

# Effect of radiation on the critical Frank – Kamenetskii parameter of thermal ignition in a combustible gas containing fuel droplets.

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## Abstract

*We investigate the effect radiation on the critical Frank-Kamenetskii parameter of thermal ignition in a combustible gas containing fuel droplets (furnaces, gas turbines and internal combustion engines). Previous works show that radiation delays ignition in a well stirred reactor. In this paper we show that, even in a non-homogeneous reaction, the critical Frank-Kamenetskii parameter increases as the radiation parameter increases when the activation energy is high. This confirms the experimented observation in diesel engines.*

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## 1.0 Introduction

The problem of thermal explosion in a combustible gas mixture containing fuel droplets has applications in furnaces, gas turbines and internal combustion engine [5]. The development of equations governing combustion involved derivation of the equations of motion of a chemically reacting gaseous mixture and a judicious simplification to render them tractable while retaining their essential characteristics [2]. Some of the models assume that the reactor is well – stirred so that the only independent variable is time [4, 5]. Even with this assumption the resulting equations are non-linear and usually analytical solutions do not exist. One may then use asymptotic technique [3, 4, 5, and 6] while one bears in mind that each technique may have advantages and disadvantages.

In the present work we model a non-homogeneous reactor where the independent variables are time and a space variable. As it is the case in many combustion problems, we assume that the activation energy is large.

## 2.0 Mathematical Modelling

Extending the model in [4] to a situation where the reactor is not well-stirred, we obtain

$$\rho_g c_{pg} \alpha_g \frac{\partial T_g}{\partial t} = \lambda_g \frac{\partial^2 T_g}{\partial x^2} + c_f Q_f \alpha_g \mu_f A \exp \left( - \left( \frac{E}{R} T_g \right) \right) - 4\pi R_d \lambda_g n_d (T_g - T_0) - 4\pi R_d^2 \sigma_1 n_d (T_g^4 - T_0^4) \quad (2.1)$$

$$\frac{d(R_d^2)}{dt} = - \frac{2\lambda_g}{\rho L} (T_g - T_0) - \frac{4\pi R_d}{L\rho} \sigma_1 (T_g^4 - T_0^4) \quad (2.2)$$

$$\alpha_g \frac{\partial c_f}{\partial t} = D_f \frac{\partial^2 c_f}{\partial x^2} - c_f \alpha_g \mu_f A \exp \left( - \left( \frac{E}{R} T_g \right) \right) + \frac{4\pi R_d \lambda_g n_d (T_g - T_0)}{L\mu_g \alpha_g} + \frac{4\pi R_d^2 \sigma_1 n_d (T_g^4 - T_0^4)}{L\mu_g \alpha_g} \quad (2.3)$$

together with the initial and boundary conditions

$$T_g = T, \quad c_f = c_{fo}, \quad R_d = R_{do} \quad (2.4)$$

$$T_g = T_0, \quad c_f = c_{fi} \text{ at } x = \pm 1, \quad i=1, 2 \quad (2.5)$$

Here, as in [4],

T – Temperature,

E – activation energy,

L – liquid evaporation energy,

C – reactant

$R_d$  – radius of drops,

Q – heat release per unit mass,

$$\sigma_1 = 2\sigma \varepsilon_d / (2 - \varepsilon_d),$$

$\sigma$  is the Stefan–Boltzman constant,

$\varepsilon_d$  is the emissivity of the droplet's surface,

$\mu$  – molar mass,

$\rho$  – density,

$\alpha$  – volumetric phase constant,

$\lambda$  – thermal conductivity,

n – number of drops per unit volume

A – pre-exponential factor,

R – universal gas constant.

Subscripts: g – gas mixture,

L – liquid

f – combustible gas component of the mixture

d – liquid drops.

### 3.0 Method of solution.

We assume, as it is usually the case [1], that before ignition  $c_f \approx c_{fo}$  and  $R_d \approx R_{do}$ . We solve the steady problem in (2.1).

Let

$$\theta = (T_g - T_0) \frac{E}{RT_0^2}, \quad \varepsilon = \frac{RT_0}{E}.$$

Then the steady part of (2.1) becomes

$$\frac{d^2\theta}{dx^2} + \delta \exp\left(\frac{\theta}{1+\varepsilon}\right) - a\theta - b(6\varepsilon\theta^2 + 4\varepsilon^2\theta^3 + \varepsilon^3\theta^4) \quad (3.1)$$

together with the boundary conditions

$$\theta(-1) = \theta(1) = 0, \quad (3.2)$$

where

$$\delta = \frac{RT_0^2 c_f Q_f \alpha_g \mu_g}{\left( \lambda_g \exp\left(\frac{E}{RT_0}\right) E \right)} \quad (3.3)$$

is the so called Frank–Kamenetskii parameter,

$$a = 4\pi R_d n_d + 4\pi R_d^2 \sigma_1 n_d \frac{T_0^3}{\lambda_g} \quad (3.4)$$

is the radiation parameter and

$$b = 4\pi R_d^2 \sigma_1 n_d \frac{T_0^3}{\lambda_g} \quad (3.5)$$

is the higher order radiation parameter. In a high activation energy situation  $\epsilon \rightarrow 0$  and equation (3.1) becomes

$$\frac{d^2\theta}{dx^2} + \delta \exp(\theta) - a\theta = 0 \quad (3.6)$$

We are to solve equation (3.6) using equation (3.2).

Following [1], we seek an approximate polynomial solution

$$V(x) = c_1 (x + 1)^2 (x - 1) + c_2 (x + 1) (x - 1)^2 \quad (3.7)$$

which satisfies the boundary conditions.

Thus

$$\frac{d^2V}{dx^2} + \delta \exp V - aV = R(x) \quad (3.8)$$

By symmetry

$$\frac{d\theta}{dx}(0) = 0.$$

Thus

$$c_1 = -c_1.$$

$$V(0) = V_{\max} \text{ and } \frac{d^2V}{dx^2} = 4c_1.$$

Thus

$$-2V_{\max} + \delta \exp(V_{\max}) - aV_{\max} = R(0) \quad (3.9)$$

The critical Frank-Kamenetskii  $\delta_{cr}$  occurs, when

$$\frac{d\delta}{dV_{\max}} = 0.$$

Hence

$$\delta_{cr} = \frac{(a+2)}{e} \quad (3.10)$$

#### 4.0 Conclusion

Equation (3.10) implies that the radiation parameter increases the critical Frank-Kamenetskii parameter.

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