, which is an update of Michael Weiss' work on Mercury orbital precession, General Relativity and the solar Buge [11]. The solar Heliospheric Observatory (SoHO) board computed the difference between solar equatorial radil and polar radil associated with the static oblateness of the sun as 8.07 $\pm$ 0.58 milliarsec [12], in March 2005. Consequently spheroidal geometry will have effects in the motions of all particles in the gravitational fields of the astronomical bodies. The oblate spheroidal coordinate of space, (η, ξ, ϕ) are defined in terms of Cartesian coordinates (x, y, z) as [3]:

$$\left.\begin{array}{lll} x & = & a\left(1-\eta^2\right)^{\frac{1}{2}}\left(1+\xi^2\right)^{\frac{1}{2}}\cos\phi \\ y & = & a\left(1-\eta^2\right)^{\frac{1}{2}}\left(1+\xi^2\right)^{\frac{1}{2}}\sin\phi \\ z & = & a\eta\xi \end{array}\right\} \qquad (1.1)$$

where a is a constant parameter of a particular oblate body and $-1 \le \eta \le 1$; $0 \le \xi < \infty$, $0 \le \phi \le 2\pi$
with the transformation in (1.1) Newton's universal gravitational scalar potential exterior to a stationary homogenous oblate spheroidal body has been derived [5] as:

$$\Phi(\eta,\, \xi,\, \phi) = A_0 Q_0(\xi) + A_2 P_2(\eta) Q_2(\xi) \qquad (1.2)$$

where A_0, A_2 are constants and P_2 and Q_2 are corresponding pairs of linearly independent Legendre functions. Therefore the gravitational field intensity due to an oblate spheroidal massive body is given by taking negative gradient [4], $\underline{g}(\eta,\, \xi,\, \phi) = g_\eta\, \hat{\eta} + g_\xi\, \hat{\xi} + g_\phi\, \hat{\phi}$, where

$$g_\eta = \frac{B_2^+}{a}\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{\frac{1}{2}} Q_2(-i\xi)\frac{d}{d\eta}P_2(\eta) \qquad (1.3)$$

$$g_\xi = \frac{1}{a}\left(\frac{1+\xi^2}{\eta^2+\xi^2}\right)^{\frac{1}{2}}\left\{B_2^+ P_0(\eta)\frac{d}{d\eta}Q_0(-i\xi) + B_2 + P_2(\eta)\frac{d}{d\xi}Q_2(-i\xi)\right\} \qquad (1.4)$$

and

$$g_\phi = 0 \qquad (1.5)$$

and B_0^+ and B_2^+ are constants. Using (1.3), (1.4), (1.5) and some computation we obtain Newton's equations of motion for a test particle in the gravitational field exterior to a spheroidal oblate body as [4]:

$$\ddot{\eta} + \left[\frac{\eta\left(1+\xi^2\right)}{\left(1-\eta^2\right)\left(\eta^2+\xi^2\right)}\right]\dot{\eta}^2 + \left[\frac{2\xi}{\left(\eta^2+\xi^2\right)}\right]\dot{\eta}\dot{\xi} - \left[\frac{\eta\left(1-\eta^2\right)^{\frac{1}{2}}}{\left(1-\xi^2\right)\left(\eta^2+\xi^2\right)}\right]\dot{\xi}^2$$
$$+ \left[\frac{\eta\left(1-\eta^2\right)\left(1+\xi^2\right)}{\left(\eta^2+\xi^2\right)}\right]\dot{\phi}^2 = \frac{-B_2^+}{a^2}\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)Q^2(-i\xi)\frac{d}{d\eta}P_2(\eta) \qquad (1.6)$$

$$\ddot{\xi} + \left[\frac{\xi\left(1-\eta^2\right)}{\left(1+\xi^2\right)\left(\eta^2+\xi^2\right)}\right]\dot{\xi}^2 + \left[\frac{2\eta}{\left(\eta^2+\xi^2\right)}\right]\dot{\xi}\dot{\eta} - \left[\frac{\xi\left(1-\xi^2\right)^{\frac{1}{2}}}{\left(1-\eta^2\right)\left(\eta^2+\xi^2\right)}\right]\eta^2 - \left[\frac{\xi\left(1-\eta^2\right)\left(1+\xi^2\right)}{\left(\eta^2+\xi^2\right)}\right]\dot{\phi}^2 \qquad (1.7)$$
$$= \frac{\left(1+\xi^2\right)}{a^2\left(\eta^2+\xi^2\right)}\left[B_0^+ P_0(\eta)\frac{d}{d\xi}Q_0(-i\xi) + B_2^+ P_2(\eta)\frac{d}{d\xi}Q^2(-i\xi)\right]$$

and

$$\ddot{\phi} + \left[\frac{2\xi}{\left(1+\xi^2\right)}\right]\dot{\phi}\dot{\xi} - \left[\frac{2\eta}{\left(1-\eta^2\right)}\right]\dot{\phi}\dot{\eta} = 0 \qquad (1.8)$$

Equations (1.6), (1.7) and (1.8) are Newton's equations of motion for particles of non – zero rest masses in the gravitational field exterior to a stationary homogenous oblate spheroidal body. However, these equations of motion do not depend on the rest mass of the particle in any way. Therefore we shall assume that the photon (a particle of zero rest mass). And hence we shall apply along a radial line in the equatiorial plane of a homogenous spheroidal massive body and hence deduce Newton's theory of radar sounding in the gravitational field of the sun regarded as an oblate spheroidal body.

2.0 Theory

Consider an observer at position $\underline{r}_1$ sending radar signals or pulses in a radial direction towards a small body at position $\underline{r}_2$ within the gravitational field established by the homogenous oblate spheroidal massive sun of mass, M such that $r_1 > r_2$ as shown in Fiqure 2.1

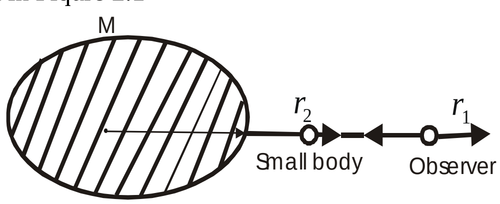

Figure 2.1: "Radar Sounding Experiment":

We are interested in computing the total time needed for the radar pulses to travel from r_1 to r_2 and back to r_1 in a radial direction within the gravitation field created by the stationary homogenous oblate sun.

For the radial motion of a photon in the equatorial plane of the oblate sun, $\eta = 0,\ l = 0$ hence equation (1.7) turn out to be

$$\ddot{\xi} + \frac{1}{\xi\left(l + \xi^2\right)}\dot{\xi}^2 = \left[-\frac{\left(1 + \xi^2\right)}{a^2\xi^2}\right]\frac{d}{d\xi}Q(\xi) \tag{2.1}$$

where

$$Q(\xi) = B_0^+ Q_0(-i\xi) - \frac{1}{2}B_2^+ Q_2(-i\xi) \tag{2.2}$$

By the transformation $\dot{\xi}(\xi) = w(\xi)$ and $w^2 = z(\xi)$, (2.1) integrates exactly as:

$$\dot{\xi}^2 = A\left(1 + \frac{1}{\xi^2}\right)^{-1} - \frac{2}{a^2}\left(1 + \frac{1}{\xi^2}\right)Q(\xi) \tag{2.3}$$

where *A* is an arbitrary constant. Also for motion of photon along a radial line, $U_\xi^2 \to c^2, \xi \to \infty$. Hence

$$A = \frac{c^2}{a^2} \tag{2.4}$$

Thus (2.3) becomes explicitly

$$\dot{\xi}^2 = \frac{c^2}{a^2}\left(1 + \frac{1}{\xi^2}\right)^{-1} - \frac{2}{a^2}\left(1 + \frac{1}{\xi^2}\right)Q(\xi) \tag{2.5}$$

It has been shown that the radial velocity in the $\hat{\xi}$ direction is given in oblate spheroidal coordinate as [5]:.

$$U_\xi^2 = \frac{a^2\xi^2}{\left(1 + \xi^2\right)}\dot{\xi}^2 \tag{2.6}$$

substituting (2.5) into (2.6) we have

$$U_\xi^2 = \frac{a^2\xi^2}{\left(1 + \xi^2\right)}\left\{\frac{c^2}{a^2}\left(1 + \frac{1}{\xi^2}\right)^{-1} - \frac{2}{a^2}\left(1 + \frac{1}{\xi^2}\right)Q(\xi)\right\} \tag{2.7}$$

After some arrangement in equation (2.7) it follows that:

$$\frac{d\xi}{dt} = \pm\frac{a}{c}\left(1 + \frac{1}{\xi^2}\right)^{-\frac{1}{2}}\left\{1 - \frac{2}{c^2}\left(1 + \frac{1}{\xi^2}\right)^2 Q(\xi)\right\}^{\frac{1}{2}} \tag{2.8}$$

From equation (2.8) the time needed for the radar signals to travel from ξ_1 to ξ_2 is given as:

$$Dt = \frac{2a}{c}\int_{\xi_2}^{\xi_1}\left(1 + \frac{1}{\xi^2}\right)^{\frac{1}{2}}\left\{1 - \frac{2}{c^2}\left(1 + \frac{1}{\xi^2}\right)^2 Q(\xi)\right\}^{\frac{1}{2}} \tag{2.9}$$

By series expansion and integration, to the order of c^{-3}, equation (2.9) becomes

$$Dt \approx \frac{a}{c}\left\{\left[(\xi_1 - \xi_2) - \frac{1}{2}\left(\frac{1}{\xi_1} + \frac{1}{\xi_2}\right) + \frac{1}{c^2}\left[Q(\xi_1) - Q(\xi_2)\right]\right]\right\} \tag{2.10}$$

Equation (2.10) is the total time to the order of c^{-3} taken for the round trip of the radar signals within the gravitational field established by the homogenous spheroidal oblate sun. The expression for the radial distance in the equatorial plane from (1.1) in terms of ξ is given by $r = a\left(1 + \xi^2\right)^{\frac{1}{2}}$ Hence

and

$$\left.\begin{aligned}\xi_1 &= \left(\frac{r_1^2}{a^2} - 1\right)^{\frac{1}{2}}\\ \xi_2 &= \left(\frac{r_2^2}{a^2} - 1\right)^{\frac{1}{2}}\end{aligned}\right\} \tag{2.11}$$

Substituting equation (2.11) into equation (12.10) we have

$$Dt \approx \frac{1}{ac}\left\{(r_1 - r_2) - \frac{1}{2}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) - \frac{a^2}{c^2}\left[Q(r_1) - Q(r_2)\right]\right\} \tag{2.12}$$

Equation (2.12) is the total time for the round trip of the radar pulses in terms of a measurable distance, (r) to the order of c^{-3}.

3.0 Summary and conclusions

In this paper we formulated Newton's equations of motion for a photon in the gravitational field of an oblate spheroidal body equations (1.6), (1.7) and (1.8). Then we solved them for radial motion in the equatorial plane to obtain the instantaneous speed of the photon as equation (2.7). Finally we applied the speed to calculate the total time taken by radar signal to move from an observer to a reflecting target and back in the gravitational field of the sun treated as an oblate spheroid given in equation (2.12).

In the first place our equations (1.6) – (1.8) open the way for solution of Newton's equation of motion for photons in all directions in the gravitational field of the oblate spheroidal sun. In the second place our expression (2.12) contains the oblate spheroidal corrections to the corresponding total time for a perfectly spherical sun given by $Dt = \frac{2}{c}\left\{(r_1 - r_2) + \frac{k}{c^2}\ln\left(\frac{r_1}{r_2}\right)\right\}$ to the order of c^{-3}. Therefore the contribution of the oblateness of the sun to the Radar sounding problem may be computed theoretically via equation (2.12) to the order of c^{-3}.

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