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Predictions of total deformations in Jebba main dam by finite element analysis technique

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Abstract

This paper examined the deformations of the Jebba Main Dam, Jebba Nigeria using the finite element method. The study also evaluated the predicted deformations and compared them with the actual deformations in the dam to identify possible causes of the observed longitudinal crack at the dam crest. The Jebba dam is a sloping earth core rockfill. The study methodology consists of the modeling of the dam with the utilization of the actual dam section, development of Finite Element mathematical formulations using plain strain conditions and total stress approach, and simulation of such model using a computer program developed in Visual Basic Language. The 3-node isoparametric triangular elements were used to analyze the model dam. The dam was simulated for end of construction (no water) and full reservoir loadings. The results of the study showed that the actual crest settlement as at 2001 was 184 mm and the predicted total settlements were 280 mm and 307 mm for the two models adopted. Generally, the deformations in the dam body were higher than deformations in the dam foundation. This may have resulted from the adequate foundation compaction during construction. The cracks development criteria were based on the criteria developed by Thomas. From these criteria, the dam was found to have developed cracks due to settlements and hydraulic fracturing.

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[B]	Strain transformation matrix
[D]	Stress transformation matrix
ε	Elemental strain
σ	Elemental stress
K	Elemental stiffness
ν	Poisson ratio of material
E	Modulus of elasticity of material
F	Elemental nodal force
δ	Elemental nodal deformation
A	Elemental area

1.0 Introduction

Settlements of dams continue over time after completion. Rock fill dams are usually founded on good foundations – rock beds or consolidated soil – to forestall excessive or differential settlements. However, settlements are bound to occur. Dams are usually instrumented for monitoring. Such instrumentation gives an indication of the overall performance of the dam. Crest settlement measurements are done with inclinometers.

Predictions of settlement have been based on some empirical formulae. These formulae are either based on height of fill, post-construction period or both. Emphasis has been on the reflection of some other parameters such as the location, construction method; fill material, etc. of the rock fill dam. Also, arguments have been proffered that modeling of settlement can only be peculiar to the dam in question.

The paper studied the settlement history of the Jebba Main Dam, Nigeria. The various empirical formulae are presented and reviewed. The behaviour of the settlement history of the Jebba Main Dam is compared with the predicted deformation patterns from Finite Element analysis.

1.1 Jebba Hydroelectric Power Station

1.1.1 General Description:

Jebba Hydroelectric Power Station is located on the River Niger, about 400km North East of Lagos, Nigeria and 100km downstream of existing *Kainji dam*.[1] The site is located 3km upstream of the community of Jebba where both a highway and a rail way bridge span the Niger River.

The Jebba Hydroelectric Development involved the formation of a reservoir in the Niger River by the construction of a rock fill embankment with a slanting core of impervious material connected to an upstream impervious blanket, together with auxiliary and saddle dams on the left bank of the river. [1] The project was designed to develop a head of 30m from the main dam backing up the head pond of the tail water of the upstream development at *Kainji* and include the following features: [1]

(i) A 108m (above foundation level) high by 650m long embankment dam with an upstream blanket.

(ii) Three embankment dams with total crest length of 1,030m.

(iii) Powerhouse containing six turbine units connected to 103.5MVA generators.

(iv) An underflow spillway, located at 103m of the dam elevation, controlled by six radial gates.

The Jebba Main Dam which was constructed between 1979 and 1984 and commissioned in 1984, has the following [1]

▪ Normal maximum reservoir operating level	103.0m
▪ Minimum reservoir operating level	99.0m
▪ Reservoir draw-down	4.0m
▪ Reservoir full supply capacity	$3.880 \times 10^9 m^3$
▪ Minimum reservoir capacity	$2.880 \times 10^9 m^3$
▪ Reservoir flood level	106m
▪ Length of reservoir	100km
▪ Width of reservoir	2-5km
▪ Average reservoir run-off	25billion.m^3/day
▪ Flood control structure	(i) 6 gates underflow spillway (ii) Emergency spillway wall.
▪ Navigation canal	incorporated
▪ Installed generating capacity	560MW.

1.1.2 The Main Dam [2-6]

As seen in Figure 1 (Dams typical sections) the main dam is a zoned earth/rock fill dam with the impervious core inclined. Apart from the impervious core tied to upstream blanket, there are the filters, and also transition zones, and rock fill, shells as shown in Figure 1. The dam has a maximum height above foundation level of approximately 40m and its crest length is 650m.

Construction materials for the impervious blanket comprise lateritic clayey silty sand. The transition material processed from excavated rock. Rock fill for the shells is obtained from required structure and channel excavations, but rip rap material for the upstream slope of the dam is quarried from a granite outcrop – from a quarry located about one kilometer northwest of the site. Pressure relief wells were installed at 68m downstream from the axis.

The Jebba Main Dam is founded on river alluvium up to 70m deep comprising fine to coarse, clean sand of varying density and containing traces of gravel sizes. Seepage through this pervious foundation is controlled to conservative limits by an impervious blanket extending upstream from the impervious core of the dam for a distance of 450m beyond the toe.

The thickness of the impervious blanket decreased uniformly in the upstream direction from 4m to 1m. The alluvium was compacted to a depth of approximately 25m to minimize settlements during both construction and reservoir impoundment. This densification was achieved with conventional compaction equipment and vibro-probes [4]. An additional layer of fine sand, approximately 1m thick, was dumped on top of the blanket. In the event that the blanket does crack, this cohesionless material would then be present to fill in the resulting gaps in the blanket; thereby limiting the effect that cracking could have on seepage.

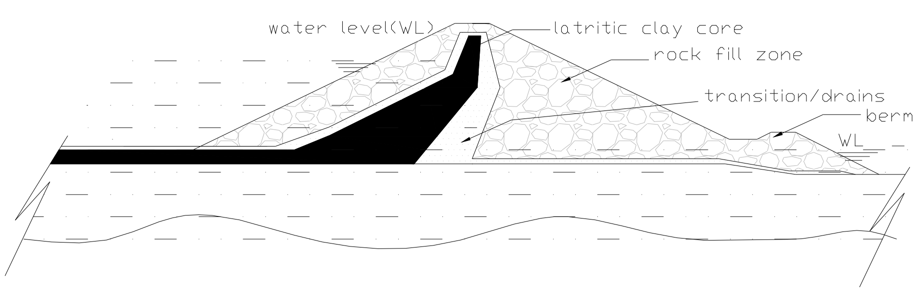

Figure 1: Jebba Main Dam Section

2.0 Prediction of crest settlement and deflections by empirical formulae

Some empirical formulas exist for the predictions of crest settlement and deformations in rock-fill dams. Such methods are based in terms of height of fill, the post-construction period or both. In such methods a single displacement for a given height and time is deduced. However, it should be noted that these method do not account for other factors such as location, fill material properties, etc., which influence the dam behaviour. Clements [7] pointed out that the error involved in this assumption is not always reported when establishing the displacement relationships.

A camber of 1% of the height of dam and anticipated foundation settlements (for dams less that 15m high) was recommended by the Bureau of Reclamation, U.S.A [8], for the design of small dams. Sowers et al [9], in their study of data from 14 dams, noting the heights, design cross-section, fill type and construction method, came up with the following relationship.

$$s = \frac{\alpha H}{100}\left(\log t_2 - \log t_1\right)$$

where, s =settlement, in meters; H = height, in meters; and t = time, in months.
The coefficient, α, was found to have values between 0.2; and 1.05 but no guidelines are given to determine appropriate values for other dams. Penman [10] observed that α might increase with time.

Parking [11, 12] believes that analysis based on total settlement is subject to uncertainty and alternative interpretation, whereas a rate analysis eliminates time-independent factors and amplifies imperfections in the data. This he observed after reviewing Sowers et al’s work using creep rate analysis. Clements has reported that the usefulness of a rate analysis is limited because it is difficult to distinguish the basic creep pattern from the irregularities during the post-construction period.

The work of Lawton and Lester [13] on 11dams, which were settling at less than 0.02% of their height per year, produced the formula below using best-fit analysis.

$$s = 0.001H^{\frac{3}{2}}$$

where, s = the total settlement; and H = the height in meters.
The time required for each dam to reach this state was not taken into account. However, they suggested 8-10year. Lawton and Lester show that the error in using this equation is up to 30% of the settlement predicted. Clements pointed out that this error is very significant for a large dam.

Soydemir and Kijaernshi [14, 15] combined displacement-time and displacement-height equations by producing displacement-height equations for different time periods. After studying 48 dams, they came out with the following equations;

$$s = \beta H^{\delta}$$

where, s = settlement, in meters, and H = height, in meters for two time periods.
The index and coefficient values were given in a table.
Soydemir and Kaernshi's work have been shown to be an over estimation of settlements [7].

Unfortunately, so many of these empirical relationships are published in some rockfill design texts and manuals [16-18]. Clements [7] suggested an alternative to simple empirical relationships with discrete solution, which is the use of a comparative prediction approach. This approach involves the prediction approach using only deformation curves of existing dams with similar characteristics to the dam under consideration. Clements arrived at this after studying 68 rockfill dams.

Consequently, it can be deduced that the use of empirical formulae in the analysis of rockfill dam can be erroneous and therefore unreliable. Therefore recourse has been taken of the Finite Element method, which is a numerical technique for solving problems that are described by partial differential equations or can be formulated as functional minimization (for instance, the variational formulation). The functional minimization is relevant in embankment analysis. [19-30]
In finite element approach, a domain of interest is represented as an assembly of finite elements. Approximating functions in finite elements are determined in terms of nodal values of a physical field, which is sought. A continuous physical problem is transformed into a discretised finite element problem with unknown values (in the embankment dam case displacements, stresses and strains). For a linear formulation, a system of linear algebraic equations should be solved. Two features of the finite element make it more reliable, viz:

(i) Piece-wise approximation of the physical field-the finite elements provide good precision even with simple approximating functions and increasing the number of elements can achieve any precision.
(ii) Locality of approximation leads to sparse equation systems for a discretised problem. This helps to solve problems with large number of nodal unknowns.
(iii) Displacements can be sought at any point in the discretised domain.

2.1 Finite Element Linear Formulations: Governing Equations for Zoned Rock Fill Dam Analysis-Plain Strain

From Variational Formulation of Finite Element [19-30]' the minimum potential energy for equilibrium conditions is given by equation (2.1)

$$\Pi = \int_V [B]^T [D][B] dV \{\delta\} - \{F\} = 0 \qquad (2.1)$$

This implies that

$$\int_V [B]^T [D][B] dV \{\delta\} = \{F\}, \qquad (2.2)$$

or

$$\{F\} = [K]\{\delta\},$$

where the stiffness matrix is given by equation (2.3)

$$[K] = \int [B]^T [D|B] dV \tag{2.3}$$

The volume integral is simply the elemental area is defined in equation (2.4)

$$\int_V dV = \int_v dxdy = \frac{1}{2}\begin{bmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_m & y_m \end{bmatrix} = A = area\ of\ triangle \tag{2.4}$$

For triangular displacement function,

$$[B] = \frac{1}{2A}\begin{bmatrix} b_i & 0 & b_j & 0 & b_m & 0 \\ 0 & c_i & 0 & c_j & 0 & c_m \\ c_j & b_i & c_j & b_j & c_m & b_m \end{bmatrix} , \tag{2.5}$$

where $a_i = x_{i+1}y_{i+2} - x_{i+2}y_{i+1}$ $b_i = y_{i+2}$, $c_i = x_{i+2} - x_{i+1}$

The strain transformation matrix [*B*] is found in equation (2.5) and it is based on shape function $a_{i,}$ b_i and c_i and other shape functions that can be derived by cyclic anticlockwise order. These shape functions in turn depend on the coordinates of nodal points, making the embankment base the origin. The deformations of the dam are given by equation (2.6).

$$\{\delta\} = [K]^{-1}\{F\} \tag{2.6}$$

Equation (2.7) gives the stress transformation matrix [D]

$$[D] = \frac{E}{(1+v)(1-2v)}\begin{bmatrix} 1-v & v & 0 \\ v & 1-v & 0 \\ 0 & 0 & \frac{(1-2v)}{2} \end{bmatrix} \tag{2.7}$$

3.0 Results presentation and analysis

3.1 Settlement history and models settlements predictions

Figures 2 and 3 show the settlement history of the Jebba Main Dam. The inclinometer readings became distorted as from 2002. Therefore, only the values up to 2001 are shown. During construction, embankment materials previously put in place and compacted were subjected to increasing loads from the weight of the subsequently placed layers.

From the settlement history, most of the settlements took place during construction and immediately after impoundment. After this point, the materials are being further compressed as the dam grows.

From the deformation analysis of the four models, two models were finally adopted. Model 1 (Blanket crack at 100m from the centre of the dam and 25m Densified Foundation), had a maximum crest settlement of 280mm, model 2 (Blanket crack at 100m from the center of dam and 68m to foundation bedrock), had maximum crest settlement of 565mm, while models 3 (Blanket crack at Dam Axis and 25m Densified Dam Foundation) and 4 (Blanket crack at Dam Axis and 68m to Foundation Bedrock) had 307mm maximum crest settlement and 0.01mm heave respectively.

Only models 1 and 3 exhibit deformations close to the actual dam behaviour on site. Therefore, they were adopted for further analysis.

The maximum crest settlement as at 2001 was 0.184m (184mm), while the predicted maximum possible crest settlements for the two models are 0.28m (280mm) for dam 1 and 0.307 (307mm) for dam 3. From these values the dam has settled up to 66% and 60% respectively considering the two models (see Figures 4 and 5). Originally, the maximum freeboard was 5m (5000mm) and this would only be reduced to about 4.7m (4700mm). This implies that overtopping is unlikely in the dam.

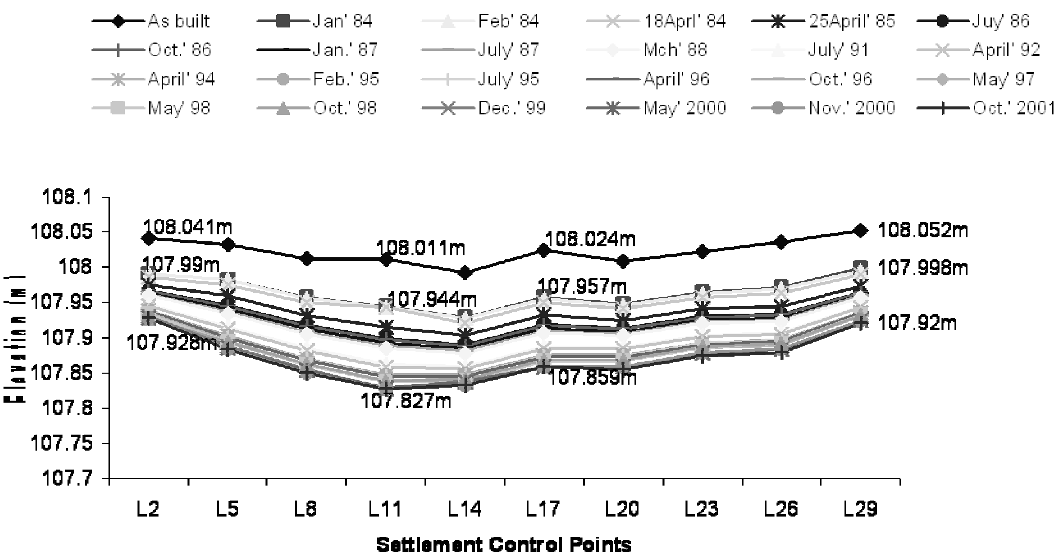

Figure 2: Changes in the Elevations of Jebba Main Dam Crest Control Points Showing Settlements

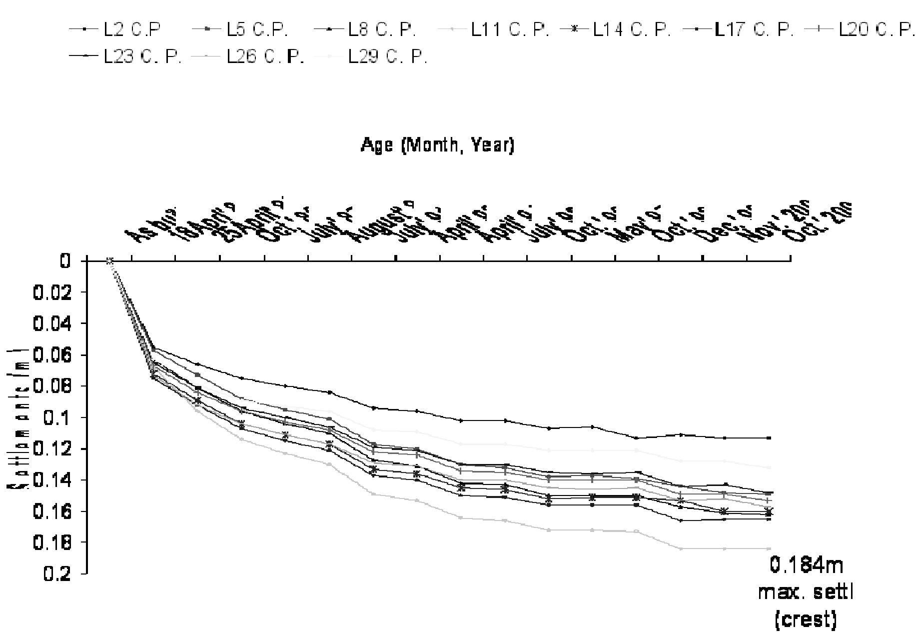

Figure 3: Settlements against Age for Jebba Main Dam Crest at Various Control Points

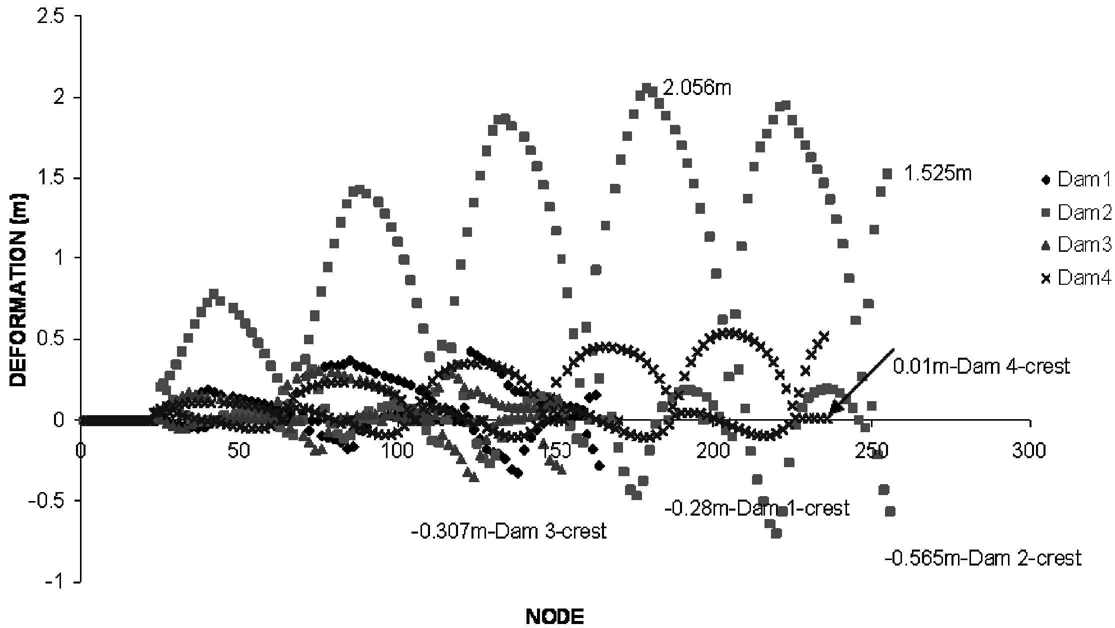

Figure 4: Predicted Crest Settlement Results for the Four Dam Models

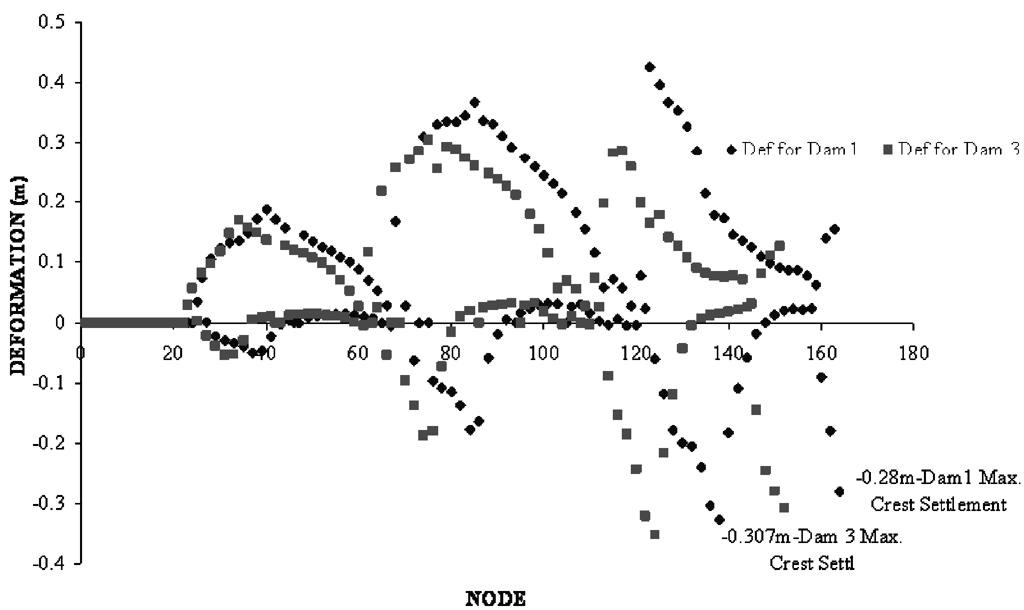

Figure 5: Settlements Results for Dams 1 and 3

3.2 Predicted Deformation Results

3.2.1 Dam Foundation

Deformation can be seen to alternate from positive to negative values, as the dam is tranversed from left abutment to the right abutment. This occurred at 5m from the foundation base (Figures 6 and 7 for vertical and horizontal deformations of Dam 1 and Figures 8 and 9 for vertical and horizontal deformations of Dam 3)

At 15m from 25m foundation base settlements and horizontal shifts, decreased from the left abutment (the upstream) to the berm (the downstream). The heave and horizontal shift (towards the upstream) was 0.187m (187mm occurring at 60m and 70m from the centre of the dam respectively. From Figures 3 and 5, the maximum foundation settlements were 0.05m and 0.053 for Dams 1 and 3 models. These occurred at 30m and 110m downstream from centre of dams respectively.

In dam 3, a maximum heave of 0.17m occurred at 40m downstream the centre of the dam. Also the maximum horizontal shift towards the berm was 0.17m at 50m from the centre of the dam. The horizontal deformation towards the upstream was observed to be 0.055m for dam 3 at 20m from the centre of dam. Dam 1 has a horizontal upstream shift of 0.05m at 40m from the centre of the dam.

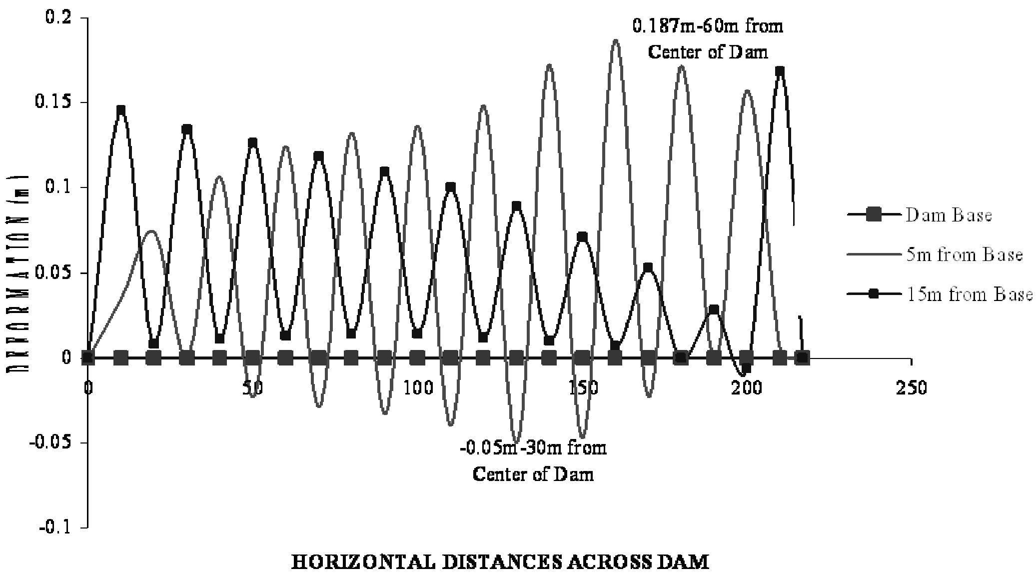

Figure 6: Vertical Deformations in the Dam Foundation- Dam 1

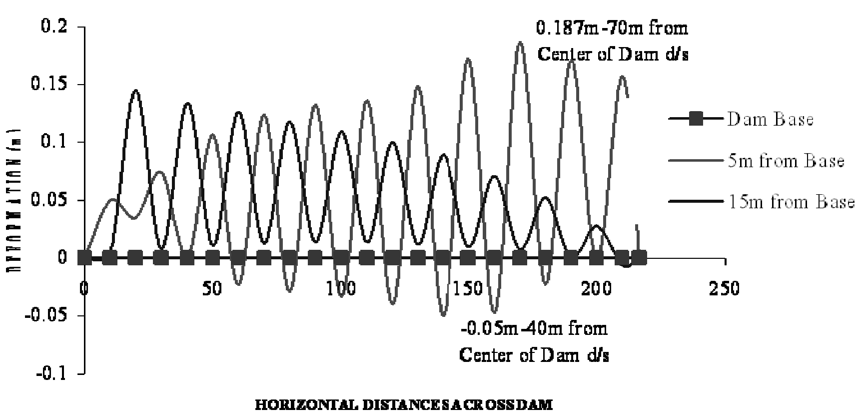

Figure 7: Horizontal Deformation Distributions in the Foundation of Dam 1

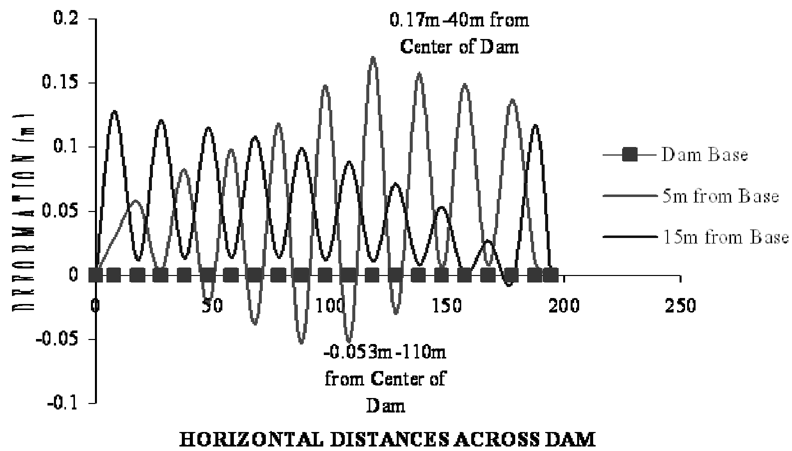

Figure 8: Vertical Deformations in the Dam Foundation- Dam 3

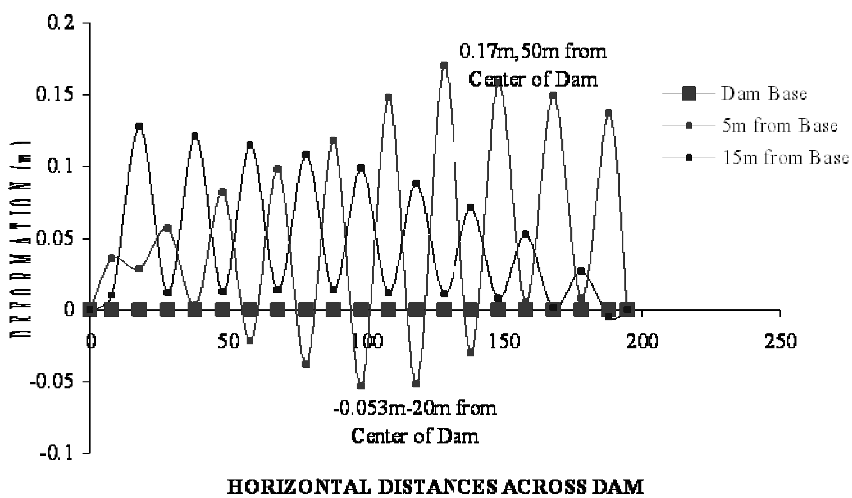

Figure 9: Horizontal Deformation Distributions in the Foundation of Dam 3

The small values of settlements shown reflect the high level of compaction achieved in the dam foundation. As observed 5m from the foundation base, the alternations of signs in the deformation values indicate high stress reversals towards the foundation base. Owing to the reservoir push, the horizontal shifts tend to be more at the beam, than the upstream. This was observed to be at the middle of dam foundation. At the base of the body of dam, the trend is reversed, the horizontal shift being more at the upstream.

It can be observed that the upstream horizontal deformation in the dam foundation is the same as the vertical (settlement) deformation for the two dam models. In dam 1, the horizontal upstream shift was 0.05m (Figures 6 and 7), which is exactly the same value as that of settlement. Similarly, in dam 3, the horizontal upstream shift is 0.053m and is the same as settlement (see Figures 8 and 9). However, these values did not occur at the same locations. A close look at the deformation of the dam foundation reveals that the maximum heave is the same with the maximum downstream shift for the tow model. Dam 1 has a 0.187m maximum horizontal shift while the maximum heave has the same value (found in Figures 6 and 7). Dam 3 has a maximum horizontal shift of 0.17m, which is same with the maximum heave (Figures 8 and 9).

3.2.2 **Dam Body**

Figures 10, 11, 12 and 13 show the variations of the vertical and horizontal deformations in dam models 1 and 3 against height. Generally, the deformation was more within the dam and reduces towards the berm. This was expected because of the greater load at the dam due to water pressure and self-weight. As can be seen from Figures 10 and 11, the maximum settlement of 0.351m occurred at 50m downstream of the centre of dam and 43.4m from the foundation base.

Also, the maximum heave expected within the lifetime of the dam was 0.426m (426mm), which occurred at 29.95m downstream of dam centre, and 35m from the base of the dam.

From the horizontal deformation values, a shift of 0.426m occurred at 40m downstream of dam centre and 35m from the foundation base. The maximum backward movement occurred at 10m downstream of dam centre and 45m from the base of dam and was 0.351m (351mm).

It can be deduced that the deformations of the dam foundation and dam body followed expected patterns. No extraordinary or alarming values were observed. However, the alternating values (from positive to negative values) observed at 5m from the foundation base and the body of the dam can be attributed to serious stress reversals. Although this is not unusual considering the loading of dam due to water loading, it can pose some distress if negative stress values are observed at the dam core.

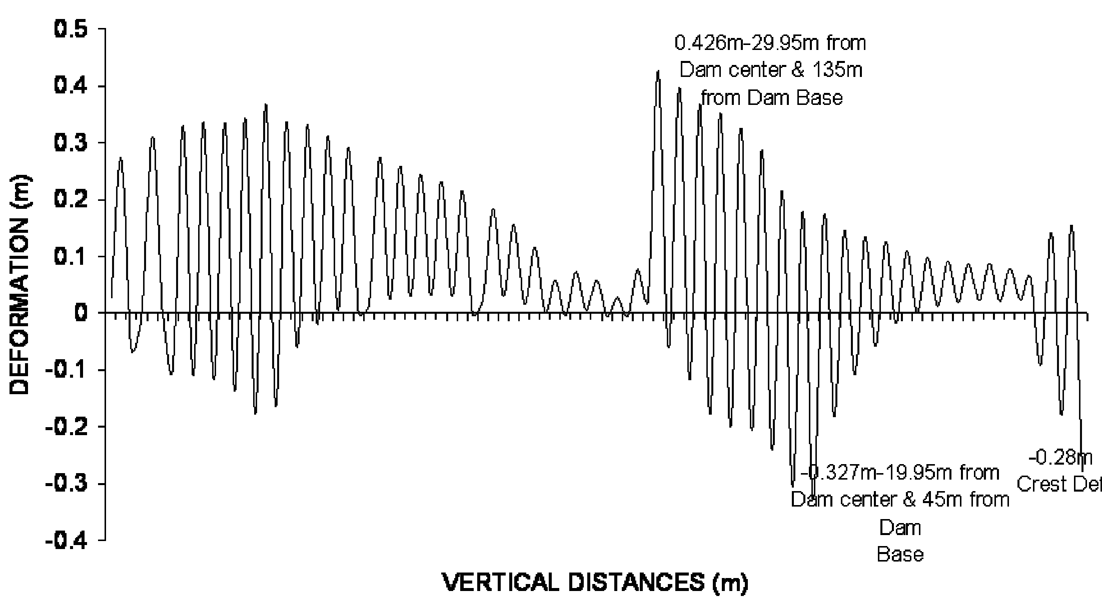

Figure 10: Vertical Deformation Distributions in the Body of Dam 1

Thomas [31] (Table 1) showed that longitudinal cracks could develop as a result excessive settlements as large as 220mm or more. Already, the 2002 inclinometer reading, which is distorted, showed a crest settlement of 0.28m (280mm) in the Jebba Main Dam. Also from the table, it can be deduced that hydraulic fracturing is likely.

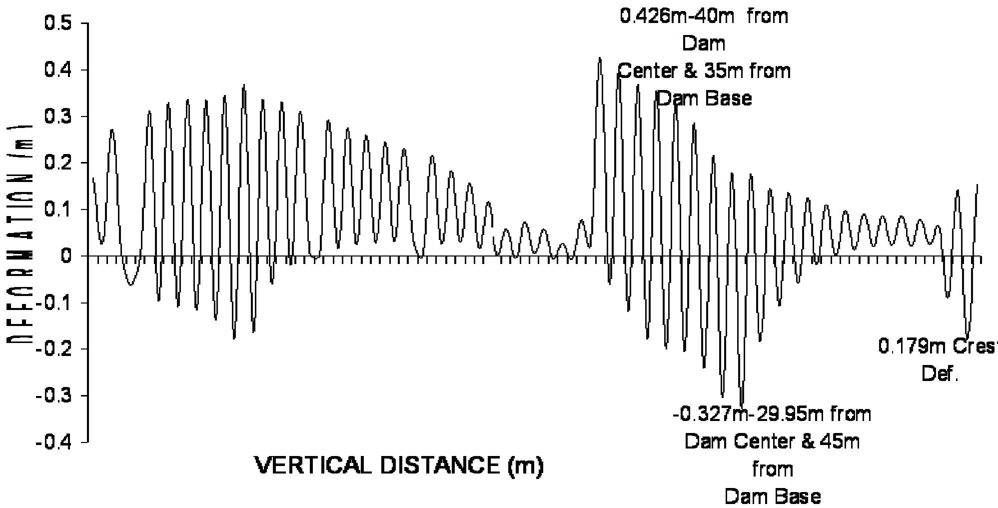

Figure 11: Horizontal Deformation Distributions in the Body of Dam 1

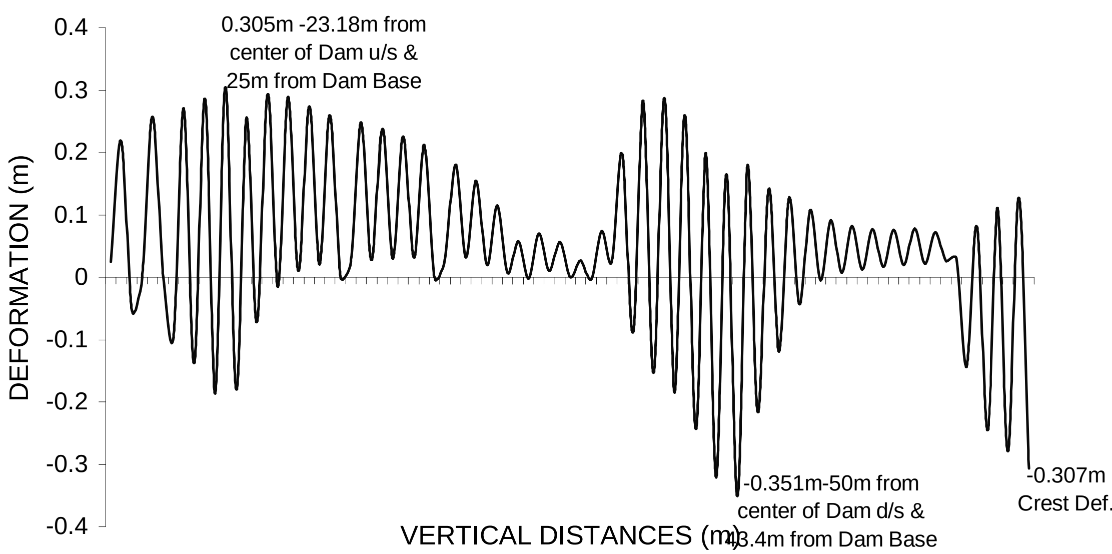

Figure 12: Vertical deformations Distribution in the Body of Dam 3

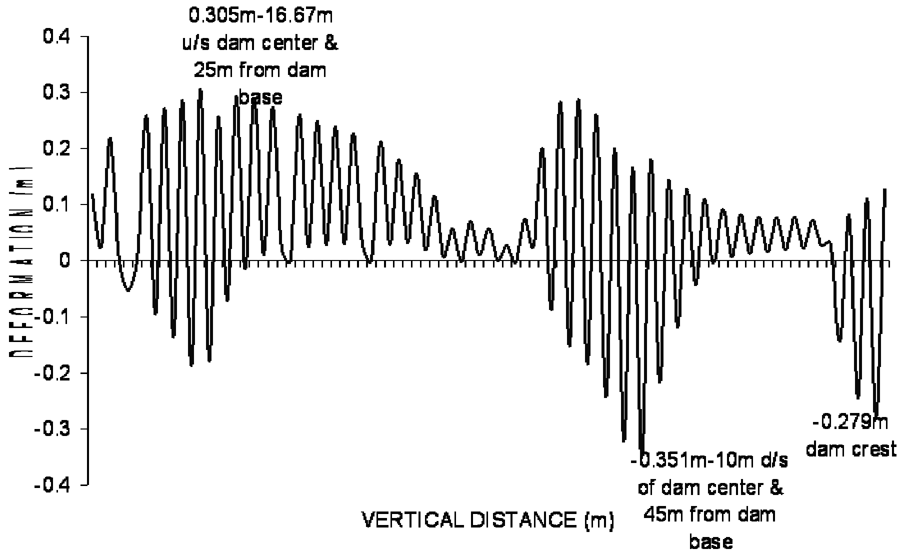

Figure 13: Horizontal Deformation Distributions in the Body of Dam 3

Table 1: Influence of Post Construction Settlement at Crest on Cracking (*After Thomas*) [31]

Crest Settlement (mm)	Kind of Cracking
Less than 50	No cracking of dams
Equal or greater than 50	Transverse cracking of dams compacted dry may appear
Greater than 100	Reinforced concrete facing without perimetral joint may crack
Equal or greater than 130	Longitudinal cracking between core and shell may appear
Greater than 160	Longitudinal cracking of core compacted dry may appear
Greater than 180	Hydraulic Fracturing may appear
Equal or greater than 220	Transverse cracking of core compacted wet may appear. Longitudinal cracking between core compacted wet and shell may appear
Equal or greater than 350	Asphaltic concrete facing may crack (for settlement of 350mm)
Greater than 400	Longitudinal cracking of core compacted wet may appear. Reinforced concrete facing with perimetral joint will crack
Greater than 1000	No uncracked dams in those studied
Greater than 1200	All dams exhibit transverse cracking
Equal or greater than 1400	Serious cracking of asphaltic concrete facing
Equal or greater than 3800	Cracking needing substitution of reinforced concrete facing

4.0 Conclusions

The finite element method of deformations distribution analysis using the total stress analysis approach was employed in the analysis of the *Jebba Main Dam* section for predictions of deformations. The actual fill properties were utilized and the two loading scenarios – no water and full reservoir operations – were investigated. At full reservoir, the actual seepage line, as recorded over the years was used.

Following the finite element mathematical modeling and computer simulations of the models, deformations were determined. The foundation deformations were found to be relatively small; this is due primarily to the level of densification achieved in the foundation during construction. This would have probably reduced the general settlements in the dam. The predicted settlements showed that the *total stress finite element analysis* carried out compared favourably with the actual settlement history of the dam structure.

The Main Dam impervious core is inclined upstream to allow the dam section to deform under its own weight without the development of discontinuities. Cracks frequently occur in earthfill dam and in cores of rockfill dams. The Jebba Main Dam has shown from actual settlement results and predicted values to be distressed. From Thomas guidelines (Table 1), crest settlements may have led to the longitudinal cracks observed at the core of the Jebba Main Dam. The observed cracks at the crest of the dam also may have resulted from hydraulic fracturing.

It can therefore be deduced that the crack at the Jebba Main Dam started from the core and extended to the crest. It is believed that the crack may not have stabilized because the size, as observed in 2002, was slightly increasing.

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