Journal of the Nigerian Association of Mathematical Physics, Volume 9 (November 2005)

Performances of estimators of linear auto-correlated error model with exponential independent variable.

J. C. Nwabueze
Dept of Statistics, Abia State University, Uturu, Nigeria.

Abstract

The performances of five estimators of linear models with autocorrelated disturbance terms are compared when the independent variable is exponential. The results reveal that for both small and large samples, the Ordinary Least Squares (OLS) compares favourably with the Generalized least Squares (GLS) estimators in respect of bias property. On the basis of variance and root mean square error property, OLS compares favourably with maximum likelihood (ML) and Maximum Likelihood Grid (MLGRID) estimators for small autocorrelation coefficient of the error term ρ but it appears uniformly superior to Cochrane-Orcutt (COC) and Hildreth and LU (HILU) estimators especially when ρ is large.

Keywords: Auto-correlated, Disturbance, Model, Estimators, Exponential.

pp 385 - 388

1.0 Introduction

Autocorrelation of the error terms included in econometric models has remained a major characteristic of most time series data. Many models in corporating auto-correlated error terms have been discussed in the literature. The variety of scenarios in which time series observations can be plagued by auto-correlated disturbances are so many that inspite of numerous analytical and empirical contributions already made on this subject, the available diagnostic procedures and competing corrective estimation methods leave many questions yet unanswered.

Although some authors like Chipman (1979) [1], Kadiyala (1968) [4] have argued that the efficiency of the estimators at the finite level depends much on the specification of the independent variable used in the experiment, there is still much need to investigate the finite sampling properties of these estimators.

Some researchers like Godfrey (1978) [2] have tried to give a general approach to the treatment of autocorrelation when they occur in linear models. However, the treatment of each type as it occurs specifically in a model has always given better results. Iyaniwura and Nwabueze (2004) [3] in their work estimated the Auto-correlated error linear model with Gross national product (GNP) data as the independent variable and found out that the estimators COC and HILU performed worse than OLS while MLDRID and ML performed better than OLS. Also, Nwabueze (2005) [5] in a work on Auto-correlated error linear model discovered that when the independent variable is autoregressive, the variance of the slope coefficient of the OLS increases very sharply with increasing value of ρ, the autocorrelation coefficient of the error term and it is always larger than the variance of GLS.

Therefore, this study shall have as its main focus, the performances of estimators of linear models with first order auto-correlated disturbance terms when the independent variable is exponential.

Rao and Grilliches (1969) [6] gave one of the earliest known Monte Carlo works on this study. They used a model of the type

$$\begin{aligned} &Y_t = \beta X_t + U_t,\ \ X_t = \lambda X_{t-1} + V_t,\ \ U_t = \rho U_{t-1} + \varepsilon_t, \\ &E(V_t) = E(\varepsilon_t) = E(V_t\varepsilon_t) = \sum(\varepsilon_t\varepsilon_{t-1}) = E(V_tV_{t-1}) = 0 \qquad (1.1) \\ &E(V_t^2) = \sigma_v^2,\ E(\varepsilon_E^2) = \sigma\varepsilon^2,\ |\lambda| \prec 1,\ |\rho| \prec 1,\ t = 1,\cdots,T \end{aligned}$$

Where, Y_t is the dependent variable, β is the regression parameter, U_t is the disturbance term of Y_t, X_t is the independent variable, V_t is the disturbance term of X_t, λ is the autocorrelation coefficient of X_t, ρ is the autocorrelation coefficient of U_t, ε_t is the disturbance term of U_t and $E(\ )$ stands for the expected value of the variable in the bracket.

2.0. MODEL SPECIFICATION

The following single equation linear econometric model with autocorrelated disturbances was used: $$Y_t = \beta_1 + \beta_2 X_t + U_t,\ U_t = \rho U_{t-1} + \varepsilon_t,\ |P| \prec 1,\ t = 1,2,\cdots,T \qquad (2.1)$$

The independent variable X_t was assumed to be exponential given as X_t= exp (0.4t).

3. SIMULATION PROCEDURE

In econometrics, while asymptotic properties of estimators obtained by using various econometric techniques are deduced from postulates, an approach that is often described as analytical, small sample properties of such estimators have always been studied from simulated data in a method known as Monte Carlo studies. This work uses Monte Carlo approach.

The parameter values of β_1 and β_2 in the model were fixed at (1,1). To generate the multivariate normal vectors used for this study, the autocorrelated error term $U_t = \rho U_{t-1} + \varepsilon_t$ was first generated. Then the independent variable X_t = Exp (0.04t) was also computed. Thereafter, the multivariate normal dependent vector Y was computed using equation (2). The generation of the error terms, the independent variable and the computation of the dependent variable are made using a package for econometric studies called time series processor (TSP).

The simulation experiment was replicated 50 times. The sample sizes were varied from 20, 40 to 60 in order to study the effect of sample size on the performance of the estimators. Since the study is investigating the performances of the estimators when the error term is autocorrelated, three different estimates of the autocorrelation coefficient of the error term ρ= 0.4, 0.8 and 0.9 were used. After the generation of the data, different estimation methods were applied to the data using the AR (1) functions of TSP software package on an IBM computer at the center for econometric and Allied research (CEAR) University of Ibadan. The deviations of the simulated values from the original data series based on the estimators were then assessed using simulation statistics. The simulation statistics used in assessing the performances of the estimators in this study are bias, sum of bias of both the intercept and the slope coefficients (SBIAS), the variance, sum of variance of both the intercept and the slope coefficients (SVARS), the root mean square error (RMSE), and the sum of root mean square error (SRMSE) of both the intercept and the slope coefficients. Five estimators were used for this study namely OLS, HILU, COC, ML and MLGRID. The four other estimators apart from the OLS are called Generalized Least squares estimators (GLS).

4.0 Results of the simulation experiment

Table 1 reveals that on the basis of Bias property, the estimators compare favourably with one another for instance at a sample size of 20, when the autocorrelation coefficient is 0.4, all the estimators overestimated the intercept β_1 and also underestimated the slope coefficient β_2. A more remarkable result is seen in table 2, where the SVAR of COC and HILU are much higher than the SVAR of OLS while the SVAR of OLS is higher than either the SVAR of MLGRID or that of ML. And this observation cuts across all the sample sizes considered in this experiment. Table 2 shows the SVARS of the estimators as 3.377270, 4.479924, 4.312851, 3.085169 and 3.010210, 20.275750, 28.411510, 30.012930, 15.361710 and 15.197930, 38.571220, 59.903180, 58.387400, 31.279830 and 3.690590 when T = 20 and ρ = 0.4, 0.8 and 0.9 for OLS, COC, H1LU, MLGRID and ML respectively. This result shows that OLS is preferred to COC and H1LU even for large samples.

Therefore, our results suggest that from cost/benefit of view, we should not estimate this model using either COC or H1LU even when the sample size T is large. From table 3, we also observe that the SRMSE of COC and H1LU are much higher than the SRMSE of OLS both for small and large autocorrelation coefficient ρ. As the degree of autocorrelation coefficient ρ increases, table 3 also shows that the SRMSE of OLS is higher than either the SRMSE of MLGRID or that of ML.

5.0 Conclusion

In summary, the major conclusions which could be drawn from our experiment are the following.

For both small and large samples, OLS compares favourably with the GLS estimators in respect of BIAS properties. On the basis of VAR and RMSE properties, OLS compares favourably with ML and MLGRID for small ρ but it appears uniformly superior to COC and H1LU especially when ρ is large. However, for large autocorrelation of the error term, MLGRID and ML dominate OLS.

Table: 1 The Use Of Bias To Compare The Estimators

		T = 20			T = 40			T = 60		
ρ	Estimators	β_1	β_2	SBIAS	β_1	β_2	SBIAS	β_1	β_2	SBIAS
	OLS	-0.185509	0.105000	0.290509	-0.067968	0.038473	0.106441	0.003960	-0.000274	0.004234
	COC	0.216399	0.062795	0.279194	-0.047372	0.023931	0.071302	-0.006690	0.001544	0.008234
0.4	HILU	-0.214580	0.121521	0.336101	-0.047270	0.019351	0.066621	-0.019360	0.001540	0.020900
	MLGRID	-0.220608	0.126495	0.347103	-0.023787	0.017620	0.041407	-0.006399	0.001490	0.007889
	ML	-0.193240	0.127190	0.320430	-0.042800	0.014620	0.057420	-0.04460	-0.000655	0.005115
	OLS	0.169905	-0.084700	0.254605	-0.140350	0.052700	0.193050	0.0515041	-0.029690	0.018010
	COC	-0.274413	0.143459	0.417872	-0.012100	0.038201	0.050301	0.000491	0.000341	0.000832
0.8	HILU	0.336209	0.172009	0.508218	-0.098278	0.043390	0.141668	-0.426200	-0.000495	0.426695
	MLGRID	-0.369410	-0.130079	0.499489	-0.048974	0.022391	0.071365	0.010638	0.002680	0.013318
	ML	-0.223170	0.127192	0.350362	-0.052400	0.023820	0.076220	0.011502	0.001530	0.013032
	OLS	-0.052185	0.047897	0.100082	-0.184890	0.083236	0.268126	-0.004390	0.006044	0.010434
	COC	-1.185330	0.311500	1.496830	-0.143660	0.056600	0.200260	-0.094860	0.001357	0.096217
0.9	HILU	0.525850	0.307560	0.833410	-0.188530	0.066760	0.255290	0.066270	-0.006788	0.073058
	MLGRID	0.048486	-0.007442	0.055928	-0.050230	0.028501	0.078731	0.040778	-0.018592	0.059370
	ML	0.155310	-0.012560	0.167870	-0.080460	0.040850	0.121310	-0.008963	-0.006300	0.015263

Table 2: The Use of Variance to Compare the Estimators

		T = 20			T = 40			T= 60		
ρ	Estimators	β_1	β_2	SVAR	β_1	β_2	SVAR	β_1	β_2	SVAR
	OLS	2.406800	0.970470	3.377270	0.229740	0.012820	0.242560	0.091800	0.004879	0.096679
	COC	3.112170	1.367754	4.479924	0.284569	0.041884	0.326448	0.093950	0.004880	0.098830
0.4	HILU	3.138259	1.174592	4.312851	0.2845471	0.040850	0.326321	0.095415	0.004880	0.100295
	MLGRID	2.203972	0.881197	3.085169	0.222670	0.035598	0.258268	0.093877	0.004920	0.098797
	ML	2.137080	0.873130	3.010210	0.235946	0.034800	0.270746	0.093560	0.005350	0.098910
	OLS	15.046600	5.229150	20.275750	1.726640	0.214560	1.941200	0.714265	0.032025	0.746290
	COC	21.614620	6.796890	28.411570	2.272247	0.363177	2.635424	0.791930	0.033000	0.824930
0.8	HILU	22.960910	7.052020	30.012950	2.415090	0.288857	2.703947	0.801612	0.033060	0.834672
	MLGRID	1.374570	3.987140	15.361710	1.684909	0.213866	1.898777	0.703618	0.029040	0.732658
	ML	11.428610	3.769320	15.197930	1.039490	0.207096	1.246586	0.708740	0.030050	0.738790

ρ	Estimators	β1	β2	SBIAS	β1	β2	SBIAS	β1	β2	SBIAS
	OLS	0.225080	8.346140	38.571220	4.726377	0.498291	5.224668	2.536550	0.092020	2.628570
	COC	7.592600	12.310580	59.903180	7.222000	0.672180	7.894180	3.106450	0.090630	3.107080

0.9 HILU	6.243220	12.144180	58.387400	6.725300	0.637500	7.362800	2.683700	0.087320	2.771020
MLGRID	4.748960	6.530870	31.279830	4.569444	0.431770	5.001214	2.357850	0.072850	2.430700
ML	4.253050	6.437540	30.690590	4.517200	0.424280	4.941480	2.273050	0.072400	2.345450

Table 3: The Use of RMSE to Compare the Estimators

		T = 20			T = 40			T= 60		
ρ	Estimators	β_1	β_2	SRMSE	β_1	β_2	SRMSE	β_1	β_2	SRMSE
	OLS	1.562438	0.990704	2.553142	0.484107	0.119583	0.603690	0.303011	0.069850	0.372861
	COC	1.777357	1.171195	2.948662	0.535548	0.206050	0.741598	0.306586	0.069874	0.376460
0.4	HILU	1.784462	1.090578	2.875040	0.536382	0.203038	0.739420	0.309499	0.069873	0.379372
	MLGRID	1.500880	0.947205	2.448085	0.472478	0.189495	0.6611973	0.306460	0.070159	0.376619
	ML	1.474592	0.943031	2.417623	0.487625	0.187120	0.6747440	0.305908	0.073147	0.379055
	OLS	3.882714	2.288302	6.171016	1.321491	0.466195	1.787686	0.845276	0.178980	1.024256
	COC	4.657244	2.611029	7.268273	1.507446	0.603851	2.111297	0.889905	0.181659	1.071564
8.0	HILU	4.803535	2.661129	6.464664	1.557160	0.539203	2.096363	0.991594	0.181825	1.173419
	MLGRID	3.392791	2.001015	5.393806	1.298964	0.462998	1.761962	0.838887	0.170432	1.009319
	ML	3.387981	1.945636	5.333617	1.020899	0.455701	1.476600	0.841946	0.173356	1.015302
	OLS	5.497982	2.89360	8.387342	2.181871	0.710788	2.892659	1.592661	0.303408	1.896069
	COC	6.999829	3.522444	10.522273	2.691215	0.821817	3.513032	1.739382	0.301051	2.040433
0.9	HILU	6.820538	3.498396	10.318934	2.660162	0.801222	3.401384	1.639540	0.295578	1.935118
	MLGRID	4.975069	2.555568	7.530637	2.138216	0.657710	2.795926	1.536071	0.270547	1.806618
	ML	4.927187	2.537262	7.464449	2.126893	0.652648	2.779541	1.507690	0.269146	1.776836

References

[1] Chipman, J.S. (1979). Efficiency of least Squares Estimation of linear Trend when Residuals Are Autocorrelation Econometrica. Vol. 47 PP 115 – 127.

[2] Godfrey, L.G.C. (1978). Testing Against General Autoregressive and Moving Average Error Model When The Regressors include Lagged Dependent Variables. Econometrica. Vol. 46 PP 1293 – 1301.

[3] Iyaniwura, J.O and Nwabueze, J.C. (2004). Estimating the Autocorrelated Error model with GNP data. Journal of the Nigerian Statistical Association. Vol. 17, PP 29 – 39.

[4] Kadiyala, K. R. (1968). A Transformation used to Circumvent The Problem of Autocorrelation Econometrica. Vol 36, pp 93-96.

[5] Nwabueze, J. C. (2005). Performances of Estimators of linear Models with Autocorrelated error terms when the independent variable is Autoregressive. Global Journal of Pure and Applied Sciences. Vol. 11, No. 1. pp 131-135.

[6] Rao, P and Grilliches, Z. (1969). Small sample properties of several two stage Regression methods in the Context of Autocorrelated Errors. Journal of American Statistical Association. Vol. 64, pp 251 – 272.