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Sheared semi-infinite crack originating at the boundary of a circular hole in a non-homogeneous infinite solid

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Abstract

The configuration studied is that of a non-homogeneous infinite solid containing a central hole and a semi-infinite crack, originating from one side of the hole. Longitudinal shear loads of magnitude T_j, j = 1, 2 are applied on parts of the crack surface. It is found that the dominant fracture characteristic is that of a hole or semi circular notch. The maximum stress $\sigma_{\psi z}(R, 0)$ expected at the hole-interface junction, where further cracking is likely to commence, is derived in a closed form. The case of the stress when the lower crack surface is not loaded ($T_2 = 0$) is presented in a graph to enable understanding of the stress ratio $\sigma_{\psi z}(R, 0)/T_1$ as the radius of the hole grows and/or as the load site varies.

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1.0 Introduction

A non-homogenous infinite solid is made from two half planes of different isotropic and elastic materials each having a semicircular notch that form a central hole of radius a when the materials are bonded along the real axis. The left side detaches completely thereby forming a semi-infinite crack that terminates at one side of the hole while the interface on the right side remains perfectly bonded. A pair of longitudinal shear loads of magnitudes T_j, j = 1, 2 are applied along the cracked surface on intervals [$-b_j$, $-a_j$], j = 1,2 not necessarily equal nor symmetric about the crack line as depicted in Figure 1.1(a)

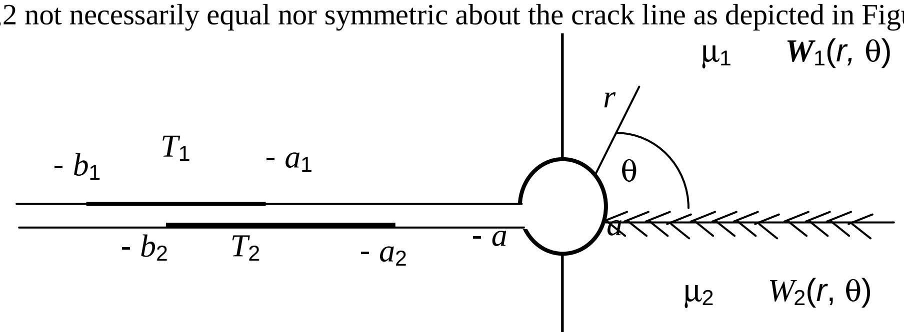

Figure 1.1(a): Geometry of the Problem On The z – plane Showing the Circular Hole and Load sites [$-b_j$, $-a_j$], j = 1, 2

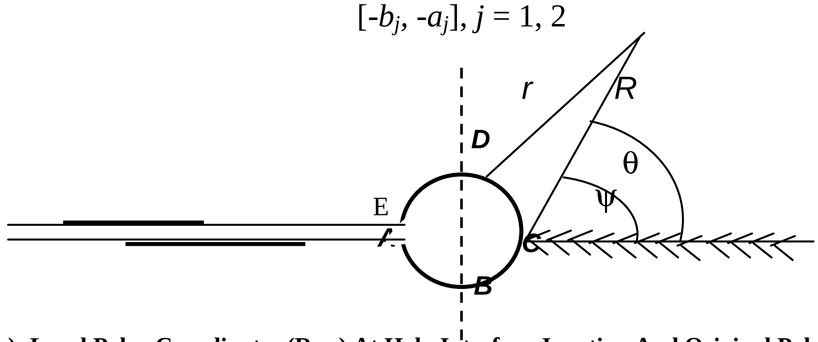

Figure 1(b): Local Polar Coordinates (R, ψ) At Hole-Interface Junction And Original Polar Coordinates (r, θ)

The fields near the hole-interface junction are investigated for crack initiation features. Our method uses conformal mapping, Mellin transform and residue theory to analyse the governing boundary value problem. This technique has been successfully applied in the homogeneous complement of the solid being studied [1]. Homogeneous infinite solids with holes of various shapes and finite line cracks have been investigated by several authors (see for example [2, 3, 4, and 5]. In [3] Bowie studied the plane problem of finite radial cracks emanating from the boundary of a circular hole in an infinite homogeneous elastic plate under uni-axial or biaxial tension employing Muskhelishvili's [6] complex variables method. Rice [5] investigated an elliptical hole in an infinite homogeneous solid under remote biaxial in plane tensions and anti-plane shear. The endeavour here is to investigate the effect of the presence of a semi-infinite crack and a hole on the fields.

2.0 Mathematical Formulation

The convention followed is that the subscript 1 refers to the material occupying the upper half plane while subscript 2 refers to that occupying the lower half plane. The governing boundary value problem is then derived in polar coordinates as:

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial\theta}\right)W_j(r,\theta)=0 \quad r\ge a,-\pi\le\theta\le\pi,\ j=1,2 \tag{2.1}$$

$$W_1(r,0)=W_2(r,0),\ \mu_1\frac{\partial \mathrm{W}_1}{\partial\theta}(r,0)=\mu_2\frac{\partial \mathrm{W}_2}{\partial\theta}(r,0) \quad \mathrm{r}\ge \mathrm{a} \tag{2.2}$$

$$\frac{\partial \mathrm{W}_j}{\partial\theta}(r,\pm\pi)=\frac{-rT_j}{\mu_j},\ \mathrm{a}_\mathrm{j}\le r\le b_j,\ j=1,2 \tag{2.3a}$$

$$=0 \text{ otherwise} \tag{2.3b}$$

$$\frac{\partial \mathrm{W}_j}{\partial r}(a,\theta)=0 \qquad -\pi\le\theta\le\pi \tag{2.3c}$$

use has been made of the relations

$$\sigma_{j\theta z}(r,\theta)=\frac{\mu_j}{r}\frac{\partial \mathrm{W}_j}{\partial\theta}(r,\theta),\sigma_{jrz}(r,\theta)=\mu_j\frac{\partial \mathrm{W}_j}{\partial r}(r,\theta),\quad j=1,2 \tag{2.3d}$$

The original z – plane of analysis is transformed onto a plane with a semi-infinite crack terminating at the origin (Figure 2) by the holomorphic mapping function

$$\xi(z)=\frac{1}{2}\left(\frac{z}{a}+\frac{a}{z}\right)-1,\quad \mathrm{z}=\mathrm{re}^{\mathrm{i}\theta},\ \xi(z)=\rho e^{i\phi} \tag{2.4}$$

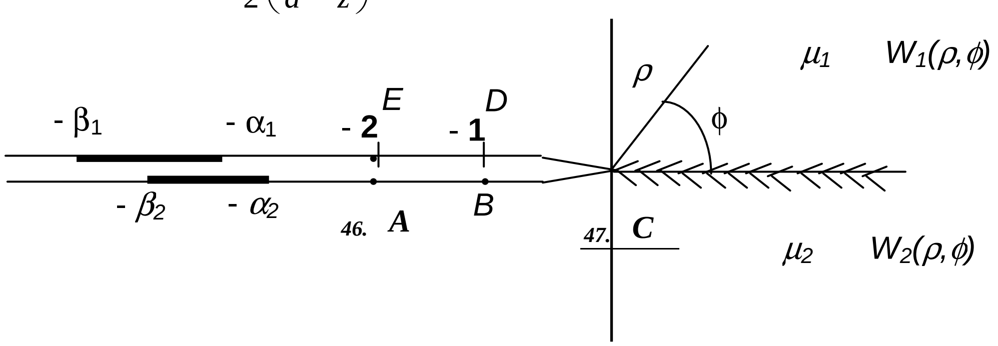

Figure 2: The ρϕ - plane showing some correspondences from Figure 1

Writing $\xi(z)=u(r,\theta)+iv(r,\theta)$ leads to

$$\rho\cos\phi = \mathrm{u}(\mathrm{r},\theta) = \frac{1}{2}\left(\frac{r}{a}+\frac{a}{r}\right)\cos\theta\,,\ \rho\sin\phi = \mathrm{v}(\mathrm{r},\theta) = \frac{1}{2}\left(\frac{r}{a}+\frac{a}{r}\right)\sin\theta$$

$\rho = \left\{\mathrm{u}^2(\mathrm{r},\theta)+\mathrm{v}^2(\mathrm{r},\theta)\right\}^{\frac{1}{2}}$, $\quad \tan\phi(\mathrm{r},\theta) = \dfrac{\mathrm{v}(\mathrm{r},\theta)}{\mathrm{u}(\mathrm{r},\theta)}$. Hence, for r ≥ a, -π ≤ θ ≤ π

$$\frac{\partial\rho}{\partial\theta}(r,\pm\pi) = 0\ ;\ \frac{\partial\rho}{\partial\theta}(r,0) = 0\,;\frac{\partial\rho}{\partial r}(a,\theta) = 0 \qquad (2.5a)$$

$$\frac{\partial\phi}{\partial\theta}(r,\pm\pi) = \left(\frac{r}{a}-\frac{a}{r}\right)\Big/2\rho = \frac{\partial\phi}{\partial\theta}(r,0)\quad ;\frac{\partial\phi}{\partial r}(a,\theta) \neq 0 \qquad (2.5b)$$

In view of W (r, θ) ≡ W_j (ρ, θ) j = 1,2 we use (2.5a,b) to get

$$\frac{\partial W_j}{\partial\theta}(r,\pm\pi) = \frac{\partial W_j}{\partial\phi}(\rho,\pm\pi)\ \frac{\partial\phi}{\partial\theta}(r,\pm\pi), \qquad \frac{\partial W_j}{\partial\theta}(r,0) = \frac{\partial W_j}{\partial\theta}(\rho,0)\ \frac{\partial\phi}{\partial\theta}(r,0)$$

$$\frac{\partial W_j}{\partial r}(a,\theta) = \frac{\partial W_j}{\partial\phi}(\rho,\pm\pi)\frac{\partial\phi}{\partial r}(a,\theta).$$

Since $\rho = \frac{1}{2}\left(\frac{r}{a}+\frac{a}{r}+2\right), \phi = \pm\pi, \theta = \pm\pi\ \ and\ \ r \geq$ a , it is seen that

$$\frac{r}{a} = \rho - 1 + \sqrt{\rho(\rho-2)}\ ,$$

and

$$\frac{a}{r} = \rho - 1 - \sqrt{\rho(\rho-2)}\ , \rho\ >\ 2$$

With these relations the ρ ϕ - plane equivalent of the problem is obtained as:

$$\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\right)W_j(\rho,\phi) = 0\,,\quad \rho \geq 0\,, -\pi \leq \phi \leq \pi\,,\ \mathrm{j}=1,2 \qquad (2.6)$$

$$W_1(\rho,0) = W_2(\rho,0)\,;\mu_1\frac{\partial W_1}{\partial\phi}(\rho,0) = \mu_2\frac{\partial W_2}{\partial\phi}(\rho,0)\,, \pi \geq 0 \qquad (2.7)$$

$$\frac{\partial W_j}{\partial\phi}(\rho,\pm\pi) = -\frac{aT_j}{\mu_j}\left[\rho^{\frac{1}{2}}(\rho-1)(\rho-2)^{-\frac{1}{2}}+\rho\right], \alpha_j \leq \rho \leq \beta_j, \alpha_j \geq 2,\ \mathrm{j}=1,2 \qquad (2.8a)$$

= 0 otherwise. (2.8b)

$$\rho(a_j,\pm\pi) = \alpha_j = \tfrac{1}{2}\left(\frac{a_j}{a}+\frac{a}{a_j}+2\right),\ \rho(b_j,\pm\pi) = \beta_j = \tfrac{1}{2}\left(\frac{b_j}{a}+\frac{a}{b_j}+2\right), \qquad (2.8c)$$

The asymptotic behaivours are W_j(ρ ϕ) = O($\rho^{1/2}$) as $\rho \to 0$ and Wj(ρ ϕ) = O($\rho^{-1/2}$) as $\rho \to \infty$, j = 1, 2.

3.0 Analysis of the Transformed Problem

Next, the Mellin integral transform is applied to (2.6) – (2.8a,b,c) and the differential equation derived is:

$$\left(\frac{d^2}{d\phi^2}+s^2\right)\hat{W}_j(s,\phi) = 0\,, -\tfrac{1}{2} < \mathrm{Re}\, s < \tfrac{1}{2}\,,\ j = 1, 2 \qquad (3.1)$$

$$\hat{W}_1(s,0) = \hat{W}_2(s,0)\ ;\ \mu_1 \frac{\partial \hat{W}_1(s,0)}{\partial \phi} = \mu_2 \frac{\partial \hat{W}_2(s,0)}{\partial \phi} \tag{3.2}$$

$$\frac{\partial \hat{W}_j}{\partial \phi}(s,\pm\pi) = \frac{aT_j}{\mu_j} g_j(a_j,b_j;s) \tag{3.3}$$

where

$$\hat{W}_j(s,\phi) = \int_0^\infty W_j(\rho,\phi)\rho^{s-1}d\rho\ ,\ -\tfrac{1}{2} < \operatorname{Re} s < \tfrac{1}{2},\ j = 1,2$$

is the Mellin transform of $W_j(\rho,\phi)$

$$g_j(a_j,b_j;s) = \int_{\beta_j}^{\alpha_j}\left[\rho^{1/2}(\rho-1)(\rho-2)^{-1/2} + \rho\right]\rho^{s-1}d\rho \qquad j = 1,2 \tag{3.4}$$

(β_j and α_j are given in (2.8c)). Let the solution of (3.1) be

$$\hat{W}_j(s,\phi) = A_j(s)\sin s\phi + B_j(s)\cos s\phi \tag{3.5}$$

From (3.2) and (3.5) are obtain

$$B_1(s) = B_2(s); \quad \mu_1 A_1(s) = \mu_2 A_2(s) \tag{3.6}$$

From (3.3) and (3.5) we get

$$A_1(s)\cos\pi s - B_1(s)\sin\pi s = \frac{aT_2}{\mu_1 s} g_1(a_1,b_1;s) \tag{3.7a}$$

$$A_2(s)\cos\pi s - B_2(s)\sin\pi s = \frac{aT_2}{\mu_2 s} g_2(a_2,b_2;s) \tag{3.7b}$$

Cramer's rule, (3.6) and (3.7a,b) yield, for j = 1, 2

$$A_j(s) = \frac{1}{\mu_j}\left[(1+\gamma)T_1 g_1(a_1,b_1;s) + (1-\gamma)T_2 g_2(a_2,b_2;s)\right]\frac{a}{2s\cos\pi s}$$

$$B_j(s) = \left[(1+\gamma)\frac{T_2}{\mu_2} g_2(a_2,b_2;s) - (1-\gamma)\frac{T_1}{\mu_1} g_1(a_1,b_1;s)\right]\frac{a}{2s\sin\pi s}$$

where γ is a material constant parameter given by $\gamma = (\mu_2 - \mu_1)/(\mu_2 + \mu_1)$. The solution of (2.6) – (2.8a,b,c) is given by the inverse Mellin transform denoted

$$W_j(\rho,\phi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\hat{W}_j(s,\phi)\rho^{s-1}ds\ ,\ -\tfrac{1}{2} < c < \tfrac{1}{2},\ j = 1,2$$

Consequently, residue theory is used to evaluate the integrals to get the expression:

$$W_j(\rho,\phi) = \frac{a}{2\mu_j}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left[(1+\gamma)T_1 g_1(a_1,b_1;s) + (1-\gamma)T_2 g_2(a_2,b_2;s)\right]\rho^{-s}\frac{\sin s\phi}{s\cos\pi s}ds$$

$$-\left(\frac{a}{2}\right).\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left[(1+\gamma)\frac{T_2}{\mu_2} g_2(a_2,b_2;s) - (1-\gamma)\frac{T_1}{\mu_1} g_1(a_1,b_1;s)\right]\frac{\rho^{-s}\cos s\phi}{s\sin s\pi s}ds \tag{3.8}$$

The integral in (3.4) can be evaluated by use of the convergent series [7] denoted

$$(1-t)^{-1/2} = \sum_{k=1}^{\infty} c_k t^k\ ,|\ t\ | < 1$$

where the coefficients are defined by $c_k = \dfrac{(2k)!}{2^{2k}(k!)^2}$ The integrand in (3.4) can be put as $\rho^s(1-2\rho^{-1})^{-1/2}$ - $\rho^{s-1}(1-2\rho^{-1})^{1/2} + \rho^s$. Hence, for j = 1, 2

$$g_j(a_j, b_j; s) = \sum_{k=0}^{\infty} \frac{c_k 2^k (\alpha_j^{s-k+1} - \beta_j^{s-k+1})}{s-k+1} - \sum_{k=0}^{\infty} \frac{c_k 2^k (\alpha_j^{s-k} - \beta_j^{s-k})}{s-k} + \frac{\alpha_j^{s+1} - \beta_j^{s+1}}{s+1} \tag{3.9a}$$

Components of (3.9a) may be needed in evaluating (3.8). Therefore, (3.9a) is written in the form

$$g_1(a_j, b_j; s) = m_j(\alpha_j; s)\, \alpha_j^s - m_j(\beta_j, s)\beta_j^s \tag{3.9b}$$

where

$$m_j(t; s) = \sum_{k=0}^{\infty} \frac{c_{k+1} 2^{k+1}}{s-k} t^{-k} - \sum_{k=0}^{\infty} \frac{c_k 2^k t^{-k}}{s-k} + \frac{2t}{s+1} \tag{3.9c}$$

whose singularities are simple poles at $s = -1$ and $s = n$, $n = 1, 2, 3, \ldots$. In view of (2.8a), the range $\alpha_j \le \rho \le \beta_j$, $j = 1, 2$ is considered and Jordan's lemma used to close contours in (3.8) in planes determined by joint occurrence of $\rho < \beta_j$ and $\rho > \alpha_j$. When $\rho < \beta_{jj}$, contours are closed in the left half plane $Res < 0$. Integrals to be evaluated are:

$$\mathrm{I}_{\beta_j}^{(1)}(\rho, \phi) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} - m_j(\beta_j; s) \left(\frac{\rho}{\beta_j}\right)^{-s} \frac{\sin s\phi}{s \cos \pi s} ds \quad j = 1, 2$$

$$\mathrm{I}_{\beta_j}^{(2)}(\rho, \phi). = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} - m_j(\beta_j; s) \left(\frac{\rho}{\beta_j}\right)^{-s} \frac{\cos s\phi}{s \sin \pi s} ds \quad j = 1, 2$$

For $\rho > \alpha_j$, contours are closed in the right half plane $Res > 0$. Integrals to be evaluated are:

$$\mathrm{I}_{\alpha_j}^{(1)}(\rho, \phi) = .\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} m_j(\alpha_j; s) \left(\frac{\rho}{\alpha_j}\right)^{-s} \frac{\sin s\phi}{s \cos \pi s} ds \quad j = 1, 2$$

$$\mathrm{I}_{\alpha_j}^{(2)}(\rho, \phi) = .\frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} m_j(\alpha_j; s) \left(\frac{\rho}{\alpha_j}\right)^{-s} \frac{\sin s\phi}{s \cos \pi s} ds \quad j = 1, 2$$

Therefore, the solution for $\alpha_j < \rho < \beta_j$, $j = 1, 2$ is:

$$W_j(\rho, \phi) = \frac{a}{2\mu_j} \left([1+\gamma] T_1 \left\{ \mathrm{I}_{\beta_j}^{(1)}(\rho, \phi) + \mathrm{I}_{\alpha_j}^{(1)}(\rho, \phi) \right\} + (1-\gamma) T_2 \left\{ \mathrm{I}_{\beta_2}^{(1)}(\rho, \phi) + \mathrm{I}_{\alpha_2}^{(1)}(\rho, \phi) \right\} \right)$$
$$- \frac{a}{2} \left[(1-\gamma) \frac{T_1}{\mu_1} \left\{ \mathrm{I}_{\beta_j}^{(2)}(\rho, \phi) + \mathrm{I}_{\alpha_j}^{(2)}(\rho, \phi) \right\} - (1+\gamma) \frac{T_2}{\mu_2} \left\{ \mathrm{I}_{\beta_2}^{(2)}(\rho, \phi) + \mathrm{I}_{\alpha_2}^{(2)}(\rho, \phi) \right\} \right] \tag{3.10}$$

The integrals are evaluated for $j = 1,2$ by writing (3.9c) as

$$m_j(t; s) = \sum_{k=0}^{\infty} \frac{c_k d_k}{s-k} 2^k t^{-k} + \frac{2t}{s+1},$$

where $c_k d_k = 2c_{k+1} - c_k$.

The results when $\rho < \beta_j$, $j = 1, 2$ are:

$$\mathrm{I}_{\beta_j}^{(1)}(\rho, \phi) = \frac{-1}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n - \frac{1}{2}} \left\{ \sum_{k=1}^{\infty} \frac{c_k d_k 2^k}{\frac{1}{2} - n - k} \beta_j^{\frac{1}{2} - n - k} + 2 \frac{\beta_j^{\frac{3}{2} - n}}{\frac{3}{2} - n} \right\} \rho^{n - \frac{1}{2}} \sin\left(n - \tfrac{1}{2}\right)\phi + 2\rho \sin\phi$$

$$\mathrm{I}^{(2)}_{\beta_j}(\rho,\phi) = \frac{-1}{\pi}\left\{\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sum_{k=1}^{\infty}\frac{c_k d_k 2^k}{n+k}\beta^{-n-k} + \sum_{n=2}^{\infty}\frac{(-1)^n}{n}\frac{\beta_j^{1-n}}{n-1}\right\}\rho^n\cos n\phi$$
$$-\frac{2}{\pi}\left\{\left[1 - In\left(\frac{\rho}{\beta_j}\right)\right]\cos\phi + \phi\sin\phi\right\}\rho$$

The results when $\rho > \alpha_j$, j = 1, 2 are

$$I^{(1)}_{\alpha_j}(\rho,\phi) = \sum_{n=1}^{\infty}\frac{(-1)^n}{n}\ c_n d_n 2^n \rho^{-n}\sin n\phi$$
$$+\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n-1/2}\left(\sum_{k=1}^{\infty}\frac{c_k d_k 2^k}{n-k-1/2}\alpha_j^{n-k-1/2} + \frac{2\alpha_j^{\ n+1/2}}{n+1/2}\right)\rho^{1/2-n}\sin(n-1/2)\phi$$

$$\mathrm{I}^{(2)}_{\alpha_j}(\rho,\phi) = -\frac{1}{\pi}\sum_{n-1}^{\infty}\frac{(-1)^n}{n}\left(\sum_{\substack{k=1\\k\neq n}}^{\infty}\frac{c_k d_k 2^k}{n-k}\alpha_j^{n-k} + \frac{2\alpha_j^{\ 1+n}}{1+n}\right)\rho^{-n}\cos n\phi$$
$$+\frac{1}{\pi}\sum_{n-1}^{\infty}\frac{(-1)^n}{n}c_n d_n 2^n\left[In\left(\frac{\rho}{\alpha_j}\right)\cos n\phi + \frac{\cos n\phi}{n} + \phi\sin n\phi\right]\rho^{-n}$$

The solution when $0 < \rho < 1$ is derived from (3.8) while observing that $\rho < \alpha_j$ and $\rho < \beta_j$, j = 1, 2 simultaneously. Therefore the appropriate expression for $g_j(a_j,b_j;s)$ to use is the one given in (3.9a) which has removable singularities. Jordan's lemma suggests closure of contours in the left half plane $Res < 0$. The integrals to be evaluated are:

$$\mathrm{I}^{(1)}_j(\rho,\phi) = .\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\mathrm{g}_j(\mathrm{a}_j,\mathrm{b}_j;\mathrm{s})\ \rho^{-\mathrm{s}}\ \frac{s\sin\phi}{s\cos\pi s}ds,$$
$$\mathrm{I}^{(2)}_j(\rho,\phi) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\mathrm{g}_j(\mathrm{a}_j,\mathrm{b}_j;\mathrm{s})\ \rho^{-\mathrm{s}}\ \frac{\cos\phi}{s\sin\pi s}ds.$$

The solution corresponding to $0 < \rho < 1$ is

$$W_j(\rho,\phi) = \frac{a}{2\mu_j}\left[(1+\gamma)T_1\,\mathrm{I}^{(1)}_1(\rho,\phi) + (1-\gamma)T_2\,\mathrm{I}^{(1)}_2(\rho,\phi)\right]$$
$$-\frac{a}{2}\left[(1+\gamma)\frac{T_2}{\mu_2}\mathrm{I}^{(2)}_2(\rho,\phi) + (1-\gamma)\frac{T_1}{\mu_1}\mathrm{I}^{(2)}_1(\rho,\phi)\right] \tag{3.11}$$

Residue theory is applied to obtain

$$\mathrm{I}^{(1)}_j(\rho,\phi) = \frac{1}{\pi}\sum_{\mathrm{n}=1}^{\infty}\frac{(-1)^{\mathrm{n}-1}}{\mathrm{n}-1/2}\mathrm{g}_j(\mathrm{a}_j,\mathrm{b}_j;\tfrac{1}{2}-\mathrm{n})\ \rho^{n-1/2}\sin(n-1/2)\phi,\quad j=1,2$$
$$\mathrm{I}^{(2)}_j(\rho,\phi) = \frac{-1}{\pi}\sum_{\mathrm{n}=1}^{\infty}\frac{(-1)^{\mathrm{n}}}{\mathrm{n}}g_j(a_j,b_j;-n)\,\rho^n\cos n\phi,\quad j=1,2$$

4.0 Fields at the junction of the hole and interface.

The junction of the hole and interface is approached as $\rho \to 0$. The effect is that

$\mathrm{I}_j^{(1)}(\rho,\phi) = \dfrac{2}{\pi} g_j(a_j, b_j; -1/2)\,\rho^{1/2}\sin\phi/2$, $j = 1,2$ as $\rho \to 0$ while $\mathrm{I}_j^{(2)}(\rho,\phi) \to 0$ as $\rho \to 0$.

The displacements at the junction are therefore obtained from (19) as:

$$W_j(\rho,\phi) = \frac{a}{\pi\mu_j}\left[(1+\gamma)T_1 g_1(a_1,b_1;-\tfrac{1}{2}) + (1-\gamma)T_2 g_2(a_2,b_2;-\tfrac{1}{2})\right]\rho^{1/2}\frac{\sin\phi}{2} \text{ as } \rho\to 0$$

Let (R, Ψ) be local polar coordinates introduced at the junction of the hole and interface as shown in Figure 1(b). Then $r\cos\theta = a + R\cos\Psi$ and $r\sin\theta = R\sin\Psi$. Hence $re^{i\theta} = a + Re^{i\Psi}$ and from (2.4)

$$\rho e^{i\phi} = 1/2\frac{R^2}{a^2}e^{2i\psi} + 0\left[\left(\frac{R}{a}\right)^3\right] \text{ as } R\to 0$$

implies that

$$\rho^{1/2}\sin\phi/2 = \frac{1}{\sqrt{2}}\frac{R}{a}\sin\psi + 0\left[\left(\frac{R}{a}\right)^3\right] \text{ as } R\to 0$$

Therefore, for j = 1, 2 and as $R \to 0$.

$$W_j(R,\psi) = \frac{1}{\sqrt{2}}\frac{a}{2\mu_j}\left[(1+\gamma)T_1 g_1(a_1,b_1;-\tfrac{1}{2}) + (1-\gamma)T_2 g_2(a_2,b_2;-\tfrac{1}{2})\right]R\sin\psi \qquad (4.1)$$

The corresponding stresses are derived from (4.1) by application of (2.3c). The expressions are:

$$\sigma_{j\psi z}(R,\psi) = \frac{1}{\sqrt{2\pi}}\left[(1+\gamma)T_1 g_j(a_j,b_j;-\tfrac{1}{2}) + (1-\gamma)T_2 g_2(a_2,b_2;-\tfrac{1}{2})\right]\cos\psi,\ j=1,2 \qquad (4.2a)$$

$$\sigma_{jRz}(R,\psi) = \frac{1}{\sqrt{2\pi}}\left[(1+\gamma)T_1 g_j(a_j,b_j;-\tfrac{1}{2}) + (1-\gamma)T_2 g_2(a_2,b_2;-\tfrac{1}{2})\right]\sin\psi,\ j=1,2 \qquad (4.2b)$$

For j = 1, 2 and from (3.9a) the following result is obtained:

$$g_j(a_j,b_j;-\tfrac{1}{2}) = \sum_{k=o}^{\infty}\frac{c_k 2^k(\alpha_j^{\frac{1}{2}-k} - \beta_j^{\frac{1}{2}-k})}{1/2-k} + \sum_{k=o}^{\infty}\frac{c_k 2^k(\alpha_j^{-\frac{1}{2}-k} - \beta_j^{-\frac{1}{2}-k})}{1/2+k} + 2\left(\alpha_j^{\frac{1}{2}} - \beta_j^{\frac{1}{2}}\right)$$

Utilizing the relation $(1-t^2)^{-1/2} = \sum_{k=o}^{\infty} c_k t^{2k}$, $|t|<1$ we derive

$\sum_{k=0}^{\infty}\dfrac{c_k t^{2k}}{2k+1} = \sin^{-1}t$ *and* $\sum_{k=0}^{\infty}\dfrac{c_k t^{2k-1}}{2k-1} = -\tan(\cos^{-1}t) + \sum_{k=0}^{\infty}\dfrac{c_k}{2k-1}$ $-\dfrac{\pi}{2} < \cos^{-1}t < \dfrac{\pi}{2}$. It then follows that for j = 1, 2,

$$g_j(a_j,b_j;-\tfrac{1}{2}) = 2\sqrt{2}\left\{\tan\left[\cos^{-1}\left(\sqrt{\frac{2}{\alpha_j}}\right)\right] - \tan\left[\cos^{-1}\left(\sqrt{\frac{2}{\beta_j}}\right)\right]\right\}$$

$$+\sqrt{2}\left\{\sin^{-1}\left(\sqrt{\frac{2}{\alpha_j}}\right) - \sin^{-1}\left(\sqrt{\frac{2}{\beta_j}}\right)\right\} + 2\sqrt{2}\left[\left(\frac{\alpha_j}{2}\right)^{1/2} - \left(\frac{\beta_j}{2}\right)^{1/2}\right] \qquad (4.3)$$

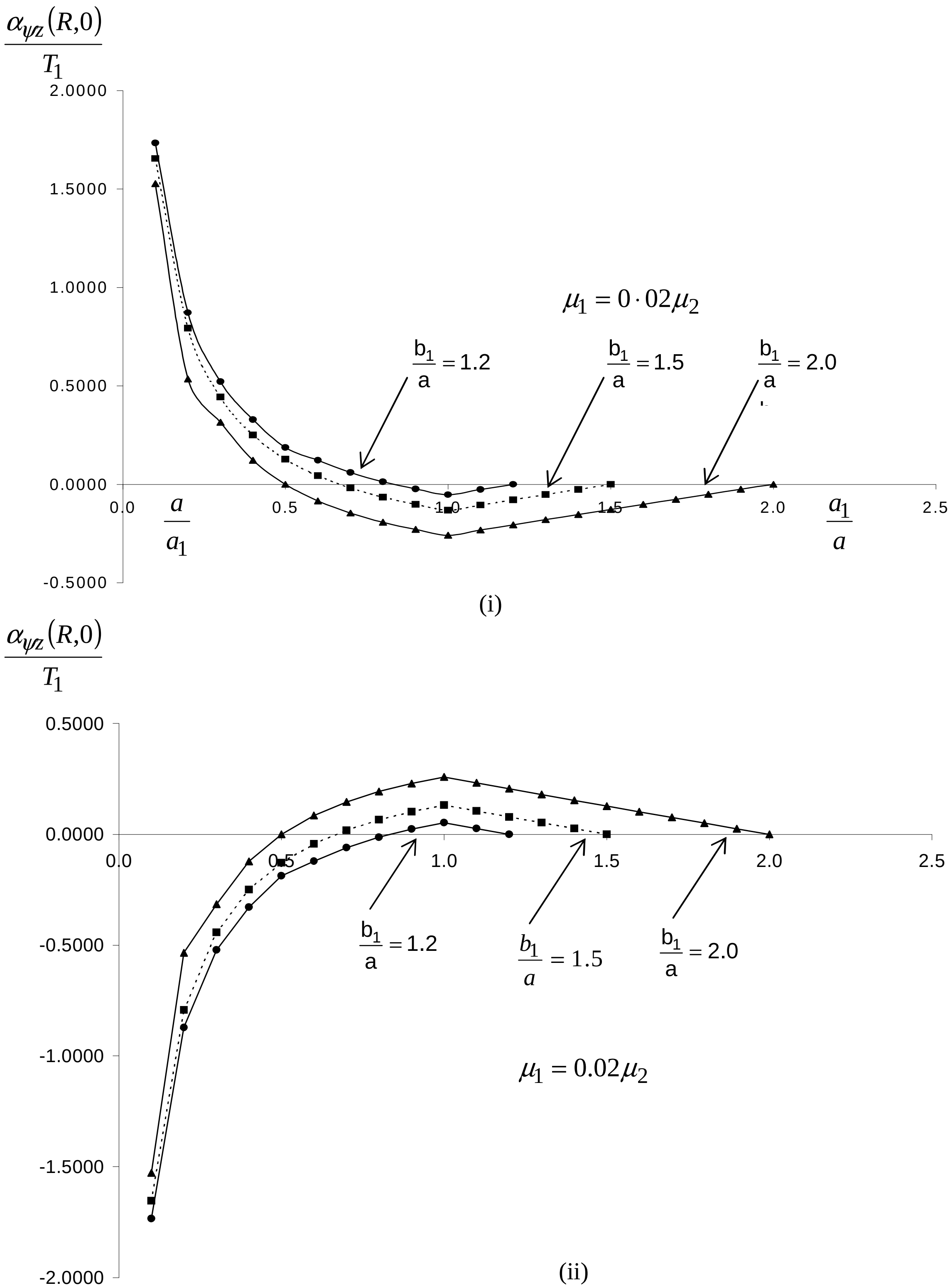

Figure 3: Variation of $\frac{\sigma_{\psi z}(R,\ 0)}{T_1}$ with $\frac{a}{a_1}$ and / or $\frac{a_1}{a}$ for various values of $\frac{b_1}{a}$ when (i) T_1 is prescribed (ii) $-T_1$ is prescribed $(a \leq a_1 < b_1)$.

where $\sqrt{\frac{2}{\beta_j}} = 2\left(\frac{b_j}{a} + \frac{a}{b_j} + 2\right)^{1/2}$ and $\sqrt{\frac{2}{\alpha_j}} = 2\left(\frac{a_j}{a} + \frac{a}{a_j} + 2\right)^{1/2}$ with $b_j > a_j \geq a$.

5.0 Conclusion

Displacement fields close to the hole-interface junction have been derived in a closed form and the stresses there shown to be non-singular as in the cases studied in [8,9] for half planes. The results indicate that the hole cannot be absent in this problem. The case for which the hole is absent (a = 0) must be treated by other methods (see for example [10, 11], homogeneous or non homogeneous without kink). The dominant fracture character of the solid is therefore that of a hole or a semi-circular notch. The normal stress along the interface and near the junction is maximum [5,8] and is expressed as

$$\sigma_{\psi z}(\mathrm{R},0) = \frac{1}{\sqrt{2}\pi}\left[(1+\gamma)T_1 g_1(a_1,b_1\ ;-\tfrac{1}{2}\ ) + (1-\gamma)T_2 g_2(a_2\ ,b_2\ ;-\tfrac{1}{2})\right]$$

The stresses are shown to depend on material constants except when $T_1 = T_2$, $a_1 = a_2$ and $b_1 = b_2$ simultaneously, in which case dependence on material constants is suppressed due to symmetric loading. The graph in Figure 3 shows the variation of $\sigma_{\psi z}(R,0)/\mathrm{T}_1$ with $\frac{a}{a_1}$ (and with $\frac{a_1}{a}$) for various values of $\frac{b_1}{a}$ when $T_2 = 0$. These enable the understanding of $\sigma_{\psi z}\left(\frac{R,0}{T_1}\right)$ in two cases:

(i) as the hole radius, a grows ($0.1\, a_1 \leq a \leq a_1$) when $\frac{b_1}{a}$ is fixed for the load site [$-b_1$,$-a_1$,];

(ii) when $\frac{a_1}{a}$ draws near $\frac{b_1}{a}$ ($b_1 > a_1 \geq a$).

Some fields for the equivalent homogeneous materials ($\mu_1 = \mu_2$) are derived from (3.10), (3.11) and (4.2a,b) with $\gamma = 0$. Whether homogeneous or non homogeneous, cracking may emanate from the hole interface junction, or its homogenous equivalent, when T_j becomes high and a_j and b_j have finite values, especially for small values of a.

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