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Mathematical model of temperature rise in three-layer skin during microwave hyperthermia

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Abstract

We discuss the existence and uniqueness of solution of mathematical model of multi-layered human skin exposed to microwave heating during cancer therapy. We show that our model has a solution and the solution is unique.

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1.0 Introduction

Ozen et al [3], examined the heat analysis of microwave exposed skin model and they predicted that rise in temperature near the skin surface depends on boundary condition, blood perfusion and thermal conductivity. Ayeni et al. [2] examined different ways of destroying or controlling the growth of tumour cells using microwave hyperthermia. Recently, Adebile and Ogunmoyela [1] examined the problem when the parameters [3] vary. They used matching analysis. The previous methods involved approximations, but wee show here that the problem in fact has a unique solution when no approximations are involved.

2.0 Mathematical formulation

The governing equations are [Adebile and Ogunmoyela 1]:

$$\rho C_p \frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} + \rho_b W_b C_p (T_b - T) + Q, \quad (x,t) \in D(O,T) \tag{2.1}$$

$$T(x,0) = \frac{T_c x}{L}, \quad x \in D \tag{2.2}$$

$$T(0,t) = T_a, \quad T(1,t) = T_c, \quad t \in (0,T) \tag{2.3}$$

where Q – electromagnetic
$\rho_b W_b$ - blood perfusion
C_b - specific heat capacity of blood
C_p - specific heat capacity of tissue
K - thermal conductivity
ρ_b - density of blood
ρ - density of the tissue
T_b – temperature of arterial blood
T – temperature

3.0 Non-dimensionalization

We seek for dimensionless variables

$$\eta = \frac{x}{L}, \quad \theta = \frac{T - T_0}{T_b - T_0} \tag{3.1}$$

such that $$T = \theta(T_b - T_0) + T_0 \quad (3.2)$$

Using equations (3.1) and (3.2) in (2.1), (2.2) and (2.3) we obtain for the steady case

$$\frac{1}{\rho_r}\frac{d^2\theta}{d^2\eta} - \alpha_0 (p + q\theta)^w (\theta - 1) + E_0 e^{-k\eta} (\beta\theta + T_0)^m = 0 \quad (3.3)$$

$$\theta(0) = \chi, \quad \theta(1) = \lambda, \quad (3.4)$$

where $$\chi = \frac{T_a - T_0}{T_b - T_0}, \quad \lambda = \frac{T_c - T_0}{T_b - T_0} \quad (3.5)$$

4.0 Existence and uniqueness of solution

Theorem 1 [4]

Let D denote the region [in (n + 1) dimensional space, one dimension for t and n dimensions for the vector x] $|t - t_0| \le a, \|x - x_0\| \le b$. If

$$x_1' = f_1(x_1, x_2, \cdots, x_n, t), \quad x_1(t_0) = x_{10} \quad (4.1)$$

$$x_2' = f_2(x_1, x_2, \cdots, x_n, t), \quad x_2(t_0) = x_{20} \quad (4.2)$$

$$\vdots$$

$$x_n' = f_n(x_1, x_2, \cdots, x_n, t), \quad x_n(t_0) = x_{n0} \quad (n)$$

Then, the system of equations (1) – (n) has a unique solution if

$$\frac{\partial f_i}{dx_j}, \quad i, j = 1, 2, \cdots, n \quad (n + 1)$$

are continuous in D.

Theorem 2

Problem (3.3) which satisfies (3.4) has a unique solution.

Proof

Let $x_1 = \eta,\ x_2 = \theta,\ x_3 = \theta'$. Then

$$x_1' = f_1(x_1, x_2, x_3) \quad (4.3)$$

$$x_2' = f_2(x_1, x_2, x_3) \quad (4.4)$$

$$x_3' = f_3(x_1, x_2, x_3) \quad (4.5)$$

where $f_1 = 1,\ f_2 = x_3,\ f_3 = \rho_r\left[\alpha_0 (p + qx_2)^w (x_2 - 1) - E_0 e^{-kx_1} (px_2 + L)^m\right]$. Clearly $\frac{\partial f_i}{\partial x_j}$ are continuous on D. By Theorem 1, problem (3.3) which satisfies (3.4) has a unique solution.

References

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