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The slide away theory of lower hybrid bursts

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Abstract

In this paper two coupled diffusion equations for resonant wave-particle interactions are solved in the slide away regime. It is found that the wave grows and damps like the lower hybrid and ion sound modes respectively; and that the electron beam velocity and the wave energy density spectrum exhibit oscillations which reproduce some of the characteristics of lower hybrid burst. The model may be used to explain the lower hybrid wave bursts observed in the upper ionosphere.

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1.0 Introduction:

We consider a configuration where there are two populations of electrons, consisting of circulating electrons and trapped electrons. The local electrical field is a finite fraction of the critical runaway field. The result is a two part electron population the parts of which are separated in velocity space, forming a region of positive slope for values of V, close to the thermal velocity and driven unstable by this positive slope. These excited modes restrain the circulating electron population, transferring their energy and momentum to the ions resulting in a stationary configuration, which is referred to as the slide away regime [1]. This situation exists in fusion plasma and in the inverted "V" and the bursty edge precipitation [2] observed in the ionosphere at the auroral heights. Lotka-Volterra type equations relating electron beam velocity and energy density of the excited wave are obtained from the well-known diffusion equations [3]. The theory presented here may be used to explain the lower hybrid bursts occurring in the ionosphere [4,5]

2.0 Derivations of Bursts equation

The quasilinear diffusion equations for ions and electrons in slide away regime can be written as [1]

$$\frac{\partial f_i^0}{\partial t} = \frac{ie^2}{m_i}\sum_k \frac{|E_k|^2}{k^2} k.\frac{\partial}{\partial v}\frac{1}{\omega - k.v} k\frac{\partial}{\partial v} f_i^0 \tag{2.1}$$

for ions and

$$\frac{\partial f_e^0}{\partial t} - \frac{e}{m_e} E\frac{\partial f_e^0}{\partial v} = \frac{ie^2}{m_e}\sum_k \frac{|E_k|^2}{k^2} k.\frac{\partial}{\partial v}\frac{1}{\omega - k.v} k\frac{\partial}{\partial v} f_e^0 \tag{2.2}$$

for electrons. Taking the first velocity moment, they become

$$m_i\frac{\partial v_i}{\partial t} = -\sum_k \left|\frac{e\phi_k}{T_i}\right|^2 kT_i \operatorname{Im} W_i \tag{2.3}$$

and

$$m_{ei}\frac{\partial v_i}{\partial t}+eE=-\sum_k\left|\frac{e\phi_k}{T_{e1}}\right|^2 KT_{e1}\left[\frac{n_1}{n}\operatorname{Im}W_{e1}+\frac{n_0}{n}\frac{T_{e1}}{T_{e0}}\operatorname{Im}W_{e0}\right] \qquad (2.4)$$

Equation (2.4) can be written as

$$\frac{\partial\left(\frac{v_{e1}}{v_{1i}}\right)}{\partial t}=\frac{\left\{eE-AkT_{e1}\left[\frac{n_0T_i}{nT_{e0}}\operatorname{Im}(x_{e0})+\frac{n_1}{n\Theta}\operatorname{Im}W(x_{e1})\right]\right\}}{mv_{i1}} \qquad (2.5)$$

where $A=\sum_k\left|\frac{e\phi_k}{T_{e1}}\right|^2$.

The subscripts 0 and 1 refer to trapped and circulating electrons, respectively, $n=n_0+n_1\approx n_0$ and t stands for thermal, while $E=E_0z$ is the applied electric field. ImW is the imaginary part of the Landau integral x_{eo} and x_{eI} are defined in [1]. The evolution of the wave amplitude is described by

$$\frac{\partial A}{\partial t}=2A\sum_i\gamma_i \qquad (2.6)$$

where γ is growth rate. An electron in motion during a time τ acquires from the E field a speed v $v\approx\tau e\frac{E}{m}$ where $\tau^{-1}=v\approx v^{ee}$. Hence equation (2.5) becomes

$$\frac{\partial v}{\partial t}=vV-\omega W(\omega,V_c)AV \qquad (2.7)$$

where $V=\frac{v_{e1}}{v_{t1}}$, and V_c is the threshold speed for the onset of instability.

Referring to Figures 8(a - f) in [1] we see that the lower branch is most unstable and suggesting in particular that growth rates, γ_{1h} and γ_{is}, of the lower hybrid mode and the ion sound mode, respectively should grow the faster. In this branch $\frac{\omega^2}{\omega_{1h}^2}<<1$ for ion should mode which responds to threshold V_c < 0.5 and therefore the instability is a damped one. Equation (2.6) becomes

$$\frac{\partial A}{\partial t}=2(\gamma_{lh}-\gamma_{is})A \qquad (2.8)$$

or

$$\frac{\partial A}{\partial t}=\alpha AV^2-\beta A \qquad (2.9)$$

where $\alpha\approx 2\omega_i\left(\frac{2\pi}{8}\right)^{1/2}\left(\frac{n_1}{n}\right)$ and $\beta\approx 2\omega_i\sqrt{\frac{m_e}{m_i}}$. Equations (2.7) and (2.8) are the system of equations that describe the bursts of electron velocity and wave energy.

To solve these systems of equations analytically, we make the following substitutions: $V' = \ln V$, $A' = \ln A$, and also $a = \left(v - \omega W e^{A}\right)$, $b = \left(\alpha e^{2\ln V} - \beta\right)\omega A$, and a normalized time $t' = 2\alpha\omega W t$ to arrive at

$$\frac{\partial a}{\partial t'} = b \tag{2.10}$$

and

$$\frac{\partial b}{\partial t'} = -a \tag{2.11}$$

The solutions to these equations are proportional to $e^{\pm it'}$ and hence periodic in time. Without loss of generality, we can choose initial conditions at $t' = 0$, $a = a_{\max}$, and $b = 0$. The solution is easily seen if we identify p = b and $q = a$ with Hamilton's equation, in the position coordinate q and the momentum coordinate p.

$$\frac{\partial H}{\partial p} = \frac{\partial q}{\partial t'} \tag{2.12}$$

$$\frac{\partial H}{\partial q} = -\frac{\partial p}{\partial t'} \tag{2.13}$$

The Hamiltonian H, which is conserved and equals energy E is given by

$$H = \frac{1}{2}a^2 + \frac{1}{2}b^2 = E \tag{2.14}$$

which gives circular trajectories in (a, b). Nothing that $a = a_{\max}$ at $b = 0$, the beam velocity is related to the wave amplitude as follows:

$$\left(\frac{1}{2}\right)a_{\max} = E$$

Therefore

$$b = \pm\sqrt{2}\sqrt{a2_{\max} - a} \tag{2.15}$$

The minimum amplitude $a_{\min} = -a_{\max}$. The period T' for Hamilton's equation is given by

$$T' = \frac{\partial J}{\partial E} \tag{2.16}$$

$J = \oint pdq$ is the action integral therefore,

$$J = \oint \sqrt{E^{e} - a^2 da} = 2\pi E, \tag{2.17}$$

$T' = 2\pi$ and $T = \dfrac{T'}{2\alpha\omega}W$.

The numerical solutions presented in ref. [6] have shown that the initial conditions affect only the amplitude while the shape and the period of the electron – wave bursts remain the same. Similar relationships can be established between equations (2.3) and (2.8) in order to account for ion-wave bursts.

3.0 Conclusions

A recent observation in the ionosphere is the burst of lower hybrid waves. A derivation is presented of two coupled differential equations which reproduce the period of the burst and the envelope of their fine structure.

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