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New One-Boson-Exchange Potential functions

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Abstract

A new one-boson-exchange potential (OBEP) is derived by fitting the oscillator matrix elements of the sum of the OBEP functions to those of the matrix elements derived by the lowest order constrained variational (LOCV) technique. The results give a reasonable fit to the OBEP model.

1.0 Introduction

For some time now a number of effective interactions have been developed by many authors to study the effective interaction between two nucleons inside the nucleus [1,2]. In particular a simple local form of the effective interaction was developed by Bertsch and collaborators [1] for the A = 16 nuclei and is called the M3Y interaction. This interaction was obtained by fitting the oscillator matrix elements of three or two Yukawa functions to those of the oscillator G-matrix derived from the Reid [3] potential. Because of its semi-microscopic nature, this interaction has turned out to be very successful in many applications [4,5]. Its density dependent variants [6,7] have been studied by many authors.

Recently [8] we derived a similarly motivated M3Y-type interaction which was however based on the lowest order constrained variational (LOCV) technique instead of the G-matrix approach. In our calculation we found several features which were very similar to the M3Y interaction developed by Bertsch and collaborators [1]. We also studied its mass dependence in detail, thus showing that the M3Y interaction should have a mass dependence. However, Hosaka et al., [9] have suggested that the simple local form of the M3Y interaction proposed by Bersch and collaborators [1] need to be improved upon by taking more realistic functional form such as the OBEP forms. With these new functional forms they found an improvement over the work of Bertsch and collaborators [1].

In this paper we have followed the procedure of Hosaka et al., [9] by choosing OBEP forms for our effective interaction. It should be noted that the work which we report here is quite different from that of the work of Hosaka et al., [9] because our matrix elements which we fit the OBEP are derived from a different calculational procedure. Thus, the results based on our OBEP should be viewed as different from those of Hosaka et al., [9].
The rest of the paper is organized as follows: In Section 2 we give a brief summary of the method used. In Section 3, we define the OBEP used in this study. In Section 4 we discuss the main results of this paper while in Section 5 we make our conclusions.

2.0 The method

In previous work [10, 11] we have discussed the LOCV technique for obtaining the matrix elements of an operator, O. In this section we shall briefly summarise this prescription which we use for the present work in generating an effective two-body interaction. According to the LOCV theory [10, 11] the matrix elements of an operator, O may be written as

$$\hat{\mathrm{O}} = \frac{\langle \psi_T \mid \mathrm{O} \mid \psi_T \rangle}{\langle \psi_T \mid \psi_T \rangle} \tag{2.1}$$

In this equation, $|\psi_T\rangle$ is the trial wavefunction defined as

$$\psi_T = UF\Phi\,, \tag{2.2}$$

where U is a unitary operator which transforms the system to the centre of mass rest-frame, leaving us with only intrinsic quantities. In eq. (2), F is a symmetric product of two-body correlation functions defined as [10]:

$$F = \prod_{i>j} f^{(2)}(ij), \qquad (2.3)$$

While Φ is a multi-dimensional product of two-body wave functions. Owing to its multi-dimensional nature, eq. (1) is difficult to determine and we must use approximate methods. The approximating technique we adopt here is the cluster expansion method. Here we divide the system into clusters. We evaluate the expectation value of each cluster starting with the two-body clusters and then sum over all the clusters to obtain the matrix elements:

$$\hat{O} = O^{(2)} + O^{(3)} + O^{(4)} + ... \qquad (2.4)$$

Our task is to minimize the two-body cluster term $O^{(2)}$ with respect to the functional variations in the two-body correlation function such that only two-body cluster terms are important, whereas contributions of $O^{(3)}$ and higher order clusters are made negligible.

In the present case the operator O, defined above is the Hamiltonian, so we need only determine the two-body energy term, $E^{(2)}$ which is given by [10]:

$$E^{(2)} = \left\langle \Phi \left| \sum_{i>j} \left(f^{(2)}(ij) \left(P_{ij}^2 / M + V_{ij} \right) f^{(2)}(ij) \right) \right| \Phi \right\rangle \qquad (2.5)$$

where $\vec{P}_{ij} = \frac{1}{\sqrt{2}}\left(\vec{p}_i - \vec{p}_j \right)$ is the relative momentum of the two-particle system; $M \approx m_N A$ is the total mass of the nucleus while for the two-body potential, V_{ij} we have used the Reid [3] soft- core potential, whereas the $f^{(2)}(ij)$ are the two-body correlation operators. The Reid [3] potential has the form:

$$V_{ij} = \sum_{\delta} V_{ij}^{\delta}, \qquad (2.6)$$

where in the different reaction channels δ we have the central, spin-orbit and tensor components. In a similar manner we allow the same flexibility to our two-body correlation functions in each reaction channel:

$$f^{(2)}(ij) = \sum_{\delta} f_{ij}^{\delta} \qquad (2.7)$$

where
$$f_{ij}^{\delta} = f_c^{\delta}(r_{ij}) + f_{LS}^{\delta}(r_{ij})\vec{L}.\vec{S} + f_Q^{\delta}(r_{ij})(\vec{L}.\vec{S})^2 + f_T^{\delta}(r_{ij})S_{ij}, \qquad (2.8)$$

with the tensor operator defined as:

$$S_{ij} = 3\vec{\sigma}_i.\hat{r}_{ij}\vec{\sigma}_j.\hat{r}_{ij} - \vec{\sigma}_i.\vec{\sigma}_j. \qquad (2.9)$$

Our parameterized form of two-body correlation functions is based on our previous studies regarding nuclear matter and finite nuclei [10]. In those studies it was found that the two-body correlation functions have three main features. These include (i) the 'wound' induced in the two-body wave function by the repulsive core of the N-N interaction, (ii) the tensor correlations which are important especially in the $^3S_1$-$^3D_1$ channel and (iii) the meson exchange corrections. Of these three features, it was found that the most important was the tensor correlations. We have thus parameterized our two-body correlation functions of eqution. [7] in the form [10, 11]:

$$f^{\delta}(ij) = 0, r_{ij} < r_c$$

$$f^{\delta}(ij) = f(r_{ij})(1 + \alpha^{\delta}(A)S_{ij}), r_{ij} \geq r_c., \qquad (2.10)$$

where
$$f(r_{ij}) = 1 - e^{-\beta(r_{ij} - r_c)^2}. \qquad (2.11)$$

Here r_c = 0.25fm and β = 25fm^{-2}. In equation. (2.10) the parameter, $\alpha^{\delta}(A)$ represented the strength of the tensor correlations. We take this to be non-zero only in the $^3S_I - {}^3D_1$ and $^3P_2$-$^3F_2$ channels. In the present analysis we use the harmonic oscillator wavefunctions to evaluate our matrix elements. We also note here that we are only interested in the effective potential matrix elements so we take only the second term of equation (2.5) i.e.,

$$E^{(2)'} = \left\langle \Phi \left| \sum_{i>j} f^{(2)}(ij) V_{ij} f^{(2)}(ij) \right| \Phi \right\rangle \qquad (2.12)$$

We next notice that the only free parameters in our effective potential are; the oscillator parameter and the strength of the tensor correlations, α^{δ} (A). Following our earlier work [8] we have separated the effective potential two-body matrix elements into the singlet-even (SE) and the singlet-odd (SO) channels denoted by ^{1}So and $^1P_1$ respectively. The triplet-even (TE) and the tensor-even (TNE)

components were taken directly from the coupled $^3S_1$-$^3D_1$ channels. Finally, we defined the triplet-odd (TO), tensor-odd (TNO) and the two components of the spin-orbit force ignoring the quadratic term as [9]:

$$V(TO) = V(^3P_0) + 2V(LSO) + 4V(TNO)$$
$$V(TNO) = -\frac{5}{72}[2V(^3P_0) - 3V(^3P_1) + V(^3P_2)]$$
$$V(LSO) = -\frac{1}{12}[2V(^3P_0) - 3V(^3P_1) - 5V(^3P_2)]$$
$$V(LSE) = \frac{1}{3}[V(TE) - 2V(TNE) - V(^3D_2)] \qquad (2.13)$$

In the following section, we shall show how the defining potential forms of equation (2.12) are used to determine the OBEP.

3.0 The effective OBEP

The objective of our paper is to define an effective OBEP whose oscillator matrix elements fit the matrix elements of an effective two-body interaction calculated from the LOCV technique. The OBEP potential is divided into the central (c), spin-orbit (ls) and tensor (*tn*) components as follows:

$$V_c(r) = \sum_j A_j Y_c(r/R_j),$$
$$V_{ls}(r) = \sum_j A_j Y_{ls}(r/R_j)\vec{L}.\vec{S},$$
$$V_{tn}(r) = \sum_j A_j Y_{tn}(r/R_j)S_{ij}, \qquad (3.1)$$

where

$$Y_c(x) = \frac{e^{-x}}{x},$$
$$Y_{ls}(x) = (1+\frac{1}{x})\frac{e^{-x}}{x^2},$$
$$Y_{tn}(x) = (1+\frac{3}{x}+\frac{3}{x^2})\frac{e^{-x}}{x}S_{ij}, \qquad (3.2)$$

In equations (3.1) and (3.2) the A_j are the strengths of the interaction which are determined by fitting the oscillator matrix elements of. (3.1) to our two-body matrix elements of. (2.12). The R_j are the ranges. The ranges for the central forces are chosen to be 0.20, 0.33, 0.50 and 1.414fm, which are motivated by the one-boson exchanges. The longest range of 1.414 fm corresponds to the one-pion exchange, while the shorter ranges correspond to heavier mesons such as σ, ρ and *w* mesons. For the spin-orbit forces and tensor channels, we choose two ranges which are 0.25 and 0.40 fm for the spin-orbit channels and 0.25 and 1.414 fm for the tensor channels in line with the work of Hosaka et al., [9]. Finally, since the pion-nucleon coupling strength is well established experimentally, we fix the longest-range interaction strength to the OBEP strength which is -10.463 MeV for the SE, TE and TNE channels. It has the value of 3.488 MeV for the TO and TNO channels. For the SO channel this strength has the experimental value of 31.388 MeV.

4.0 Results

In this section we present the results of the best-fit interaction strengths for the OBEP. We have used the results discussed in ref. [8] for the relative two-body matrix element of our effective interaction for the A = 16 system consisting of SE, SO, TE, TO, TNE, TNO, LSO and LSE channels. We use that
result in fitting the sum of the OBEP functions described in Section 3. We calculated the matrix elements in a harmonic oscillator basis with $\hbar\omega = 45A^{-1/3} - 25A^{-1/3}$ MeV so that $\hbar\omega$ takes on the value of 14.0MeV for the present case. The value of α^{δ} (A) used was equal to $\alpha^{\delta}(16) = 0.075$.

In Table 1 we present the results of the best-fit interaction strengths of the OBEP for the SE, SO, TE, TO, TNE, TNO, LSE, LSO channels. The first entry of a column is the result of the present calculation while the second entry of a column in bracket is the result of Hosaka et al [9]. For the SE and the TE channels, we chose the ranges R_j = 0.20, 0.33, 0.50 and 1.414 fm. These ranges are motivated by the one-boson exchanges as discussed in Section 3. The highest range of 1.414 fm corresponds to the OPEP whose strength is experimentally known to be –10.403 MeV. The shorter ranges of 0.33, 0.50 and 0.70 fm correspond to heavier meson exchanges such as γ, ρ and ω mesons. The shortest range of 0.20fm was included to improve the fit. For the SO and TO channels, the longest

ranges chosen are 31.389 and 3.488 MeV respectively as observed in [9]. Here also the shortest range of 0.20fm was included to improve the fit.

For the TNE and TNO channels, we simply used two ranges of 0.25 and 1.414fm as suggested by Hosaka et al [9]. Two ranges of 0.25 and 0.40 fm are also expected for the LSE and LSO channels as also discussed by Hosaka et al [9].

From Table 1, one can see that our results for the SE, TNE, TNO, LSE and LSO channels give similar values to the results of Hosaka et al [9]. As for the TE, SO and TO channels, the results do not give similar strengths but this can be attributed to the fitting routines. For this reason we have used the best-fit strengths to recalculate the relative matrix elements. The results are presented in Table 2 (in bracket) together with our original relative matrix elements which we use in fitting the OBEP. Clearly it can be seen in Table 2 that our best-fit strengths fit the relative matrix element very well.

5.0 Conclusion

We have derived a new OBEP by fitting the oscillator matrix elements of our effective interaction [8] to the OBEP functional forms as suggested by Hosaka et al [9]. This new OBEP can be seen to be quite different from the one derived by Hosaka et al [9], since, the relative matrix elements which we fit the OBEP are derived from a different calculational procedure to the ones used by Hosaka et al [9]. Since there still exists a controversy over the strengths of the OBEP and the M3Y interactions used in heavy-ion physics [12, 13], it now remains to be seen any improvement this new interaction will give in heavy-ion physics or even nuclear structure calculations. In our next article, we shall present a calculation giving an application of our new OBEP.

Table 1: Best-fit interaction strengths (in MeV) for OBEP (Eq. 14). The first entry of a column is the result of the present calculation while the second entry of a column in bracket is the result of Hosaka et al [9].

	Range						
Channel	0.20	0.25	0.33	0.40	0.50	0.70	1.414
SE	13070.25 (13015)		-1874.15 (-882.09)		-746.05 (-993.25)		-10.463 (-10.463)
TE	-8105.06 (19886)		12104.31 (-2400.9)		-3836.02 (-1262.7)		-10.463 (-10.463)
SO	-321408.0 (15991)		90614.29 (-1955.1)		-10048.3 (396.51)		31.389 (31.389)
TO	-5538.39 (26795)		4670.84 (-2801.4)		-728.50 (-142.92)		3.488 (3.488)
TNE		2674.35 (3104.9)					-10.463 (-10.463)
TNO		-1831.65 (-1382.0)					3.488 (3.488)
LSE		-1404.68 (-30963)		2280.79 (2343.4)			
LSO		12302.06 (9109.8)		-3401.54 (-2301.8)			

Table 2: **Calculated relative matrix element for TE, SO and TO channels. The first entry of a column is the result of the recalculated relative matrix elements while the second entry of a column in bracket is the original result of Fiase et al [8].**

S/S	n=0	1	2
Triplet-even			
$n^{\|} = 0$	-9.36	-7.26	-4.68
	(-8.50)	(-6.29)	(-3.83)
1		-6.05	-4.07
		(-5.22)	(-3.17)
2			-2.75
			(-1.86)

S/S	n=0	1	2
Singlet-odd			
$n^{\|} = 0$	-1.22	1.32	2.56
	(0.43)	(1.87)	(3.13)
1		2.72	4.07
		(3.29)	(4.62)
2			5.30
			(5.89)

S/S	n=0	1	2
Triplet-odd			
$n^{\|} = 0$	-0.051	0.079	0.24
	(-0.060)	(0.08)	(0.23)
1		0.25	0.43
		(0.24)	(0.42)
2			1.18
			(0.63)

References

[1] G. Bertsch, J. Borysowicz, H. McManus and W. G. Love, Nucl. Phys. A284, 399 (1977).
[2] P. J. Dortmans and K. Amos, Phys. Rev. C49 (1994).
[3] R. V. Reid, Ann. Phys. (N.Y) 50, 411 (1968).
[4] M. Ismail and Kh. A. Ramadan, J. Phys. G. Nucl. Part. Phys. 26, 1621 (2000).
[5] D. S. Delion, A. Sandulescu, S. Misicu, F. Carstoiu and W. Greiner, Phys. Rev C64, 041303 (2001).
[6] Dao T. Khoa and G. R. Sachler, Nucl. Phys. A668, 3 (2000).
[7] G. R. Satchler and W. C. Love, Phys. Repts 55, 183 (1979).
[8] J. O. Fiase, K. R. S. Devan and A. Hosaka, Phys. Rev. C66 (2002).
[9] A. Hosaka, K. I. Kubo and H. Toki, Nucl. Phys. A444, 76 (1985).
[10] J. Fiase, A. Hamoudi, J.M. Irvine and F. Yazici, J. Phys. G. Nucl. Phys. **14**, 27 (1988).
[11] J. Fiase, Phys. Rev **C63**, 037303-1 (2001).

[12] F. Petrovitch, in; the (p,n) reaction and the nucleon-nucleon force (Plenum, New York, 1980) p.115.
[13] H. Toki, Proc. Int. Symp., On Electromagnetic properties of atomic nuclei, Tokyo, Japan (1983), ed. H. Horie and H. Ohnuma.