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Explicit presentation of the Colebrook's friction factor equation

P. U. Ohirhian,
Department of Petroleum Engineering,
University of Benin, Benin City, Nigeria

Abstract

Two explicit and very accurate equations for calculating the friction factor of pipes over the entire range of relative roughness and Reynold's Number covered by the Colebrook's Equation have been developed. A rectangular array of relative Roughness and Reynold's Number was used to test the accuracy of the new equation. The first equation had over 85% of the data points exactly (up to 5 decimal places) as the Colebrook's values. Errors in the remaining table values range between **±0.0555%.** *The second equation, which is an extension of the first equation, simply duplicates Colebrook's iterative solution.*

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1.0 Introduction

In 1938, Colebrook combined two separate equations for calculating the friction factors of smooth and rough pipes into one single equation. This equation as useful as it is does not solve for friction factor directly.

Iterative solution of a non-linear equation (such as the Colebrook's equation) requires a good grasp of numerical analysis to yield an accurate solution. Further, iterative solutions are more time consuming than the direct solutions. This limitation of the Colebrook's Equation has led many researchers to the formulation of direct solutions to the original Colebrook's Equation.

Many of the direct solution to the to the Colebrook's Equation currently in use are not as accurate as the Colebrook's Equation. According to Ouyang l. and Aziz K (1996) [5], 'most of the explicit … have their own validity range of Reynolds numbers and relative pipe roughness. Thus the search for better direct equations continues.

In this work we present two explicit solutions to the original Colebrook's equation. The second equation is an extension of the first equation. The second equation simply duplicates the iterative solution of the Colebrook's equation and thus can be regarded as an explicit presentation of the Colebrook's equation.

2.0 Development of the equations

The Colebrook's Equation can be written as:

$$f(x) = x + 2\log(a + bx) = 0 \tag{2.1}$$

where

$$a = \frac{\in}{3.7d}, \; b = \frac{2.51}{R}, \; x = \frac{1}{\sqrt{f}}$$

$\frac{\in}{d}$ = relative roughness of pipe

R_N = Reynold's Number

f = friction factor of pipe

Equation (2.1) is a non-linear equation which can be solved by well known numerical methods. Examination of equation 1 shows that it is of the type $x = f(x)$. An iterative solution to this type of Equation is:

$$x_{i+1} = f(x_i) \tag{2.2}$$

The convergence criterion is:

$$\left| f(x_{i+1}) - f(x_i) \right| \leq \text{Tolerance} \tag{2.3}$$

The Newton-Raphson Algorithm can also be used to solve equation (2.1). The Newton–Raphson Algorithm is:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \tag{2.4}$$

The convergence criterion is also that given inequality (2/3). Application of the Newton–Raphson Algorithm to Equation (2.1) yields

$$f_{i+1} = \left[x_i - \frac{(c_i x_i + 2c_i \log c_i)}{(0.8686\, b + c_i)} \right]^{-2} \tag{2.5}$$

where $c_i = a + b x_i$.

Depending on the choice of x_0 needed to start Algorithm (4); it can take several iterations to converge. Ohirhian (2004) [4] found empirically that

$$x_0 = -1.14 \log\left(\frac{\in}{d} + 0.30558 \right) + 0.57 \log R_N \left(0.01772 \log R_N + 1.6093\right)$$

gave a fairly accurate solution to algorithm (4) in only one iteration. This same equation for x_0 in algorithm (2.1) also yielded another fairly accurate solution in 1 iteration. This solution is:

$$x = -2\log(a + bx_0) \tag{2.6}$$

Substitution of this solution (given in equation (2.5)) into algorithm (4) gives the first equation which is able to calculate friction factor (f) within the range – 0.0555% to 0.02828%. The equation is

$$f = \left[x - \frac{(cx + 2c\log c)}{(0.8686b + c)} \right]^{-2} \tag{2.7}$$

where:

$$c = a + bx$$
$$x = -2\log\left(a + bx_0\right)$$
$$x_0 = -1.14\log\left(\frac{\in}{d} + 0.30558\right) + 0.57\log R_N\left(0.01772\log R_N + 1.6093\right)$$
$$b = \frac{2\cdot 51}{R_N},\quad a = \frac{\in}{3\cdot 7d}$$

Equation (2.7) was programmed in visual basic over the entire range of relative roughness ($\in/d$) and Reynolds Number (R_N) for which the Colebrook's Equation is valid and the result is presented in Table1. Internal computation was 9 decimal places floating point arithmetic and final results were rounded to 5 decimal places. Application of x_0 as starter in algorithm (2), after two iterations yields:

$$x = -2\log\left(a - 2b\log\left(a + bx_0\right)\right)$$

Substitution of this new value of x as initial guess in algorithm (2) yields the super accurate equation which we can call the explicit representation of the Colebrook's Equation.
The equation is

$$f = \left[x - \frac{\left(cx + 2c\log c\right)}{\left(0.8686b + c\right)}\right]^{-2} \qquad (2.8)$$

where

$$c = a + bx$$
$$x = -2\log\left(a - 2b\log\left(a + bx_0\right)\right)$$
$$x_0 = -1.14\log\left(\frac{\in}{d} + 0.30558\right) + 0.57\log R_N\left(0.01772\log R_N + 1.6093\right)$$
$$b = \frac{2\cdot 51}{R_N},\quad a = \frac{\in}{3\cdot 7d}$$

Computations of *f* with the equation (2.8) are shown in Table 2. Internal computation was also 9 decimal place arithmetic with final results rounded to 5 decimal places.

3.0 Accuracy of the new equations

The iterative solution of the Colebrook's equation shown in algorithm (4) was programmed in visual basic with an initial guess of 4.5. It took 4 to 6 iterations to converge with a tolerance of 10^{-6}. Internal computation was also 9 decimal place arithmetic with final results rounded to 5 decimal places. The results are shown in Table 3. Table 3 was used as a standard to compare the results presented in tables 1 and 2. The errors that resulted from comparing the friction factors from equation (2.7) with the standard values in Table 3 are shown in Appendix I.

3.1 The output table of our computation

Relative roughness entered are:

0, 0.00001, 0.00005, 0.0001, 0.0004, 0.001, 0.004, 0.01, 0.02, 0.03, 0.04, 0.05,

Reynolds numbers entered are:

4000, 10000, 50000, 100000, 500000, 1000000, 5000000, 10000000, 50000000, 100000000,

The output taking one $\frac{\in}{d}$ and all R_N is displayed per line:

Relative-roughness	4E3	1E4	5E4	1E5	5E5	1E6	5E6	1E7	5E7	1E8
	Reynolds Number									
0	0.03992	0.03089	0.02089	0.01799	0.01316	0.01165	0.00898	0.00810	0.00649	0.00594
0.00001	0.03993	0.03090	0.02093	0.01805	0.01330	0.01187	0.00959	0.00900	0.00830	0.00819
0.00005	0.03997	0.03097	0.02107	0.01826	0.01384	0.01265	0.01114	0.01086	0.01061	0.01058
0.0001	0.04002	0.03104	0.02125	0.01852	0.01443	0.01344	0.01234	0.01217	0.01202	0.01200
0.0004	0.04032	0.03150	0.02225	0.01991	0.01701	0.01649	0.01602	0.01596	0.01591	0.01590
0.001	0.04092	0.03238	0.02402	0.02217	0.02024	0.01994	0.01970	0.01967	0.01964	0.01964
0.004	0.04379	0.03641	0.03048	0.02950	0.02864	0.02853	0.02844	0.02843	0.02842	0.02842
0.01	0.04908	0.04313	0.03908	0.03850	0.03803	0.03796	0.03792	0.03791	0.03790	0.03790
0.02	0.05696	0.05227	0.04941	0.04903	0.04872	0.04868	0.04865	0.04864	0.04864	0.04864
0.03	0.06408	0.06011	0.05778	0.05748	0.05724	0.05720	0.05718	0.05718	0.05717	0.05717
0.04	0.07070	0.06719	0.06519	0.06493	0.06472	0.06470	0.06468	0.06467	0.06467	0.06467
0.05	0.07699	0.07380	0.07201	0.07178	0.07160	0.07157	0.07156	0.07155	0.07155	0.07155

Table 1: Friction factors calculated by the first of the new equations

3.2 The output table of our computation

Relative roughness entered is:

0, 0.00001, 0.00005, 0.0001, 0.0004, 0.001, 0.004, 0.01, 0.02, 0.03, 0.04, 0.05,

Reynolds numbers entered are:

4000, 10000, 50000, 100000, 500000, 1000000, 5000000, 10000000, 50000000, 100000000,

The output taking one $\frac{\in}{d}$ and all R_N is displayed per line:

Relative-Roughness	4E3	1E4	5E4	1E5	5E5	1E6	5E6	1E7	5E7	1E8
	Reynolds Number									
0	0.03991	0.03088	0.02089	0.01799	0.01316	0.01165	0.00898	0.00810	0.00649	0.00594
0.00001	0.03992	0.03090	0.02093	0.01804	0.01330	0.01187	0.00959	0.00900	0.00830	0.00819
0.00005	0.03996	0.03096	0.02107	0.01826	0.01384	0.01265	0.01114	0.01086	0.01061	0.01058
0.0001	0.04001	0.03104	0.02125	0.01851	0.01443	0.01344	0.01234	0.01217	0.01202	0.01200
0.0004	0.04031	0.03149	0.02225	0.01991	0.01701	0.01649	0.01602	0.01596	0.01591	0.01590
0.001	0.04091	0.03238	0.02402	0.02217	0.02024	0.01994	0.01970	0.01967	0.01964	0.01964
0.004	0.04379	0.03641	0.03048	0.02950	0.02864	0.02853	0.02844	0.02843	0.02842	0.02842
0.01	0.04908	0.04313	0.03908	0.03850	0.03803	0.03796	0.03792	0.03791	0.03790	0.03790
0.02	0.05696	0.05227	0.04941	0.04903	0.04872	0.04868	0.04865	0.04864	0.04864	0.04864
0.03	0.06408	0.06011	0.05778	0.05748	0.05724	0.05720	0.05718	0.05718	0.05717	0.05717
0.04	0.07070	0.06719	0.06519	0.06493	0.06472	0.06470	0.06468	0.06467	0.06467	0.06467
0.05	0.07699	0.07380	0.07201	0.07178	0.07160	0.07157	0.07156	0.07155	0.07155	0.07155

Table 2: Friction factors calculated by the second of the new equations

Relative roughness entered is:

0, 0.00001, 0.00005, 0.0001, 0.0004, 0.001, 0.004, 0.01, 0.02, 0.03, 0.04, 0.05,

Reynolds Numbers entered are:

4000, 10000, 50000, 100000, 500000, 1000000, 5000000, 10000000, 50000000, 100000000,

The iterative solution taking one $\frac{\in}{d}$ and all R_N is displayed per line:

Relative- Reynolds Number

Roughness 4E3 1E4 5E4 1E5 5E5 1E6 5E6 1E7 5E7 1E8

0	**0.03991**	**0.03088**	**0.02089**	**0.01799**	**0.01316**	**0.01165**	**0.00898**	**0.00810**	**0.00649**	**0.00594**
0.00001	0.03992	0.03090	0.02093	0.01804	0.01330	0.01187	0.00959	0.00900	0.00830	0.00819
0.00005	0.03996	0.03096	0.02107	0.01826	0.01384	0.01265	0.01114	0.01086	0.01061	0.01058
0.0001	0.04001	0.03104	0.02125	0.01851	0.01443	0.01344	0.01234	0.01217	0.01202	0.01200
0.0004	0.04031	0.03149	0.02225	0.01991	0.01701	0.01649	0.01602	0.01596	0.01591	0.01590
0.001	0.04091	0.03238	0.02402	0.02217	0.02024	0.01994	0.01970	0.01967	0.01964	0.01964
0.004	0.04379	0.03640	0.03048	0.02950	0.02864	0.02853	0.02844	0.02843	0.02842	0.02842
0.01	0.04908	0.04313	0.03908	0.03850	0.03803	0.03796	0.03792	0.03791	0.03790	0.03790
0.02	0.05696	0.05228	0.04941	0.04903	0.04872	0.04868	0.04865	0.04864	0.04864	0.04864
0.03	0.06409	0.06011	0.05778	0.05748	0.05724	0.05720	0.05718	0.05718	0.05717	0.05717
0.04	0.07072	0.06720	0.06519	0.06493	0.06472	0.06470	0.06468	0.06467	0.06467	0.06467
0.05	0.07701	0.07380	0.07201	0.07178	0.07160	0.07157	0.07156	0.07155	0.07155	0.07155

1 TABLE 3: ITERATIVE SOLUTION OF COLEBROOKS EQUATION

Observation of Appendix 1 shows that:

(1) About 86.7% of the friction factor values calculated by equation 6 match those calculated by the Colebrook's Equation.

(2) The friction factor determined by equation 6 match those calculated by the Colebrook's Equation when $R_N \geq 5E5$.

(3) Errors by use of equation 6 range from – 0.05543% to 0.02828%.

Error in the friction factor by the use of equation 7 is shown in Appendix 2. Examination of the table shows that:

(1) About 95% of the friction factor values calculated by equation 6 match those calculated by the Colebrook's Equation..

(2) The errors are minor and occur in the range of $R_N \leq 5E4$ and $\in/d \geq 0.02$.

(3) The errors range from 0.01488% to 0.02828%. Such errors are negligible.

4.0 Conclusion

1. The two equations developed are quite accurate for calculating the friction factor of smooth and rough pipes.

2. The second equation, which is an extension of the first, is supper accurate and can be called an explicit form of the Colebrook's Equation.

Appendix I

Relative	**1.1.1.1 Reynolds number**									
46. Roughness	4E3	1E4	5E4	1E5	5E5	1E6	5E6	1E7	5E7	1E8
0.00000	-0.02506	-0.03238	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00001	-0.02505	0.00000	0.00000	-0.05543	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00005	-0.02503	-0.03230	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0001	-0.02499	0.00000	0.00000	-0.05402	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0004	-0.02481	-0.03176	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.001	-0.02444	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.004	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.01	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.02	0.00000	0.01913	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.03	0.01560	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.04	0.02828	0.01488	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.05	0.02597	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 4: Errors in *f* by use of the first of the new equations

Appendix 2

Relative 47. *Roughness*	1.1.1.2 **Reynolds number** 4E3	1E4	5E4	1E5	5E5	1E6	5E6	1E7	5E7	1E8
0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00001	0.00000	0.00000	0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.00005	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0001	0.00000	0.00000	0.00000	-0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.0004	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.001	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.004	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.01	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.02	0.00000	0.01913	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.03	0.01560	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.04	0.02828	0.01488	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
0.05	0.02597	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000

Table 5: Errors in *f* by use of the second of the new equations

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