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Analogy between the standard gauge model of the basic forces and hadronic mechanics

A. O. E. Animalu, ***FAS, NNOM***
The Institute for Basic Research, 4A Limpopo Street, FHA, Mmaitama, Abuja and
Department of Physics and Astronomy, University of Nigeria, Nsukka.
(e-mail: nascience@aol.com)

Abstract

In this paper we review the standard gauge model of the basic (action-at-a-distance) forces in nature characterized by the conventional gauge-invariant substitution, $\partial_\mu \psi \rightarrow \left(\partial_\mu - i(e/\hbar c)A_\mu\right)\psi$ ***for the electromagnetic field (in the Schrodinger or Dirac equation for the normal hydrogen atom in conventional quantum mechanics), and by*** $\partial_\mu \psi \rightarrow \left(\partial_\mu - it_a A_{a\mu}\right)\psi$ ***for the Yang-Mills field in the Weinberg-Glashow-Salam unification of electroweak forces (referred to as "standard" unified gauge model) which are based on conventional Einstein's special relativity theory, and indicate the analogy with the new generalizations of conventional quantum mechanics and the "standard" unified gauge model under the name "hadronic" mechanics to include not only action-at-a-distance forces but also a fifth (non-local, contact/overlap) force, represented by a Hulthen potential, in superdense matter (such as compressed hydrogen atom in a neutron star) and in condensed matter (such as a high-temperature cuprate superconductor). Hadronic Mechanics distinguishes Einstein's special relativity (for relative motion in vacuum) from "extended" relativity and general relativity, and provides a non-perturbative grand unification scheme for the basic forces in nature (including gravitation and the fifth force) that brings not only Oyibo's "grand unified theorem" but also superstring "theory of everything" within its purview. The "hadronic" mechanics representation of the neutron as a compressed H-atom*** $n = (e^-, p)_{HM}$ ***, known as***

Rutherford-Santilli neutron,

or as

$$(e^- \bar{\nu}_e p) \rightarrow (W^-, p)_{HM} = n,$$

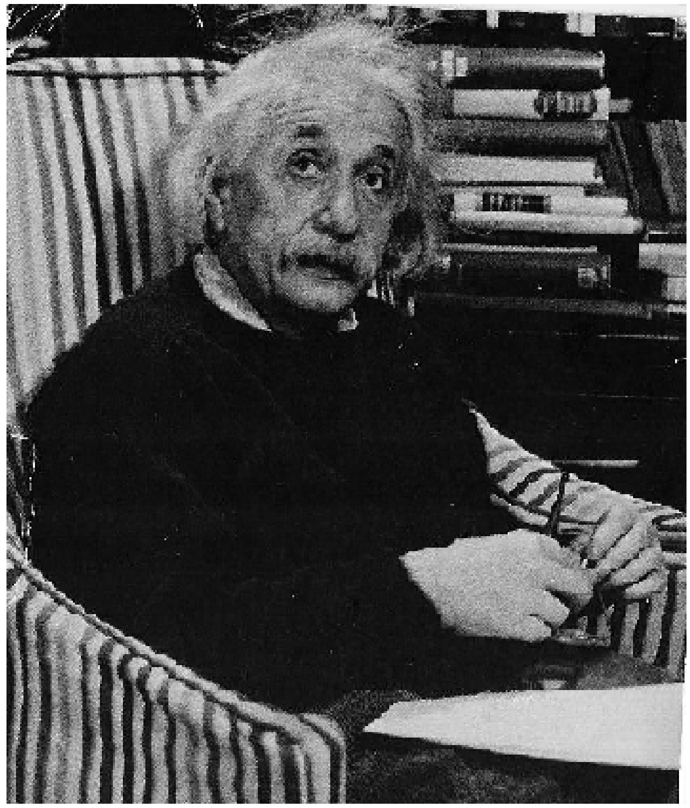
implies that the p-n, p-p and n-n binding in atomic nuclei of all chemical elements can be understood in terms of "hadronic" (strong interaction) chemistry, with potential application to development and production of clean fuels from "hadronic" energy.

pp 1- 30

1.0 A review of Einstein's Special Extended General Relativity and Unified Field Theories

According to an American picture book for inquisitive children entitled *Einstein for Beginners* [1] which I bought for my late son, Charles Animalu at about nine years of age, Albert Einstein is said to have been led to his 1905 theory of special relativity by a *fancy* he had when he was 15 years old. The fancy arose from his *experience* with flying kites in the form of the following question (p.75 of ref. [1]):

What if I were flying at the speed of light [in vacuum] and look in a plane mirror also moving at the speed of light (as in Figure 1), ***would my image disappear from the mirror?***



Albert Einstein (1879-1955)

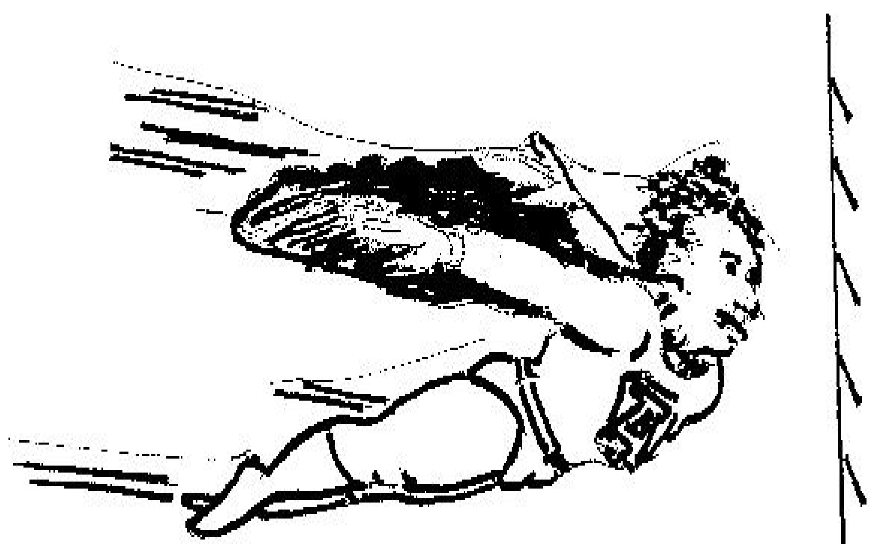


Figure.1: Einstein flying at speed of light and looking into a plane mirror.

The answer, which was finally produced by Einstein's theory of special relativity[2] based on the constancy of the speed of light (in vacuum) in accordance with the mathematical (Lorentz) transformation law,

$$g_{\mu\nu}x^{\mu}x^{\nu} \equiv (ct)^2 - x^2 - y^2 - z^2 = (ct')^2 - x'^2 - y'^2 - z'^2 \equiv g_{\mu\nu}x'^{\mu}x'^{\nu}$$
$$ct' = \gamma(ct - (v/c)x),\ x' = \gamma(x - vt),\ y' = y,\ z' = z,\ (\gamma \equiv 1/\sqrt{1 - v^2/c^2}) \quad , \quad (1.1)$$

was that one would see nothing because light leaving one's face at the speed of light would not reach the mirror. The most important consequence of this fancy was that time turned out to be a kind of illusion. A person who is moving with speed (v) will experience time dilatation, i.e., the slowing down of time by the factor, γ. It also led to the twin paradox which states that an astronaut on return to earth after space travel will be younger than his brother on earth who did not go on space travel. One of the consequences of special relativity theory was that one would never attain the speed of light, because putting $v^2 = c^2$ would make the slowing down factor, γ, infinitely large, which means that Einstein's original fancy was a fantasy; but another consequence of the theory, that mass (m) and energy (E) should be equivalent, turned out to be true in accordance with Einstein's most celebrated formula:

$$E = mc^2 \quad (1.2)$$

Before his theory of special relativity in 1905, Einstein had, (following Planck's 1900 theory of radiation based on the quantum hypothesis relating the energy and frequency of light wave ($E = h\nu$)), conceptualized light wave in terms of a particle (photon) at the centre of a wavepacket, and used the concept to explain the *photoelectric effect* (for which he received the Nobel Prize in 1921). Going back to Einstein's fancy as a child, my son and I had wondered in a joint article [3] entitled *Birkhoffian Mechanics of Velocity-Dependent Forces in the Extended Relativity of Deformable Particles,* what Einstein would have seen if he were inside such a wave packet (see, Figure 2) moving with *group* velocity v and *phase* velocity V, such that $vV = c^2 = V'^2 - v'^2$, in a physical (refractive) medium characterized by a refractive index $n = c/v$ in which both v and c were not constant, but varying, for example, with the time or temperature of the medium.

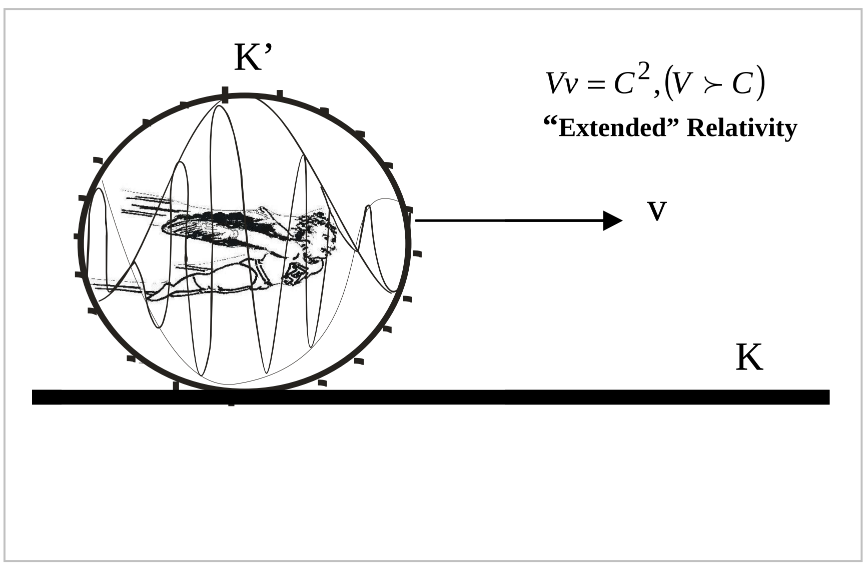


Figure 2: Einstein inside a wavepacket in a dense medium

This situation, involving motion with relative speed *V>c* is the subject-matter of "extended" relativity theory, and our interest in it arose from observation of light caustics through a louver glass – a common sight in Nigeria (see, Fig.3). The caustics are generated by the envelope of the normals to a parabola or an ellipse, and are of interest because of their recurrence as a motif in certain Nigerian cultural symbols which I had discussed in my 1989 lecture [4] to the Nigerian Academy of Science.

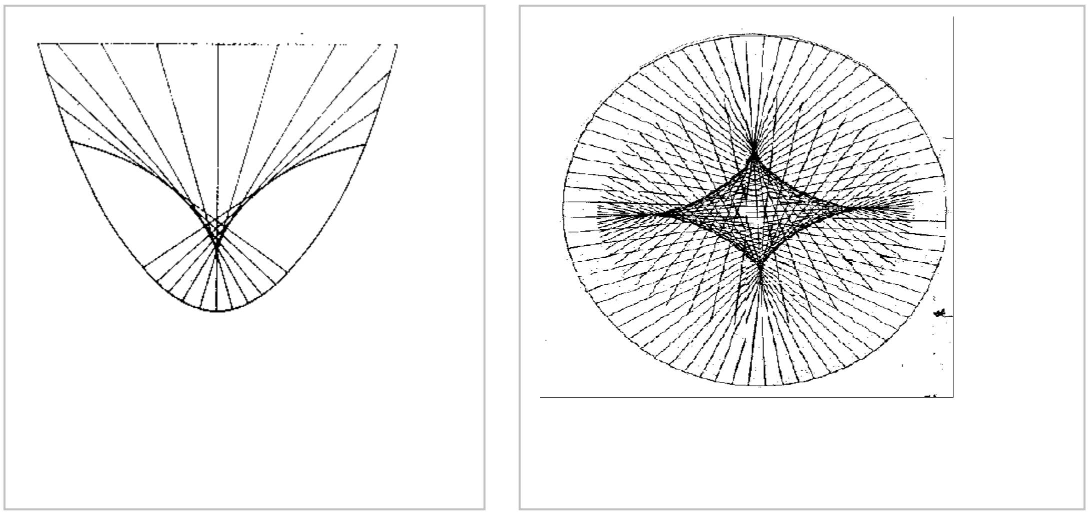


Figure 3: Envelope of the normals to (a) parabola, (b) ellipse.

My son and I sought for an explanation in the framework of Einstein's analysis of the Fizeau experiment (p. 10 of ref.[2]) on the motion of a drop of liquid (representing our wavepacket) through a tube in which the fixed wall of the tube played the part of the unprimed coordinate system (K) while the drop of liquid played the part of the moving coordinate system (K') and the ray of light played the part of a moving point whose coordinate is (ct, x, y, z) with respect to K and (ct', x', y', z') with respect to K'. However, we expressed the usual Lorentz transformation in Equation (1.1) in the form

$$n'^{-\frac{1}{2}}\, cdt' = n^{\frac{1}{2}}(cdt - (v/c)dx), \quad n'^{-\frac{1}{2}}\, dx' = n^{\frac{1}{2}}(dx - cdt), \quad dy' = dy, \quad dz' = dz \qquad (1.3a)$$

where $n = c/v$ and $n' = c/v'$ are the refractive indices for motion in K and K' respectively. Then Einstein's special relativity characterized as in Equation.(1.1) by the invariance of the expression $s^2 = (ct)^2 - x^2 - y^2 - z^2$ would hold provided that

$$1/n' = n - 1/n \equiv (c^2 - v^2)/vc\,. \qquad (1.3b)$$

Consequently, in geometric terms, if one characterizes a deformation of the "light cone" in (*c*,*v*)-space as follows,

$$0 = c^2 - v^2 \equiv (c, v)\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} c \\ v \end{pmatrix}$$

$$\to (c\ ,\ v)\begin{pmatrix} 1 & -\dfrac{dc}{dv} \\ -\dfrac{dc}{dv} & -1 \end{pmatrix}\begin{pmatrix} c \\ v \end{pmatrix} \equiv c^2 - v^2 - 2cv(\frac{dc}{dv}) = 0. \qquad (1.4a)$$

then, one would infer that

$$\frac{dc}{dv} = (c^2 - v^2)/2vc \tag{1.4b}$$

is the ratio of the velocity potential, $\phi = (c^2 - v^2)/2$, and the stream function, $S = cv$, for a hydrodynamic flow in 2-dimensional (c,v)-space. Equatopm. (1.4b) can be integrated exactly to find (with c_0^3 as a constant)

$$c^2 = \frac{v^2}{3} + \frac{c_0^3}{3v}, \text{ i.e., } 3vc^2 = v^3 + c_0^3 \tag{1.5a}$$

Consequently, if on one hand, one puts $v = r/t$ in this expression, one gets a cubic equation

$$\begin{gathered} 3r(c_0 t)^2 = r^3 + (c_0 t)^3, \\ \text{i.e., } X + Yr + r^3 = 0, \text{ where } (X, Y) = \left((c_0 t)^3, -3(c_0 t)^2\right) \end{gathered} \tag{1.6a}$$

which states that one would see in (X,Y)-space, the envelope of the normals to a parabola representing what is known as "cusp" catastrophe [5] (Figure 3a). For an ellipse the corresponding envelope of the normals can be seen as light caustics generated by passing a laser beam through a grated louver glass (Fig. 3b). If, on the other hand, one puts $v = v_0 - \omega r$, in Equation (1.5a), one gets

$$c^2 = \frac{(v_0 - \omega r)^2}{3} + \frac{c_0^3}{3(v_0 - \omega r)} \equiv \frac{c_0^3/3\omega}{(r_0 - r)} + \frac{\omega^2 (r_0 - r)^2}{3} \tag{1.6b}$$

which characterizes the so-called (Schwartzschild) singularity ("warping of space") at $r = r_0 \equiv v_0/\omega$, called a "black hole". It means that if (like Einstein in a wavepacket) one imagines being in a balloon spinning into such a black hole, one would be sucked into the hole (as depicted in Figure 4), like an airplane flying into and disappearing in the Bermuda Triangle on planet Earth.

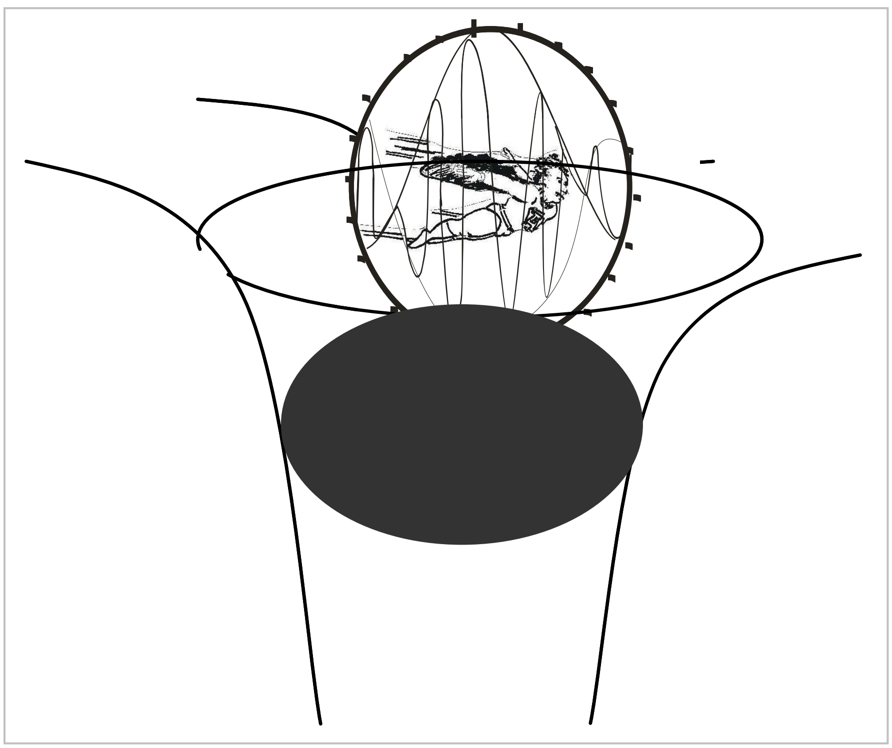


Figure 4: Warped Space (black hole singularity) of General Relativity Theory

The above formulation and consequences of “extended” relativity provide an insight into Einstein’s subsequent generalization of his special relativity theory (for which $(g_{\mu\nu}) = diag.(+1,-1,-1,-1)$ is a constant metric tensor) to a theory of the gravitation field under the name “general relativity theory” for which the $g_{\mu\nu}$ are functions of position (x^{μ}) in a 4-dimensional Riemannian (curved) space-time with line element

$$g_{\mu\nu}dx^{\mu}dx^{\nu} \equiv g_{00}(cdt)^2 + g_{11}(dx)^2 + g_{22}(dy)^2 + g_{33}(dz)^2 \qquad (1.7a)$$

and the field equation of the (general relativity) theory is:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = -\kappa\Theta_{\mu\nu}\,. \qquad (1.7b)$$

Here, $\Theta_{\mu\nu}$ is the energy momentum tensor whose (covariant) conservation implies that of the symmetric tensor, $G_{\mu\nu,}{}^{\nu} = 0$; and $R_{\mu\nu}$ is the Ricci’s tensor, and R the curvature scalar given in terms of the Riemann curvature tensor $R_{\mu\alpha\beta\nu}$ by $R_{\mu\nu} = R_{\mu\alpha\beta\nu}g^{\alpha\beta}$, $R = R_{\mu\nu}g^{\mu\nu}$ while $\kappa = 8\pi f/c^4$, f being the gravitational constant and c the speed of light in vacuum. For space-time outside a symmetric mass M equipped with the (Schwarzschild) metric [6]

$$(g_{\mu\nu}) = \begin{pmatrix} Z & 0 & 0 & 0 \\ 0 & -1/Z & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\theta \end{pmatrix}; \quad Z = 1 - \frac{2GM}{c^2 r} \qquad (1.8)$$

with respect to spherical coordinate ordering (ct, r, θ, φ), and line element

$$ds^2 = g_{00}(r)(cdt)^2 + g_{rr}(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \qquad (1.9)$$

the Schwarzschild’s “solution” of Einstein’s field equation has the form

$$g_{rr}(r) = [1 - 2m(r)/r]^{-1},\ m(r) \equiv \kappa\int_0^r 4\pi r^2\rho(r)dr\,, \qquad (1.10)$$

which satisfies a *nonlinear* (Riccati’s) equation:

$$\frac{dg_{rr}}{dr} - \frac{1}{r}g_{rr} - (8\pi\kappa\rho r - \frac{1}{r})g_{rr}^2 = 0 \qquad (1.11)$$

where $16\pi\kappa\rho$ is the scalar curvature and ρ the (curvature) energy density. We shall return to the mathematical significance of Riccati’s equation for unified gauge theories in Section 3 of this paper.

Before the “confirmation” of the general relativity theory (of gravitation?) in 1919 by the measurement of the deflection of light rays in a gravitational field and spectroscopic examination of binary stars, Einstein proposed in 1916 a “unified field theory” in which he put together the *symmetric* gravitational field (Riemannian metric) tensor $g_{\mu\nu}$ and the *antisymmetric* electromagnetic (gauge) field tensor, $F_{\mu\nu}$, in a “unified” field defined by the metric

$$\bar{g} = \bar{g}_{\mu\nu} \equiv g_{\mu\nu} + F_{\mu\nu} \qquad (1.12)$$

of differential Riemannian space-time geometry. This proposal was, however, not satisfactory for several reasons among which the most fundamental is the lack of complete symbiosis between Einstein’s theory of special relativity and conventional quantum mechanics. A mathematically self-consistent framework for tackling this symbiotic problem (to be presented in Section. 2) is by constructing a structurally more general *isomathematics* of quantum mechanics known as “hadronic” mechanics [7], which includes special, extended and general relativities in its framework. However, given the large volume of literature on “hadronic” mechanics (the 18th Workshop on Hadronic Mechanics was held in

Sweden in June 2005), the presentation in Sec. 2 will be kept at a readily understandable level of a analogy between the conventional quantum mechanics of the normal H-atom or positronium (referred to as “atomic” mechanics) and the “hadronic” mechanics of compressed H-atom in superdense state of matter such as a neutron star and Cooper pairs in superconductors.

As is well known, while the problem of unification of gravity with any other basic force (electromagnetic, weak and strong interactions) in nature remains an active area of research at present, some measure of success has been achieved with unification of electromagnetism and weak (nuclear) force in a gauge theory within the framework of conventional (special) relativistic quantum mechanics. The current-current interaction model of neutron decay and the London theory of superconductivity will be used in Sec. 3 to explore an analogy between unified electroweak gauge theory and “hadronic” mechanics and its generalization via a Weyl-like gauge principle to the strong interaction. In Section 4, we shall discuss the grand unification scheme for the basic forces in nature (including gravity and non-local integral forces). Finally in Section 5, we shall explore analogies between molecular H-H and nuclear *n-n* interaction chemistry with potential industrial applications to production of new clean fuels via submerged arcs technology and draw the attendant conclusions.

2.0 Analogy between “Atomic” Mechanics and “Hadronic” Mechanics

As stated in Section 1, a major source of the problems of Einstein’s unification scheme is a well-known historical one, namely, that because (non-relativistic and relativistic) quantum mechanics had not been fully developed in 1916, the quantum mechanical nature of the problem of unification of electromagnetism and the (general relativity) warping of space was not fully addressed by Einstein. This lack of symbiosis between special relativity and conventional quantum mechanics was identified in 1978 by R.M. Santilli [7], a professor of theoretical physics at Harvard University, USA, in a proposal for a new *isomathematics* under which one constructs an isotopic, that is, axiom preserving lifting (non-unitary transformation) of Lie’s theory that is directly applicable to nonlinear as well as nonlocal-integral systems - a theory that is today called the *Lie-Santilli isotheory*. The resulting *isomechanics* is called Birkhoffian mechanics (at the classical level) and “hadronic” mechanics (at quantum level). For simplicity, we shall distinguished “hadronic” mechanics from conventional quantum mechanics of the normal positronium and H-atom systems (to be referred to as “atomic” mechanics) by drawing an analogy between the two in the light of our present knowledge about the intrinsic properties of all known elementary particles, namely leptons $(e^-, \nu_e, \mu^-, \nu_\mu, ...)$, baryons(*p,n,* Λ,…), mesons($\pi^0, \pi^\pm, K^\pm, K^0, ...$) and intermediate bosons ($W^\pm, Z^0$) and the hypothetical quarks (*u,d,s*) and their antiparticles

According to our present knowledge, all known elementary particles and the hypothetical quarks as well as the H-atom and the anti-H-atom can be represented in a three-dimensional orthogonal (L,B,Q) lattice space, (L being lepton number, B baryon number and Q the electric charge in units of the proton charge) as shown in Fig.5 and leads to the representation of the H-atom in quark theory (Figure 6). Note that in the (L,B,Q)-space representation in Figure 5 we have used the standard (electroweak) model classification of (ν, e^-, μ^+) as leptons and $(\bar{\nu}, e^+, \mu^-)$ as antileptons, $\nu \equiv (\nu_e, \nu_\mu^C)$ being a 4-component neutrino. As is well-known, the usual assignments of the lepton number $L = (1,1,1)$ and electric charge $Q_L = (0,-1,-1)$ to the lepton triplet (ν_e, e^-, μ^-), and the baryon number $B = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and electric charge $Q_B = (\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ to the quark triplet (u, d, s) obey the relations[8,9], $L + Q_L = B + Q_B$, $B = L - \frac{2}{3}L$, $Q_B = Q_L + \frac{2}{3}L$, from which it follows that the hypothetical quarks may be obtained from the physical leptons by shifting $\frac{2}{3}$ of the lepton number to the lepton charge. This led to Barut’s proposal (see Figure 7a) to build mesons from lepton-antilepton pairs, e.g.

$\pi^0 = \sqrt{\frac{1}{2}}(\nu_e\overline{\nu}_e - e^- e^+)$ and baryons from proton, leptons and antileptons, e.g. $n = pe^-\overline{\nu}_e$. However, because of "mutation" of intrinsic properties, like the spin, of particles in the transition from point like particles in "atomic" mechanics to extended isoparticles in "hadronic" mechanics (HM), Santilli [7)] proposed elimination of the neutrinos/antineutrinos as constituents to reach the compressed positronium atom model for $\pi^0 = (e^+, e^-)_{HM}$ and Rutherford's compressed H-atom model of the neutron, $n = (p, e^-)_{HM}$ shown in Figure7b. This was the conceptual physical framework that guided the development of the *Lie-Santilli isotheory* based on the (classical [Birkhoffian] and HM) analogies summarized in Tables 1a and 1b.

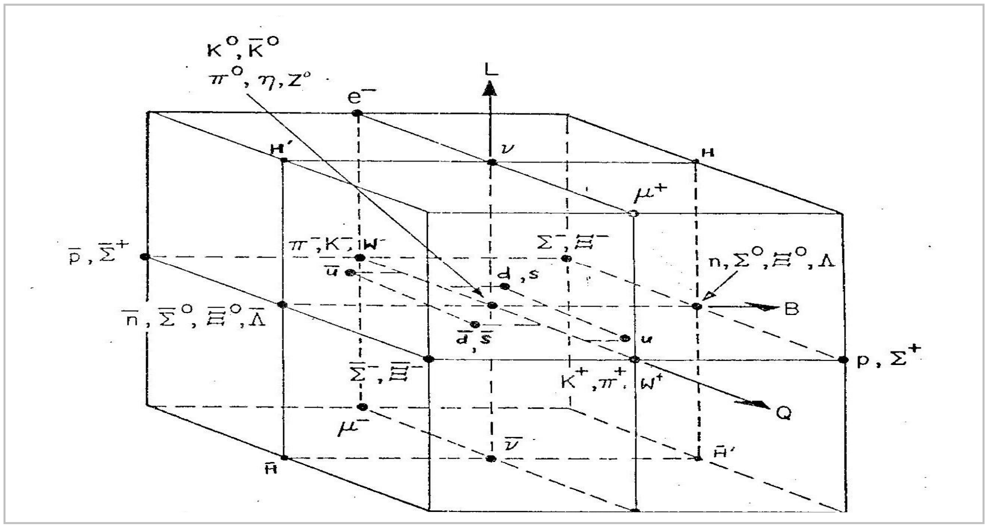


Figure 5: Representation of particles in three-dimensional (L,B,Q) space lattice.

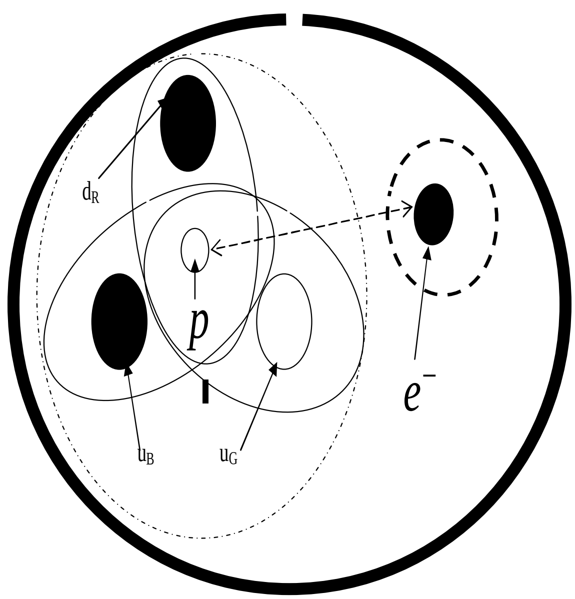


Figure 6: Representation of the H-atom in quark theory.

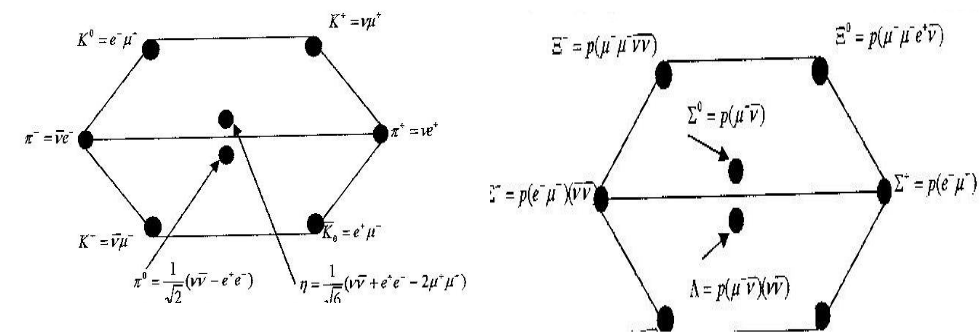


Figure 7a: Barut's representation of mesons and baryons (ref.[9]).

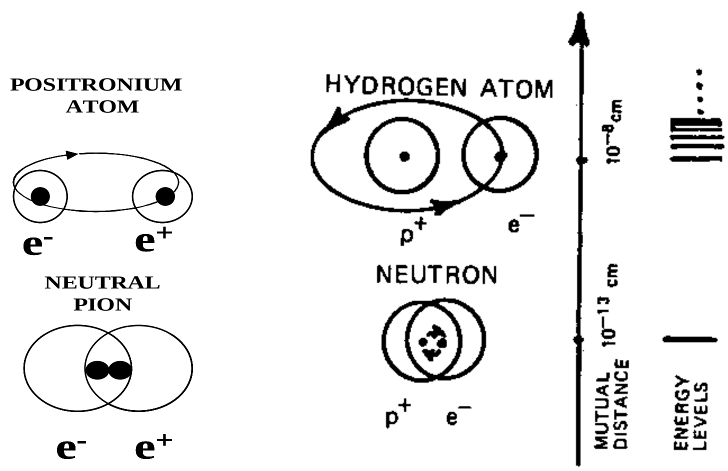


Figure 7b: Santilli's representation (ref. [7]) of the transition from the "atomic" mechanics of the normal positronium atom to the neutral pion and of the normal H-atom to the neutron in "hadronic" mechanics.

Table 1a: Analogy between Thermodynamics, Classical Mechanics and Birkhoffian Mechanics

	Classical Thermodynamics	**Classical Mechanics**	**Birkhoffian Mechanics**
1.	(V, S, T, P)	$(q, p, \dot{q}, \dot{p})$	$(q, p, \dot{q}, \dot{p})$
2.	Internal Energy $U = U(V,S), \quad dU = -PdS + TdS$ $P = -\frac{\partial U}{\partial V}, T = \frac{\partial U}{\partial S}$	Hamiltonian $H = H(q,p) : dH = -\dot{p}dq$ $\dot{q} = \frac{\partial H}{\partial p}, \dot{p} = -\frac{\partial H}{\partial q}$	
3	Gibb's free Energy, $G = G(p,T)$ $\quad dG = d(U - TS + pV) = VdP - SdT$ $V = \frac{\partial G}{\partial P}, S = -\frac{\partial G}{\partial T}$	Hamiltonian+External Terms: $\dot{q} = \frac{\partial H}{\partial p} + f_1,$ $\dot{p} = \frac{\partial H}{\partial q} + f_2$ $(a_1, a_2) = (q, p)$ $(f_1, f_2) = -(\dot{q}, \dot{p})$	Birkhoffian $B = B(\dot{p}, \dot{q})$ $dB = d(H - \dot{q}p + q\dot{p})$ $= qd\dot{p} - pd\dot{q}$ $q = \frac{\partial B}{\partial \dot{p}}, \; p = -\frac{\partial B}{\partial \dot{q}}.$ $\dot{a}_\mu = S^{\mu\nu}\frac{\partial B}{\partial a^\nu}$

4. Liouville Equation

$$\dot{A} = \dot{q}\frac{\partial A}{\partial q} + \dot{p}\frac{\partial A}{\partial p} = \frac{\partial A}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial H}{\partial q}\frac{\partial A}{\partial p} \equiv \frac{\partial A}{\partial a^{\mu}}\omega^{\mu\nu}\frac{\partial H}{\partial a^{\nu}},$$

$$(\omega_{\mu\nu}) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\dot{A} = \frac{\partial A}{\partial a^{\mu}} S^{\mu\nu}\frac{\partial B}{\partial a^{\nu}},$$

$$\det\left\|(S^{-1})_{\mu\nu}\right\| = \frac{\partial \dot{q}}{\partial q}\frac{\partial \dot{p}}{\partial p} - \frac{\partial \dot{p}}{\partial q}\frac{\partial \dot{q}}{\partial p} \equiv [\dot{q}, \dot{p}]_C \neq 0$$

Table 1b: Analogy between "Atomic" and "Hadronic" Mechanics

		"Atomic" Mechanics	**"Hadronic" Mechanics**
1.	Characteristics of constituents	Point-particles	Wavepackets
2.	Nature of Interaction	Action-at-a-distance force	Action-at-a-distance + contact/overlap forces
3.	Equation of motion	Heisenberg's Equation. $ih\frac{dA}{dt} = AH - HA,$	Iso-Heisenberg eqn. $ih\frac{dA}{dt} = ATH - HTA,$ $T^{-1} = g$ ("metric" tensor)
4.	Wave equation	Schrodinger's equation $H\psi \equiv [p^2/2m + V_C = i\hbar\frac{\partial \psi}{\partial t}$	Iso-Schrodinger's eqn. $(HT)\hat{\psi} = i\hbar\frac{\partial \hat{\psi}}{\partial t},$ e.g. $T = 1 - \lvert\psi\rangle\langle\psi\rvert$
5.	Interaction potential of (e^-, p^+) in the H-atom	Coulomb potential, $V_C = -e^2/r$	Hulthen potential, $\hat{V} = V_C - \frac{H\lvert\psi\rangle\langle\psi\lvert\hat{\psi}\rangle}{\hat{\psi}} \rightarrow -V_0/(e^{kr} - 1) \equiv V_H$
6.	Relation to Gauge principle	$V_C = eA_0$, where A_0, for a static Coulomb potential, $V_C = -e^2/r$, satisfies the diff. eqn. $\frac{\partial A_0}{\partial r} = (1/e)A_0^2$	$V_H = e\hat{A}_0$, where $\hat{A}_0$ obeys Riccati's diff. eqn. $\frac{\partial \hat{A}_0}{\partial r} = \frac{1}{e}\hat{A}_0^2 + \varsigma\hat{A}_0 - \kappa$ related to Yang-Mills field.

Observe in Figure 7b that the point-particle characteristics and the consequential action-at-a-distance (Coulomb) force between constituents apply to inter-particle separation of order 10^{-8} cm in "atomic" mechanics while the wavepacket characteristics and the consequential activation of additional contact/overlap forces between constituents apply to inter-particle separation of order 10^{-13} cm in "hadronic" mechanics as elaborated further in Figure 8.

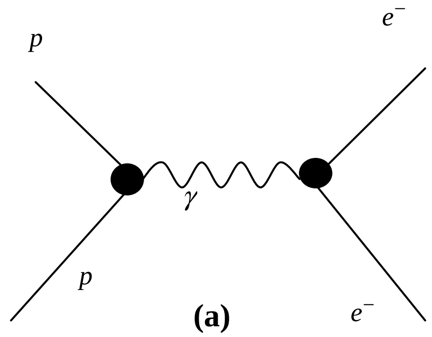


Figure 8(a): Conventional representation of the proton (p) and electron (e-) in "atomic" mechanics of the normal H-atom as point-particles bound by action-at-a-distance (Coulomb) force mediated by exchange of a virtual photon.

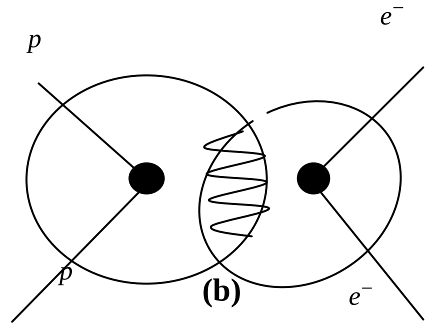


Figure 8b: New representation of p and e- in "hadronic" mechanics as overlapping extended deformable objects with consequential activation of non-local effects in compressed H-atom in neutron star.

Observe in Table 1a that the analogies between the Gibbs function, the Hamiltonian with external terms and the Birkhoffian at classical level lead to the operator images in Table 1b. Thus, the usual Lie-algebraic structure of the conventional Heisenberg's equation of motion for an operator, $A = q$ or p , in a system characterized by a Hamiltonian *H*,

$$i\hbar \frac{dA}{dt} = AH - HA \tag{2.1a}$$

is generalized to the Lie-Santilli algebraic structure of the iso-Heisenberg equation

$$i\hbar \frac{dA}{dt} = ATH - HTA, \quad T^{-1} = g \quad (\text{"metric" tensor}) \tag{2.1b}$$

which permits an underlying "metric" tensor to be introduced. This provides the required symbiosis between special/extended/general relativities and quantum mechanics. Before proceeding to drive the point home, we note in Table 1b that the corresponding generalization of the conventional Schrodinger's wave equation

$$H\psi \equiv \left(\frac{\hbar^2 p^2}{2\overline{m}} + V_C \right)\psi = i\hbar \frac{\partial \psi}{\partial t}, \tag{2.2a}$$

to the iso-Schrodinger equation

$$HT\hat{\psi} \equiv \left(\frac{b^2\hbar^2 p^2}{2\overline{m}} + V_H \right)\hat{\psi} = i\hbar \frac{\partial \hat{\psi}}{\partial t}, \quad . \tag{2.2b}$$

where *b* is a scaling (effective mass) parameter, is achieved for $T = 1 - |\psi\rangle\langle\psi^*|,$ by the replacement of V_C by a strong "pseudopotential" [10] having the form of a Hulthen potential:

$$V_C(r) - \frac{\langle \psi^* | \hat{\psi} \rangle H\psi(r)}{\hat{\psi}(r)} \to \frac{-V_0}{\left(e^{kr} - 1\right)} \equiv V_H(r)\,. \tag{2.3a}$$

As a result, whereas the energy eigenvalue Equation (2.2a) leads to the usual discrete spectrum of "atomic" energy levels, the spectrum is suppressed to only one stable "hadronic" energy level for (2.2b), as indicated in Figure7b. Moreover, we note that the transition in Equation (2.3a) from Coulomb to Hulthen potential can be expressed in the form of a progressive generalization of nonlinear Ricatti's equation for the time-component of the electromagnetic 4-vector potentials, $A_0 = (1/e)V_{\mathrm{C}}$ to the Riccati equation for the time-component of a Yang-Mills field 4-vector potential, $\hat{A}_0 = (1/e)V_{\mathrm{H}}$, as follows:

$$0 = \frac{\partial A_0}{\partial r} - \frac{1}{e} A_0^2 \rightarrow \frac{\partial \hat{A}_0}{\partial r} - \frac{1}{e} \hat{A}_0^2 - \xi \hat{A}_0 + \kappa = 0 . \tag{2.3b}$$

We shall return to the significance of this feature in Section 3. Now, in order to make the symbiosis between special-extended- general relativities and quantum mechanics evident, we combine Equation (2.2a) with the Hermitian conjugate of Equation (2.2b) in isospin representation introduced in ref.[10b] as follows:

$$i\hbar \frac{\partial \Psi(\vec{r},t)}{\partial t} = H\Psi(\vec{r},t), \quad H \equiv \frac{1}{2m} \vec{p}^{\,2} \hat{\tau}_3 + V_C; \; \Psi = \begin{pmatrix} \psi \\ \hat{\psi}^* \end{pmatrix}, \; \Psi^+ = [\psi^*, \quad \hat{\psi}]. \tag{2.4a}$$

This involves a lifting of the underlying "metric" tensor

$$g \equiv \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \rightarrow \hat{\tau}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -T \end{pmatrix} \equiv \hat{g} \tag{2.4b}$$

characterized by a pseudopotential type of integral operator (T) defined as follows:

$$T\hat{\psi}(\vec{r}) = \int d\vec{r}\,'\left[\delta(\vec{r} - \vec{r}\,') - \psi^*(\vec{r})\psi(\vec{r}\,')\right]\hat{\psi}(r'),$$
$$\text{i.e.,} \quad T = 1 - |\psi\rangle\langle\psi^*|, \quad T^2 = T, \text{ when } \langle\psi^*|\psi\rangle = 1, \tag{2.5a}$$

and definig the overlap integral, $Z^{\frac{1}{2}} = \int d\vec{r}\,'\hat{\psi}^*(\vec{r}\,')\psi(\vec{r}\,') \equiv \langle\hat{\psi}^*|\psi\rangle$, we have

$$\langle\hat{\psi}^*|T|\hat{\psi}\rangle = 1 - Z; \quad \langle\psi^*|T|\psi\rangle = 0, \quad \text{when } \langle\hat{\psi}^*|\hat{\psi}\rangle = 1. \tag{2.5b}$$

It is thus apparent that in the absence of overlap as in Figure 8(a), i.e., when $Z=0$, the operator T reduces to unity; but in the presence of overlap as in Figure 8(b), i.e., when $Z \neq 0$, the physical effect of T is that the charge on $\hat{\psi}$ represented by $\langle\hat{\psi}^*|T|\hat{\psi}\rangle$ is "depleted" by an amount Z (called "orthogonalization charge") whereas the isocharge on ψ represented by $\langle\psi^*|T|\psi\rangle$ is zero.

I would like to drive home the geometrical significance of Equation (2.4b) with the visual image of the isosymmetries of a unit sphere in 3-dimensional space, $1 = x^2 + y^2 + z^2 \equiv x_i g_{ij} x_j$ obtained with an isounit defined in 1995 by Tepper L. Gill and co-workers[11] at Howard University, USA:

$$T = T_\xi \equiv \frac{1}{\sqrt{a(\xi)x^2 + b(\xi)(y^2 + z^2)}}, \; \left(a(\xi) = 1 + 3\xi, \; b(\xi) = 1 - \xi\right) \tag{2.6a}$$

as follows:

$$1 = x^2 + y^2 + z^2 \equiv x_i g_{ij} x_j \rightarrow x_i T \xi g_{ij} x_j \equiv \frac{x^2 + y^2 + z^2}{\sqrt{a(\xi)x^2 + b(\xi)(y^2 + z^2)}} = 1 \tag{2.6b}$$

This leads to a representation of the isosymmetries of the unit sphere by a composite pair of quadric surfaces defined by the quartic form $a(\xi)x^2 + b(\xi)(y^2 + z^2) = \left(x^2 + y^2 + z^2\right)^2$

i.e., $(x^2+y^2+z^2)+\xi(3x^2-y^2-z^2)=\left(x^2+y^2+z^2\right)^2$. (2.7)

Such a pair evolves, as shown in Figure 9 for ξ in [0,1], from a pair of unit and point sphere (Figure 9a):

$$\xi=0,\Rightarrow (x^2+y^2+z^2)(x^2+y^2+z^2-1)=0 \qquad (2.8)$$

through various degrees of overlap/interpenetration (Figure 9b and c) to a pair of unit spheres in contact (Figure 9d):

$$\xi=1,\Rightarrow ((x-1)^2+y^2+z^2-1)((x+1)^2+y^2+z^2-1)=0\,. \qquad (2.9)$$

It follows from the visual representation in Figure 9 that isotopic lifting is a geometrical process characterizing not only the deformation of extended geometrical object(s) but also their mutual contact/penetration or fusion into a single composite object, as envisaged in "hadronic" mechanics. It goes beyond the usual interpretation of Lorentz transformation of special relativity theory as a rotation, or interpretation of general relativity (theory of gravitation) in terms of curvature. Mathematically, we may re-interpret the (isotopic lifting) transformation defined by (2.6b) as an abstract scale transformation of the "metric" tensor

$$g\to\hat{g}=T_\xi g \qquad (2.10a)$$

with index ξ, and the isosymmetries of a unit sphere from Equation.(2.7) as a representation given by the solution for a given (invariant) index:

$$\xi=\frac{\left((x^2+y^2+z^2)-1\right)(x^2+y^2+z^2)}{(3x^2-y^2-z^2)}\,. \qquad (2.10b)$$

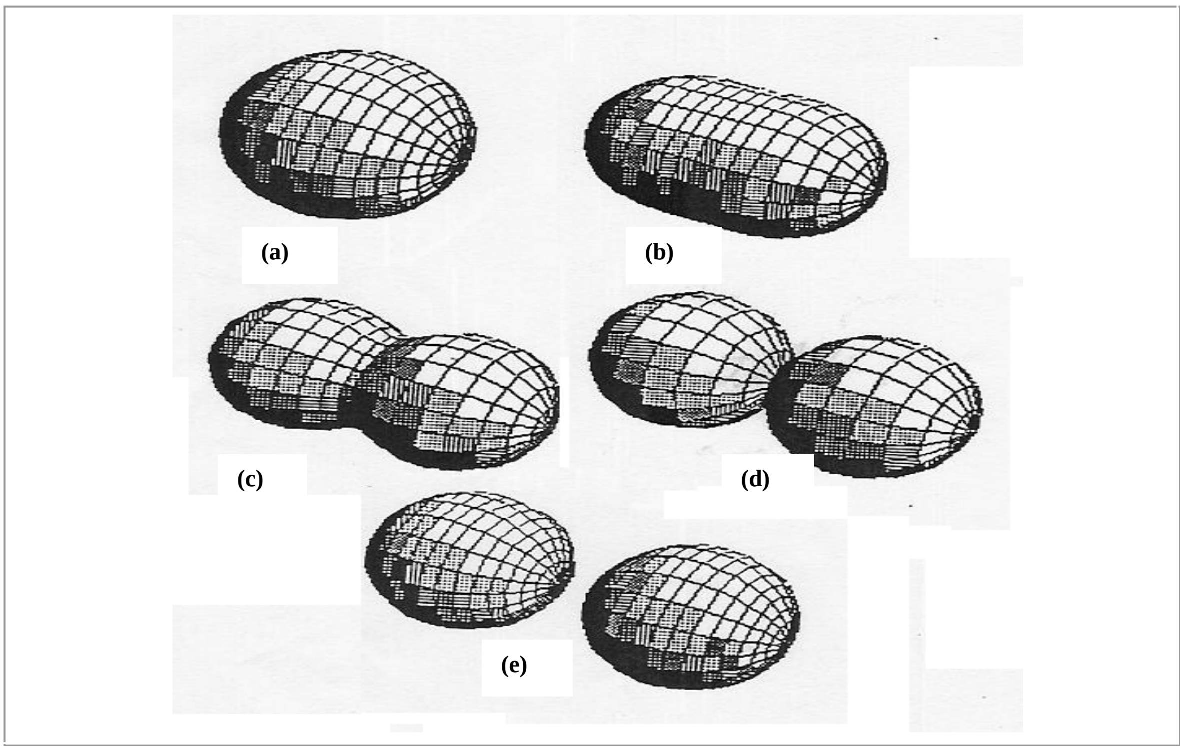


Figure 9: Visual image of Isosymmetries of a sphere (a) as a pair of quadric surfaces in various states of mutual overlap (b) and (c) , contact (d), and separation (d).

3.0 Analogy between unified electroweak gauge model and "Hadronic" Mechanics

Having constructed the nonrelativistic "hadronic" mechanics of the *Rutherford-Santilli neutron*, $n = (e^- \downarrow - p)_{HM}$, as a compressed H-atom in Section 2, a question naturally arises as to what analogy exists between "hadronic" mechanics of such a structure model of the neutron (which includes nonlocal-integral forces) and

(1) Fermi current-current interaction model of neutron decay, $n \rightarrow p + e^- + \bar{\nu}_e$, involving an antineutrino ($\bar{\nu}_e$), based on conventional relativistic quantum mechanics, and

(2) Weinberg-Glashow-Salam unified gauge model of electroweak (i.e. electromagnetic and weak) interactions [12] in which the neutron mass is computed from renormalization theory of conventional relativistic quantum mechanics.

This question is tackled in this section.

3.1 Quantum Electrodynamics and Current-Current Inter-action Model of Neutron Decay and Superconductivity

As is well-known, the standard procedure or paradigm for putting together classical special relativity and classical electromagnetic (gauge) theory in (special) relativistic quantum mechanics, under the name "quantum electrodynamics" (QED), involves three steps. Firstly, one writes down the relativistic Dirac equation in the Minowski (flat) space-time metric, $\eta = diag(1,-1,-1,-1)$ of special relativity theory

$$(\gamma_0 p_0 - \gamma_1 p_1 - \gamma_2 p_2 - \gamma_3 p_3 - mc)\psi \equiv (\gamma_\mu p_\mu - mc)\psi = 0 \ , \qquad (3.1a)$$

where $p_0 = -i\hbar\,\partial/\partial t,\ p_j = -i\hbar\,\partial/\partial x_j\ \ (j = 1,2,3),\ \gamma_0 = \beta,\ \gamma_j = \beta\alpha_j\ \ (j = 1,2,3)$ and

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\eta_{\mu\nu} \ . \qquad (3.1b)$$

Secondly, one introduces interaction with the classical electromagnetic field via the gauge-invariant substitutions:

$$\vec{p}\psi \rightarrow (\vec{p} - (e/c)\vec{A})\psi, \quad \frac{\partial\psi}{\partial t} \rightarrow \left(\frac{\partial}{\partial t} - (ie/\hbar c)A_0\right)\psi \qquad (3.2)$$

where the electromagnetic field, tensor, $F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$ obeys Maxwell's equation,

$$\partial^\nu F_{\mu\nu} = J_\mu \equiv ie(\bar{\psi}\gamma_\mu\psi) \ . \qquad (3.3a)$$

and may be rewritten in terms of Moller's potential

$$\partial^2 A_\mu(x) = J_\mu(x); \quad \text{i.e.,} \quad q^2 A_\mu(q) = J_\mu(q) \ . \qquad (3.3b)$$

Finally, one uses perturbation theory to calculate the effects of electromagnetic interaction,

$$J_\mu A^\mu = \frac{J_\mu J^\mu}{q^2} \qquad (3.4a)$$

which implies that the interaction is mediated by the massless photon ($m_\gamma = 0$) quantum of the electromagnetic field. The Feynman graph for this process is as represented for $e^- - p$ interaction in Figure 10(a).

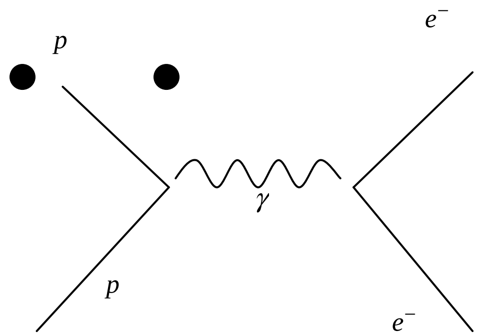


Figure 10(a): Representation of $e^- - p$ electromagnetic interaction in QED.

The Fermi weak current-current interaction model of β – decay of the neutron, $n \to p + e^- + \bar{\nu}_e$, given by $\frac{G_F}{\sqrt{2}} \bar{\psi}_n \gamma^\mu (1-\gamma^5)\psi_e \bar{\psi}_{\nu_e} \gamma_\mu (1-\gamma_5)\psi_p$, which is usually expressed in the form in Figure 10b as an action-at-a-distance interaction mediated by massive W-boson exchange, has an analogous form:

$$\frac{J_\mu J^\mu + (q_\mu J^\mu)^2 / M_W^2}{q^2 + M_W^2} \to \frac{J_\mu J^\mu}{M_W^2} \text{ for } q^2 << M_W^2, \qquad (3.4b)$$

and hence, $e^2 / M_W^2 \approx G_F \Rightarrow \mathrm{M_W} \approx 50\,\mathrm{GeV}$.

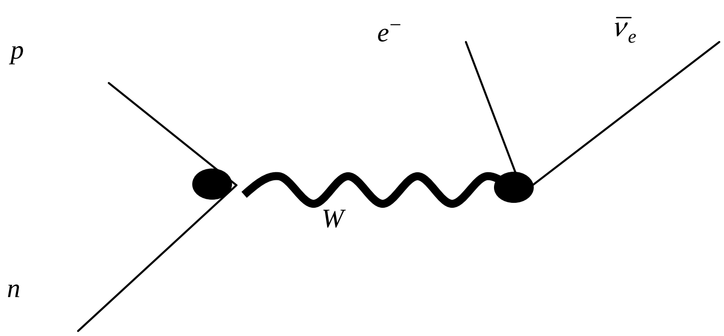


Figure 10(b): Weak current-current interaction via W-boson exchange.

In order to construct an analogy with the "hadronic" mechanics structure of the neutron, we require a model of the neutron as a drop of superfluid H-atom [13] in a neutron star, $n = (e^- \downarrow - p - \bar{e}^0 \uparrow)_{HM}$, formed by Cooper-like pairing of $e^- \downarrow$ and $\bar{e}^0 \uparrow$ around a p "trigger" as shown in Fig. 10c, where $\bar{e}^0$ is a massive neutral spin-$\frac{1}{2}$ antiparticle. This model is comparable to Barut's model of the neutron, $n = pe^- \bar{\nu}_e$, based on the decay process, $n \to p + e^- + \bar{\nu}_e$, but differs from it inasmuch as $\bar{e}^0$ is an anti-isoparticle, i.e., a massive "mutation" of the antineutrino ($\bar{\nu}_e$) analogous to the "spinion" in *iso-*

superconductivity theory [10]; and because of the weak process, $e^- \to W^- + \nu_e$, we have the alternative model, $n = (W^-, p)_{HM}$.

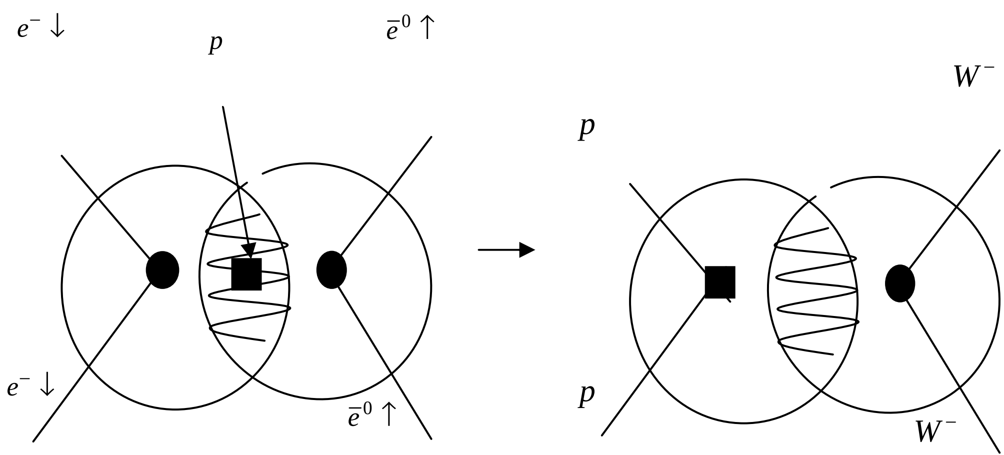


Figure 10(c):"Hadronic" mechanics model of the neutron as a superfluid state, $n = (e^- \downarrow - p - \bar{e}^0 \uparrow)_{HM}$, based on Cooper pairing of $e^- \downarrow$ and $\bar{e}^0 \uparrow$ around a *p* "trigger", but because of the weak process $e^- \to W^- + \nu_e$, may be represented as a two-body system, $n = (W^- - p)_{HM}$.

An analogy now exists between current-current interaction of neutron decay and electromagnetic interaction given by the London equation for the diamagnetic current in a superconductor,

$$\vec{J} = (2eh/mc)\vec{A}, \qquad \text{i.e.,} \qquad \vec{J}.\vec{A} = \frac{\vec{J}.\vec{J}}{(mc/2e^2\hbar)} \tag{3.4c}$$

Note that this equation can be recast (as shown by London in his classic 1961 Dover-published book entitled *Superfluids, Microscopic Theory of Superconductivity* [14] p. 64, in the (special) relativistic form

$$\frac{\partial p_\mu}{\partial x_\nu} - \frac{\partial p_\nu}{\partial x_\mu} = 0, \;\; p_k = mu_k + \frac{e}{c}A_k, \;\; \frac{e}{i}p_0 = \frac{mc^2}{\sqrt{1 - v^2/c^2}} + eA_0 \tag{3.4d}$$

As argued in iso-superconductvity theory [10], Pippard's nonlocal generalization of London's equation (3.4c) to

$$\vec{J}(\vec{r}) = -(2e^2\hbar/mc)\int \hat{I}(\vec{r},\vec{r}\,')\vec{A}(\vec{r}\,')d\vec{r}\,' \tag{3.5a}$$

where the integral operator ($\hat{I}$) appearing in the kernel function has an explicit form

$$\hat{I}(\vec{r},\vec{r}\,') \equiv \begin{cases} \delta(\hat{r} - \vec{r}\,')\;, & \text{London model} \\ \exp\left(-\left|\vec{r} - \vec{r}\,'\right|/\xi\right) & \text{Pippard model} \end{cases} \tag{3.5b}$$

ξ being Pippard's coherence length, may be looked upon as an isotheory resulting from the replacement, $\hbar \to \hbar\hat{I}$, characterizing a transition from the standard (Bardeen-Copper-Schrieffer) model of superconductivity to the isostandard model, with the consequential modification of London's Equation (3.4d) based on special relativity to its isorelativistic generalization in ref. [10]. Such a generalization defined in Equation (3.5b) provides the *paradigm shift* for including nonlocal-integral forces in QED and Fermi's theory of β – decay which we are after.

3.2 **Analogy between Electroweak Gauge Theory and "Hadronic" Mechanics.**

We turn next to examine the construction of the standard (Weinberg-Glashow-Salam) unified gauge theory of electroweak interactions based on Wienberg's 1967 proposal[12], namely, to put together photons & intermediate bosons in a (unified) gauge theory (of electro-magnetic and weak interactions) that is:

- Gauge-Invariant
- Renormalizable
- Intermediate boson mass arises from spontaneous symmetry breaking.

The first step requires a replacement of the $U(1)$ gauge-invariant substitution (3.2) by the generalized $SU(2)$ gauge-invariant substitution

$$\partial_\mu \psi \to (\partial_\mu - i t_a A_{a\mu})\psi, \qquad \text{with } [t_a, t_b] = iC_{abc} t_c \qquad (3.6)$$

which involves the generalized charges, where t_a ($a = 1,2,3$), and the 4-vector potential $A_{a\mu}$ of a generalized classical antisymmetric tensor ($F_{a\mu\nu}$) obeying Yang-Mills equation:

$$F_{a\mu\nu} = \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} + C_{abc} A_{b\mu} A_{c\nu}, \quad \partial_\nu F_a^{\mu\nu} = -i\bar{\psi}\gamma^\nu t_a \psi . \qquad (3.7)$$

However, as in electromagnetic gauge invariance, this step results in zero vector boson mass. The second step requires an $SU(2)\times U(1)$ gauge model of leptons

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{matrix} T_3 = +\frac{1}{2} \\ T_3 = -\frac{1}{2} \end{matrix} \Bigg\} \; T = \frac{1}{2}, \; \mathrm{Y} = \mathrm{T}_3 - Q = +\frac{1}{2} \qquad (3.8)$$

$$e_R^- \qquad T_3 = 0 \qquad T = 0, \quad \mathrm{Y} = +1$$

so that, by noting that $Q = \frac{1}{g}(gT_3) + \frac{1}{g'}(-g'Y)$, one can define a renormalizable interaction of the form,

$$g\vec{T}.\vec{A}_\mu - g'YB_\mu , \qquad (3.9)$$

where the electroweak fields ($A_{\mu,}, W_\mu, Z_\mu$) are

$$A_\mu = \frac{1}{\sqrt{\frac{1}{g^2}+\frac{1}{g'^2}}}\left[\frac{1}{g}A_{3\mu} + \frac{1}{g'}B_\mu\right], \quad W_\mu = \frac{1}{\sqrt{2}}\left[A_{1\mu} + iA_{2\mu}\right]$$

$$Z_\mu = \frac{1}{\sqrt{\frac{1}{g^2}+\frac{1}{g'^2}}}\left[-\frac{1}{g'}A_{3\mu} + \frac{1}{g}B_\mu\right], \quad W_\mu^+ = \frac{1}{\sqrt{2}}\left[A_{1\mu} - iA_{2\mu}\right] \qquad (3.10)$$

and hence the electroweak interaction has the final form:

$$\frac{gg'(T_3 - Y)}{\sqrt{g^2+g'^2}}A_\mu + \frac{g^2T_3 + g'^2 Y}{\sqrt{g^2+g'^2}}Z_\mu + \frac{g(T_1 - iT_2)}{\sqrt{2}}W_\mu^- + \frac{g(T_1 + iT_2)}{\sqrt{2}}W_\mu^+ \qquad (3.11).$$

From the viewpoint of further generalization of the standard gauge model to the isostandard model that we are after, this step amount not only to linearization of electroweak interactions to only action-at-a-distance interaction between pointlike particles mediated by gauge vector bosons but also to *a priori* assumption of the validity of an underlying Minkowskian (flat) space-time metric of special relativity theory and conventional quantum mechanics.

In the third step regarding the generation of mass through spontaneous symmetry breaking, current algebra consideration of virtual processes (see, Figure 11) involving pion exchange, suggest use of the current density:

$$J_\mu = \overline{\psi}_p \gamma_\mu \psi_p + i(\pi^+ \partial_\mu \pi^- - \pi^- \partial_\mu \pi^+) . \tag{3.12}$$

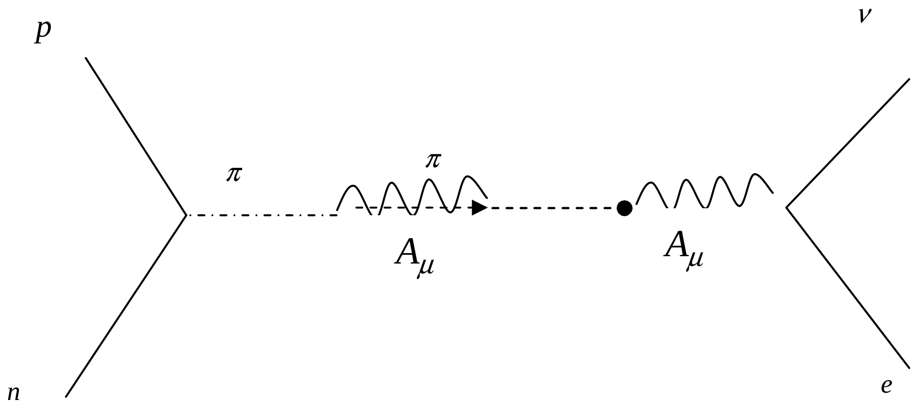


Figure 11

for investigation of spontaneous symmetry breaking in the framework of the (π, σ) model Lagrangian:

$$L = \overline{\psi}\left[i\gamma_\mu \partial^\mu - g(\sigma + i\pi\gamma_5)\right]\psi + \tfrac{1}{2}\left[(\partial\sigma)^2 + (\partial\pi)^2\right] - \tfrac{1}{2}\kappa^2\left[\sigma^2 + \pi^2\right] - \tfrac{1}{4}\lambda\left[\sigma^2 + \pi^2\right]^2 . \tag{3.13}$$

Apart from the first term, L is invariant under rotation in two-dimensional space spanned by (π, σ); and the corresponding anharmonic oscillator Hamiltonian in two-dimensional $(x, y) = (\pi, \sigma)$ space is

$$H = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + V(x, y); \quad (\lambda > 0) \tag{3.14a}$$

$$V(x, y) \equiv \tfrac{1}{2}\kappa^2\left[x^2 + y^2\right] + \tfrac{1}{4}\lambda\left[x^2 + y^2\right]^2, \tag{3.14b}$$

where the form of the potential energy $V(x, y)$ implies zero boson mass for $\mu^2 > 0$ but one zero (Goldstone) boson mass and another non-zero boson mass for $\mu^2 < 0$ (see, Figure 12). Consequently, in the standard electroweak gauge model, the neutron mass is computed from renormalization theory by generalizing the usual (electromagnetic) self-energy of the proton, as shown in Figure 13. Acoordingly, one obtains a generalization of the 1939 Wiesskopf formula for the neutron mass,

$$m_n - m_p = \left(3e^2/16\pi^2\right)m_p\left(\tfrac{1}{2} + \ln\left(\Lambda^2/m_p^2\right)\right) . \tag{3.15}$$

in which the usual cut off mass $\ln(\Lambda^2)$ is replaced typically by $\ln(m_\mu^2)$. In "hadronic" mechanics, instead of computing the neutron mass, and other intrinsic properties from electroweak renormalization theory, one should use the atomic-hadronic mechanics analogy in Figure 14, to solve the iso-Dirac equation for the relative $e^- \downarrow - p$ motion

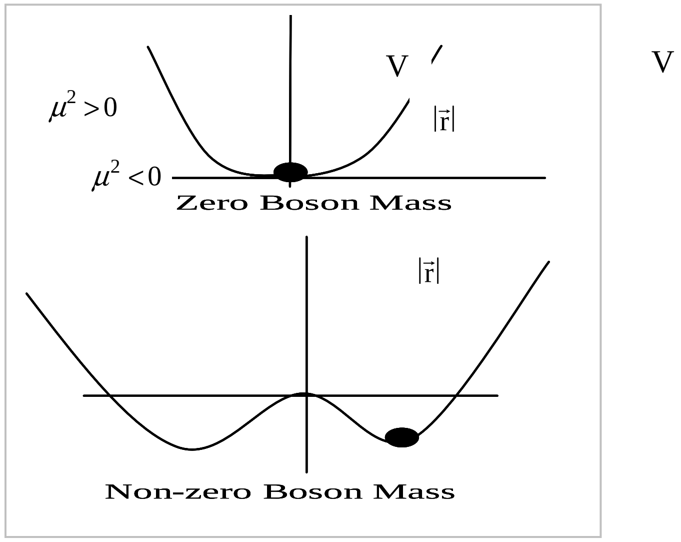


Figure 12: V($|\vec{r}|$) versus $|\vec{r}| = \sqrt{x^2 + y^2}$

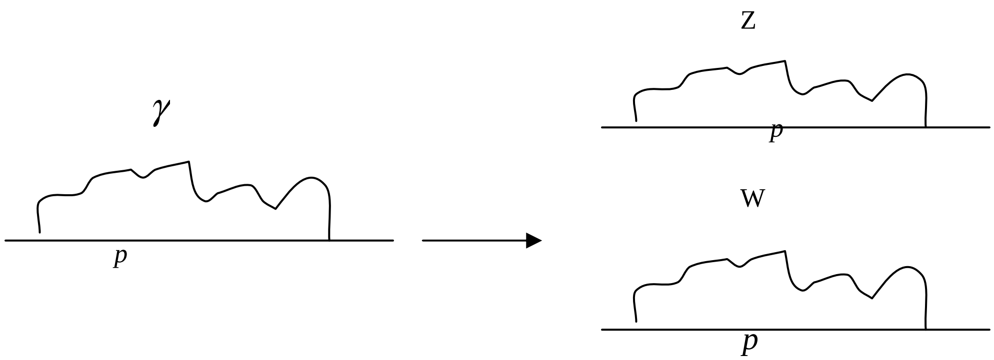


Figure 13

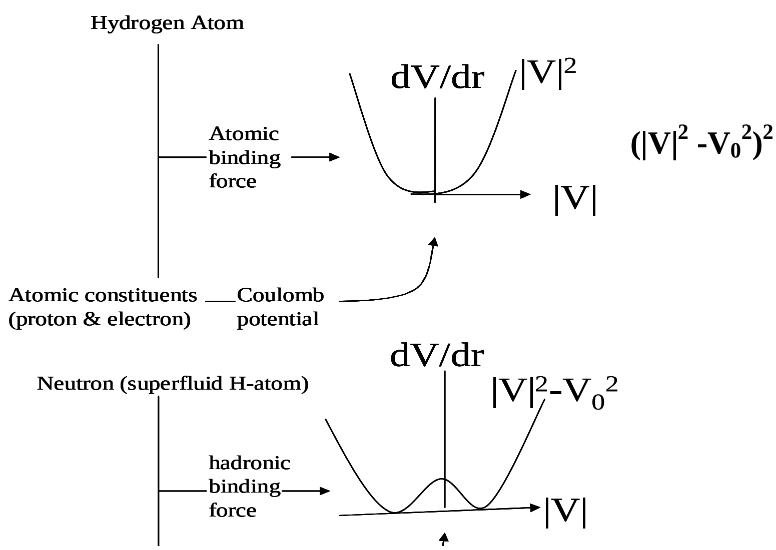


Figure 14

$$\left(c\vec{\alpha}.\vec{p} + \beta\overline{m} * c^2 + V_H\right)\hat{\psi} = \hat{E}\hat{\psi} \quad (3.16)$$

in which the effective $e^- \downarrow -p$ binding force $(-dV_H / dr)$ is a functional of the Hulthen potential, $V_H \propto t_a A_{a0}$, (A_{a0} being the time-component of the Yang-Mill field), characterized by the Riccati's Equation (2.3b). In principle, the components of the metric tensor, $g_{\mu\nu} = \frac{1}{2}(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu)$ of the underlying iso-space-time also satisfies Riccati' s Equation .(1.11).

Technically, spontaneous symmetry breaking demands the generalization of the Hulthen potential indicated in the general structure of the hadronic binding force given in Figure 14, for which an exact solution of Equation (3.16) is not available. In practice, considerable simplification of Equation (3.16) arises from the fact that in the Yukawa limit given by the expansion,

$$V_H(r) = -\frac{Me^{-m_0 r}}{1-e^{-m_0 r}} \approx -\frac{Me^{-m_0 r}}{m_0 r} = -\frac{g_0^2 e^{-m_0 r}}{r} \equiv \phi(\mathrm{r}), \quad (3.17a)$$

(in units such that $\hbar = c = 1$), the Hulthen potential obeys an *explicit* Riccati's equation

$$\frac{dV_H}{dr} = (1/e^2)V_H^2 - \frac{2}{r}V_H - (m_\phi^2 / e^2), \quad (M = \tfrac{1}{2}m_\phi) \quad (3.17b)$$

However, this *nonlinear* differential equation can be transformed via a Weyl-like gauge principle

$$\phi = \exp\left(-(1/e^2)\int_0^r V_H(r)dr\right)$$

$$\text{i.e.,} \quad V_H = -e^2 \frac{d\log(\phi)}{dr} \equiv -e^2 \frac{1}{\phi}\left(\frac{d\phi}{dr}\right) \quad (3.18a)$$

into a *linear* second-order (static Klein-Gordon) equation for a Higgs (scalar) field, ϕ, of mass m_ϕ.

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - m_\phi^2\phi \equiv \left(\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr} - m_\phi^2\right)\phi(r) = 0 \quad (3.18b)$$

Consequently, as the full Klein-Gordon equation conserves a "convective" current density, $i\phi^*(x)\overleftrightarrow{\partial}_\mu\phi(x)$, a further substitution, $\phi = m_\phi^{-\frac{1}{2}}\hat{\psi}$, in this current density and inclusion of the conserved (Dirac) current part of Equation (3.16) lead to the generalization of the electroweak gauge current that we are after, namely an ***isocurrent density*** (analogous to the current density Equation (3.12) used in defining the (π,σ) - model):

$$\hat{j}^\mu(x) = \hat{\overline{\psi}}(x)\left(\gamma_\mu - im_\phi^{-1}\overleftrightarrow{\partial}_\mu\right)\hat{\psi}(x) \quad (3.19a)$$

This is the most general current in the O(4,2) algebra of Dirac matrices[15a] conserved by the *exactly soluble* wave equation[15b]

$$\left(i\gamma_\mu\partial^\mu - (M + m_\phi) - (1/m_\phi)\partial_\mu\partial^\mu\right)\hat{\psi} = 0, \quad (M = \tfrac{1}{2}m_\phi) \quad (3.19b)$$

This is the equation characterizing the analogy between the unified electroweak gauge theory and "hadronic" mechanics that we are after.

Two important observations can now be made. Firstly, Equation (3.19b) is derivable from a Lagrangian density of the form [15b]

$$L = -\tfrac{1}{4}\hat{\overline{F}}_{\nu\mu}\hat{F}^{\mu\nu} + (k_0 - k)\hat{\overline{\psi}}\hat{\psi} \quad (3.20a)$$

where the generalized tensor field $\hat{F}^{\mu\nu}$ analogous to Maxwell's free electromagnetic field tensor, $F^{\mu\nu}$, associated with the gauge-invariant substitution (3.6a), is given by

$$\hat{F}^{\mu\nu} = \sqrt{k_0}\eta^{\mu\nu}\hat{\psi} - 2i\frac{1}{\sqrt{k_0}}\gamma^{\mu}(\partial^{\nu}\hat{\psi}); \quad \hat{\bar{F}}_{\nu\mu} = \sqrt{k_0}\hat{\bar{\psi}}\eta_{\nu\mu} + 2i\frac{1}{\sqrt{k_0}}(\partial_{\mu}\hat{\bar{\psi}})\gamma_{\nu} \qquad (3.20b)$$

and $k = (M + m_{\phi})$, $k_0 = m_{\phi}$ are characteristic inverse lengths (in units such that $\hbar = c = 1$), while $\eta^{\mu\nu}$ is the symmetric metric tensor of the underlying Minkwoskian (flat) space-time. For, on using the fact that $\eta_{\mu\nu} = \eta_{\nu\mu}$, $\eta^{\mu\nu}\eta_{\mu\nu} = 4$ and the properties of the $\gamma-$matrices in Equation (3.1b), we find on substituting (3.20b) in (3.20a) that

$$L = \tfrac{1}{2}i\left[\hat{\bar{\psi}}\gamma^{\mu}(\partial_{\mu}\hat{\psi}) - (\partial_{\mu}\hat{\bar{\psi}})\gamma^{\mu}\hat{\psi}\right] - k\hat{\bar{\psi}}\hat{\psi} - \frac{1}{k_0}(\partial_{\mu}\hat{\bar{\psi}})(\partial^{\mu}\hat{\psi}). \qquad (3.20c)$$

This differs from the conventional Dirac Lagrangian density for a massive spin$-\tfrac{1}{2}$ particle only through the last (convective current) term which is related to the Pauli intrinsic moment.

Secondly, suppose that instead of starting with the $SU(2)\times U(1)$ gauge model of the electroweak forces, we go to the Gell-Mann-Oakes-Renner [16] model of chiral $SU(3)\times SU(3)$ symmetry-breaking down to the $SU(3)$ symmetry of strong interactions by selecting [15b]

$$M = \left(\sqrt{\frac{3}{2}}\lambda_0\right)m_0 \equiv diag(m_0, m_0, m_0), \quad m_{\phi} = \left(-\sqrt{3}\lambda_8\right)m_0 \equiv diag(-m_0, -m_0, +2m_0) \; (3.21)$$

$$\frac{1}{m_{\phi}} \equiv \left(\frac{-1}{m_0\sqrt{3}}\right)\lambda_8^{-1} \equiv \left(\frac{-1}{2m_0}\right)\left(\sqrt{\frac{3}{2}}\right)\left(\lambda_0 + \sqrt{2}\lambda_8\right) \equiv diag(-1/m_0, -1/m_0, +1/2m_0)$$

Then Equation (3.19b) would reduce to two identical equations for an isospin doublet,

$$\left(i\gamma_{\mu}\partial^{\mu} + m_0^{-1}\partial_{\mu}\partial^{\mu}\right)\hat{\psi}_d = 0 \qquad (3.22a)$$

with two mass eigenvalues, $m = m_0$ or 0; and a third equation for a singlet

$$\left(i\gamma_{\mu}\partial^{\mu} - 3m_0 - (2m_0)^{-1}\partial_{\mu}\partial^{\mu}\right)\hat{\psi}_s = 0. \qquad (3.22b)$$

with two mass egenvalues determined by the roots of the equation,

$$(2m_0)^{-1}m^2 + m - 3m_0 = 0, \qquad (3.22c)$$

i.e., $m_{\pm} = m_0(\pm\sqrt{7} - 1)$, or $m_+ = 1.65m_0$, and $m_- = -3.65m_0$, leading to the two mass ratios

$$\frac{m_+}{m_0} = 1.65, \quad \frac{|m_-|}{m_0} = 3.65 \qquad (3.23)$$

These ratios have a simple physical meaning. If, on one hand, in the *additive* constituent quark model of the spin $\tfrac{1}{2}$ baryon octet (to which the neutron belongs), we put $m_u = m_d = m_0$, then we may determine from the observed masses of the neutron (as input), $m_n = 3m_0 = 939.6$ MeV and the other members of the octet, $m_{\Lambda'} = 2m_d + m_s = 1142 \approx \tfrac{1}{2}(m_{\Lambda} + m_{\Sigma^0}) = \tfrac{1}{2}(1116 + 1192)$ the two mass ratios:

$$\frac{m_s}{m_d} = 1.65, \quad \frac{m_{\Lambda'}}{m_d} = 3.65, \qquad (3.24)$$

which are in agreement with those in Equation (3.23). In other words, the predicted mass ratios of the members of the SU(3) baryon octet are in agreement with experiment, in accordance with the equal-spacing rule, when splitting due to electromagnetic interactions is ignored. Moreover, the second mass ratio in Equation .(3.24) is related to the mass of the strange particle, Λ, as an excited state of the neutron described by Equation (3.19b) which can be rewritten in terms of the neutron mass ($3m_0 \equiv m_n$) in the form:

$$\left(i\gamma_\mu \partial^\mu - m_n - (\tfrac{2}{3} m_n)^{-1} \partial_\mu \partial^\mu \right)\hat{\psi}_n = 0\,. \qquad (3.25)$$

On the other hand, if in Equation (3.23), $m_0 = m_{e^-}$ and $m^+ = m_{e^0}$ characterize the e^- and $\bar{e}^0$ masses in our neutron model, $n = (e^- \downarrow - p - \bar{e}^0 \uparrow)_{HM}$, i.e.,

$$\frac{m_+}{m_0} \equiv \frac{m_{e^0}}{m_{e^-}} = 1.65\,, \qquad (3.26)$$

then the rest mass of the neutron predicted by "hadronic" mechanics,

$$m_n = m_p + m_{e^-} + m_{e^0} + B,$$

leads to a small *negative binding energy*,

$$B = (m_n - m_p - m_{e^-} - 1.65 m_{e^-}) = -0.12 m_{e^-}\,,$$

also consistent with experiment**.** We are, therefore, led to the conclusion that our "hadronic" mechanics analogy with the unified electroweak gauge model not only extends the model to include Weyl-like gauge theory of the strong force but also lifts the $SU(2) \times U(1) \times O(3,1)$ current algebra of the standard gauge theory to the $SU(3) \times SU(3) \times O(4,2)$ dynamical group current algebra of Dirac matrices. It remains to discuss grand unification of all forces including the warping of space-time by gravity and nonlocal forces.

4.0 Grand Unification of Basic Forces Including Gravity and Non-Local Forces

4.1 A Statement of the Problem

In 1999 *Our Time Press* New York [17] announced that a New-York-based Nigerian, Professor Gabriel Oyibo had written an article entitled *Generalized Mathematical Proof of Einstein's theory Using a New [Conformal] Group Theory* published in American and Russian journals which he subsequently developed further in a book published by Nova Science Publishers, New York, USA under the title [18] "*The Grand Unified Theorem"* (GUT), alternatively called *"God Almighty's Grand Unified Theorem"* (GAGUT). When in November 2004, Oyibo came on a lecture tour of Nigeria, he jolted the Nigerian scientific community with two conclusions from his GAGUT, namely, that:

- The H-atom is the only "basic" element in the Periodic Table of chemical elements from which all other elements (and their isotopes) could be built.
- Mass can be transformed not only into energy (as "standard" Einstein states) but also into momentum (as "Oyibo" states [19] or "isodual" Einstein states).

The first conclusion would be true *if the neutron were the Rutherford-Santilli neutron, i.e .the compressed H-atom envisaged in "hadronic" mechanics*, in which case the (conformal) transformation law defined by GAGUT (as stated in 4.2 below) must lie within the *isomathematics* purview of "hadronic" mechanics. But, what is equally important, it would have practical application to production of "hadronic" energy from stimulated disintegration of the neutron into electron and proton to which we shall return in Section 5. The second conclusion (to be derived in

(4.3)) also lies in the purview of *isodual* generalization of isomechanics recently proposals by Santilli[20] for grand iso-unification of all forces in nature (including gravity and no local-integral forces) – a generalization that includes treatment of antimatter at all (classical and quantum) levels without recourse to second quantization:

4.2 The Grand Unified Theorem

According to Oyibo [18] a function, $G = G(Y_1, Y_2, ..., Y_p)$ is said to be conformal invariant under a given group transformation

$$T_k : Y_i = f_i(y_1, y_2, ..., y_p, k) \quad (4.1a)$$

if T_k is the group of the transformation and

$$G(Y_1, Y_2, ..., Y_p) = F_i(y_1, y_2, ..., y_p, k) \bullet G(y_1, y_2, ..., y_p) \quad (4.1b)$$

where $F_i(y_1, y_2, ..., y_p, k)$ is a function of y_i and k the single group parameter. On the basis of this definition, he derived a set of *conservation equations*

$$(G_{0n})_t + (G_{1n})_x + (G_{2n})_y + (G_{3n})_z = 0, \quad (n = 0,1,2,3,4), \quad (4.2a)$$

which may be rewritten in the Einstein-like form:

$$G_{ij,j} = 0. \quad (4.2b)$$

The function,

$$G_{mn} = G_{mn}(x, y, z, t, \dot{x}, \dot{y}, \dot{z}, \rho, \mu, T, P, ...)$$

defined mathematically as a set of "generic" quantities, are arbitrary functions of space and time coordinates *(x,y,z,t)*, velocities ($\dot{x}, \dot{y}, \dot{z}$), density ($\rho$), fluid or gas viscosity (μ), temperature (*T*), pressure (*P*), etc. By generalizing the conformal transformation (4.1a) to a system of partial differential equations of order n given by

$$G_j(x^1 ... x^p, .y^1 ... y^q, \frac{\partial^n y^1}{(\partial x^1)^n} ..., \frac{\partial^n y^q}{(\partial x^p)^n}) = 0, \quad (4.3a)$$

he derived "solutions" of Equation .(4.2a) in terms of the absolute invariants η_n of the subgroup of transformations for the independent coordinate variables in the form:

$$\eta_n = g_{n0}t^{n+1} + g_{n1}x^{n+1} + g_{n2}y^{n+1} + g_{n3}z^{n+1}. \quad (4.3b)$$

A basic problem in applying GAGUT to physical problems lies in the apparent difference between the above definition of conformal invariance and the usual one. For, according to MIT Professor Roman Jackiw [21] in an article on *Scale Symmetry* published in *Physics Today* (January 1970), conformal and scale or dilatation symmetries of a (classical or quantum) fields constructed in the usual Minkowski space-time of special relativity theory are characterized by dilatation and conformal currents,

$$D_\mu = x^\nu \Theta_{\mu\nu} \text{ and } K_{\mu\nu} = (2x_\mu x_\lambda - g_{\mu\lambda}x^2)\Theta_\nu{}^\lambda \quad (4.4a)$$

and their conservation laws

$$\partial^\mu D_\mu = \Theta_\mu{}^\mu = 0 \text{ and } \partial^\nu K_{\mu\nu} = 2x_\mu \Theta_\lambda{}^\lambda = 0. \quad (4.4b)$$

where $\Theta_{\mu\nu}$ is the energy-momentum tensor and $L = \Theta_\mu{}^\mu$ is the Lagrangian of the field.

Consequently, if conformal and scale symmetries were not broken in the universe, the Lagrangian

$L = \Theta_\mu{}^\mu$ must vanish in Einstein's general relativity equation (1.7a), which would therefore constrain all particles to have *zero mass* . But, as pointed out by Animalu [22] in a letter to *Physics Today* in 1972, such a constraint could be removed by working with a Lagrangian of the form in Eq. (3.20c) with the parameters in Equation .(3.21). However, we observe that when *n=0*, Equation (4.3b) leads to the relations

$$\eta_0 = g_{00}t + g_{01}x + g_{02}y + g_{03}z \sim x^\nu \Theta_{0\nu} \equiv D_0 \tag{4.5a}$$

which imply that η_0 may be interpreted as the dilaton charge D_0 if

$$g_{00} : g_{01} : g_{02} : g_{03} = \Theta_{00} : \Theta_{01} : \Theta_{02} : \Theta_{03} \tag{4.5b}$$

i.e., if $\Theta_{\mu\nu} \propto g_{\mu\nu}$. But this is the solution of Equation (1.7b) in a space of zero curvature ($R_{\mu\nu} = 0$). For this reason, the real challenge is to find corresponding meaningful realizations of GAGUT for $n \geq 1$.

For $n = 1$, Gill's characterization of the group of the isosymmetries of a unit sphere [11] defined earlier by Equations (2.6a and b), is an explicit realization of Oyibo's conformal transformation in Equations (4.1a and b) in the form:

$$F(x, y, z, k) \equiv F_k = \frac{1}{\sqrt{a(k)x^2 + b(k)(y^2 + z^2)}}, \ \left(a(k) = 1 + 3k,\ b(k) = 1 - k\right)$$
$$1 = x^2 + y^2 + z^2 \equiv x_i g_{ij} x_j \rightarrow x_i F_k g_{ij} x_j \equiv \frac{x^2 + y^2 + z^2}{\sqrt{a(k)x^2 + b(k)(y^2 + z^2)}} = 1 \tag{4.6a}$$

Accordingly, by selecting $k = \eta_1$ (an absolute invariant), the "solution" envisaged in Equation (4.2b) is given by Equation (2.10b) in the form:

$$\eta_1 = \frac{\left((x^2 + y^2 + z^2) - 1\right)(x^2 + y^2 + z^2)}{(3x^2 - y^2 - z^2)} \equiv g_{10} + g_{11}x^2 + g_{12}y^2 + g_{13}z^2 \tag{4.6b}$$

where $g_{1\nu}$ are functions of (*x,y,z*). This leads to the visual image of isosymmetries of the unit sphere shown earlier in Figure 9, which is within the purview of isomechanics.

For $n = 2$, a realization of Equation (4.3b) is provided by the canonical cusp catastrophe in Equations (1.6a) and in Figures 3a and b which are associated with deformation of the light cone defined by Equations (1.4a and b) in accordance with "extended" relativity principle.

The realization of GAGUT for $n = 3$ was the subject of my 2000 review 19] of GAGUT, dealing with Oyibo's conclusion that mass can be transformed not only into energy (as standard Einstein states) but also into momentum (as "isodual" Einstein states, or if one wishes, "Oyibo states"). As a matter of fact, an illuminating analogy exists between "isodual" states in extended relativity and strong coupling superconductivity theories, with experimental verification of the latter, as we now proceed to show.

4.3 Analogous Isodual states in extended relativity and strong coupling superconductivity theories.

It is apparent that an analogy exists between the standard quasi-particle energy dispersion relation for a Cooper pair in a superconductor, $E_k = \sqrt{\varepsilon_k^2 + \Delta_k^2}$, and the standard energy-momentum-mass relation, $E/c \equiv p_0 = \sqrt{p^2 + (mc)^2}$, in special relativity theory, i.e., the quasiparticle energy E_k is analogous to $p_0 = E/c$, ε_k (the kinetic energy measured relative to the Fermi level) is analogous to $p = \sqrt{p_x^2 + p_y^2 + p_z^2}$, and Δ_k (the energy gap in a superconductor) is analogous to the rest mass term *mc*. However, whereas Einstein's celebrated formula ($E = mc^2$) results from putting $p = 0$, it is customary to ignore the implication of putting $p_0 = E/c = 0$, to find $mc = \pm ip$, i.e. equivalence between mass and momentum, because one wishes to avoid pure-imaginary momentum. But mathematically, a geometrical interpretation for the pure imaginary momentum exists: one treats the analogous relations

$$(\Delta_k^2 + \varepsilon_k^2 = 0 \quad \text{or} \quad \Delta_k = \pm i\varepsilon_k) \Leftrightarrow ((mc)^2 + p^2 = 0 \quad \text{or} \quad mc = \pm ip) \qquad (4.7a)$$

as point circles in a projective planes (over the complex number field) which are represented as shown in 15 and b by equivalent pairs of straight lines with pure imaginary slopes, $\pm i$.

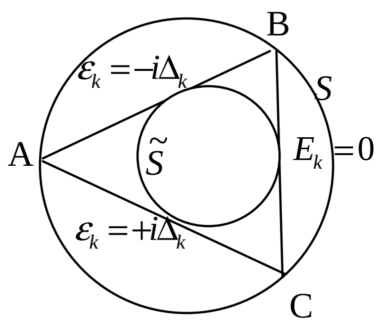


Figure 15a

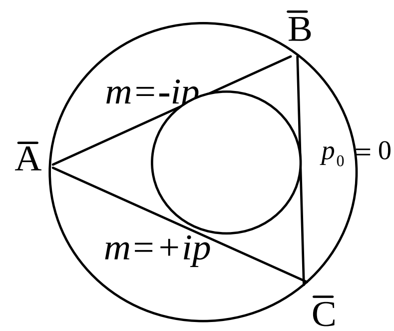


Figure 15b

This representation leads to the geometric *principle of duality* between the inscribed and the circumscribed circles, S .and $\tilde{S}$, in which the point-circle represented by Equation (4.7) is both a point (A) and the intersection of two non-physical straight lines, AC and AB. And for $E_k \neq 0 \Leftrightarrow p_0 \neq 0$, we infer by rewriting the equation of the circle in the form,

$$E_k^2 - (\Delta_k - i\varepsilon_k)(\Delta_k + i\varepsilon_k) = 0 \quad \Leftrightarrow \quad p_0^2 - (mc - ip)(mc + ip) = 0 \qquad (4.7b)$$

that the pair of imaginary lines defines by the linear homogeneous equations

$$\begin{pmatrix} E_k & -(\Delta_k - i\varepsilon_k) \\ -(\Delta_k + i\varepsilon_k) & E_k \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0 \Leftrightarrow \begin{pmatrix} p_0 & -(mc - ip) \\ -(mc + ip) & p_0 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \qquad (4.7c)$$

are associated with the corresponding circles, ABC in Figure 15a and $\bar{A}\bar{B}\bar{C}$ in Figure15b. Moreover, by the geometric principle of duality, the lines AB, AC and BC envelope a conic $\tilde{S}$ (the dual of S with respect to the triangle ABC) whose equation has the general form

$$\begin{gathered} (a\Delta_k^2 + 2f\Delta_k\varepsilon_k + b\varepsilon_k^2 = 0 = E_k) \Leftrightarrow \\ (a(mc)^2 + 2fmcp + bp^2 = 0 = p_0) \end{gathered} \qquad (4.7d)$$

and is a hyperbola, a parabola, or an ellipse according as $f^2 - ab$ is greater than, equal to, or less than zero. Equations (4.7d) are the "isoduals" of Equations (4.7a).

Bearing the above geometrical meaning in mind, one may represent and at the same time *generalize* the standard Einstein relation, $p_0^2 - p_x^2 - p_y^2 - p_z^2 = 0$, by associating it with the secular equation for a set of four homogenous equations,

$$0 = \det\left\| - p_0\delta_{\mu\nu} + p\eta_{\mu\nu} + (mc)\tilde{\eta}_{\mu\nu} \right\|$$

$$\equiv \begin{vmatrix} p - p_0 & 0 & -mc & 0 \\ 0 & -p - p_0 & 0 & -mc \\ -mc & 0 & -p - p_0 & 0 \\ 0 & mc & 0 & -p - p_0 \end{vmatrix} \qquad (4.8a)$$

$$\equiv \left(p_0^2 - p^2 - (mc)^2\right)\left((p + p_0)^2 + (mc)^2\right)$$

where, η and $\tilde{\eta}$, are the usual Minkowskian (flat) space-time metricand its isotopy defined in terms of an antisymmetric 4×4 matrix,

$$\tilde{\beta} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, \tilde{\beta}^2 = -I \ , \qquad (4.8b)$$

as follows:

$$\eta = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \to \tilde{\eta} = \tilde{\beta}\eta = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \qquad (4.8c)$$

This is an explicit realization of the transformation law of GAGUT defined by Equation (4.1b), as a *Lax pairing* [23] of the symmetric and antisymmetric tensors, η and $\tilde{\beta}$. The generalization in Equation (4.8a) lies in the fact that, if *p=0*, the first factor yields the standard Einstein's relation between mass and energy, $E = mc^2$, while the second factor yields a different Einstein's state characterized by $E = \pm imc^2$ .which is usually associated with a faster-than-light particle (tachyon).

The new ("isodual") states envisaged by Oyibo in which momentum can be transformed into mass are obtained by interchanging $-p_0$ and *mc* in Equation (4.8a) to get

$$0 = \det\left\| (p + \tilde{\beta}p_0)\eta_{\mu\nu} + mc\delta_{\mu\nu} \right\|$$

$$\equiv \begin{vmatrix} p + mc & 0 & -p_0 & 0 \\ 0 & -p + mc & 0 & -p_0 \\ -p_0 & 0 & -p + mc & 0 \\ 0 & p_0 & 0 & -p + mc \end{vmatrix} \qquad (4.9)$$

$$\equiv [(mc)^2 - p^2 - p_0^2][(mc - p)^2 + p_0^2].$$

which leads to the two possibilities,

$$(mc)^2 - p^2 - p_0^2 = 0, \qquad \text{i.e., } mc = \pm\sqrt{p^2 + p_0^2}, \tag{4.10a}$$

$$(mc - p)^2 + p_0^2 = 0, \quad \text{i.e., } mc = p \pm ip_0 \text{ or, } |mc| = \sqrt{p^2 + p_0^2} \tag{4.10b}$$

And since $p = \sqrt{p_1^2 + p_2^2 + p_3^2}$, we can rewrite these in the respective standard forms

$$(mc)^2 - p_0^2 - p_1^2 - p_2^2 - p_3^2 = 0, \tag{4.11a}$$

$$(mc \pm ip_0)^2 - p_1^2 - p_2^2 - p_3^2 = 0, \tag{4.11b}$$

from which it follows that when $p_0 = 0$, one gets, $p = mc$, i.e. transformation of mass into momentum!

Moreover, by noting that

$$(mc \pm ip_0)^2 = [(mc)^2 + p_0^2]e^{\pm 2i\varphi} \equiv (m\hat{c})^2 + \hat{p}_0^2 \tag{4.12}$$

where $\varphi = \tan^{-1}(p_0 / mc)$, we can rewrite Equation (4.11b) in the form:

$$(m\hat{c})^2 = -\{\hat{p}_0^2 - p_1^2 - p_2^2 - p_3^2\}, \tag{4.13}$$

which is the mass-energy-momentum relation of "extended relativity" theory defined by Recami [24] or *isodual space* in Santilli's terminology [20] in which $\hat{p}_0 = e^{\pm i\varphi} p_0 \equiv E / \hat{c}$, and $\hat{c} \equiv ce^{\pm i\varphi}$ is a complex velocity of light. Such a complex velocity of light implies that the underlying medium is dispersive. Accordingly, the analogy between $m\hat{c}$ and the complex energy gap ($\hat{\Delta}$) of a strong coupling superconductor such as *Pb* has physical significance insofar as the real and imaginary parts of $\hat{\Delta} = \Delta_1 + i\Delta_2$ have been measured experimentally as functions of the energy [25] (see, Figure 16).

It is the principle of duality represented geometrically by Figures 15a and b and analytically by Equations (4.8) and (4.9) that brings GAGUT within the purview of Santilli's iso-unification [20]. This is the result we are after.

5.0 Strong Interaction Chemistry and applications

A chemical synthesis of the neutron from protons and electrons was done by don Borghi (an Italian priest and physicist), Glori and Dall'olio [26]. If confirmed, it would not only support "hadronic" mechanics but also explain why nuclear *(n-n) binding force in atomic nuclei, when appropriately scaled in energy, is analogous to molecular H-H binding force in molecular systems*, as shown in Figure17. This is but one example of strong interaction ("hadronic") chemistry [27], the most promising consequence of which is the possibility of a new source of subnuclear energy, called "hadronic" energy. Essentially, because the neutron is naturally unstable, according to the exothermic reaction, $n \to p + e^- + \bar{\nu}_e$, which produces about 1.3 MeV, one could expect that such a decay could be stimulated. That is, there might exist a means for artificial decay of the neutron, for example, via bombardment with a gamma radiation ($\hat{\gamma}$) of a frequency suitable to stimulate the emission of the electron. Indeed, as pointed out by Santilli in ref. [27], a light stable natural element (A, Z) is predicted by relativistic "hadronic" mechanics and isospecial relativity to undergo the stimulated transmutation

$$\hat{\gamma} + (A, Z) \to (A, Z+1) + e^= + \bar{\nu}_e$$

e. g. $$1.3MeV + Mo(100,40) \to Tc(100,41) + e^= + \bar{\nu}_e .$$

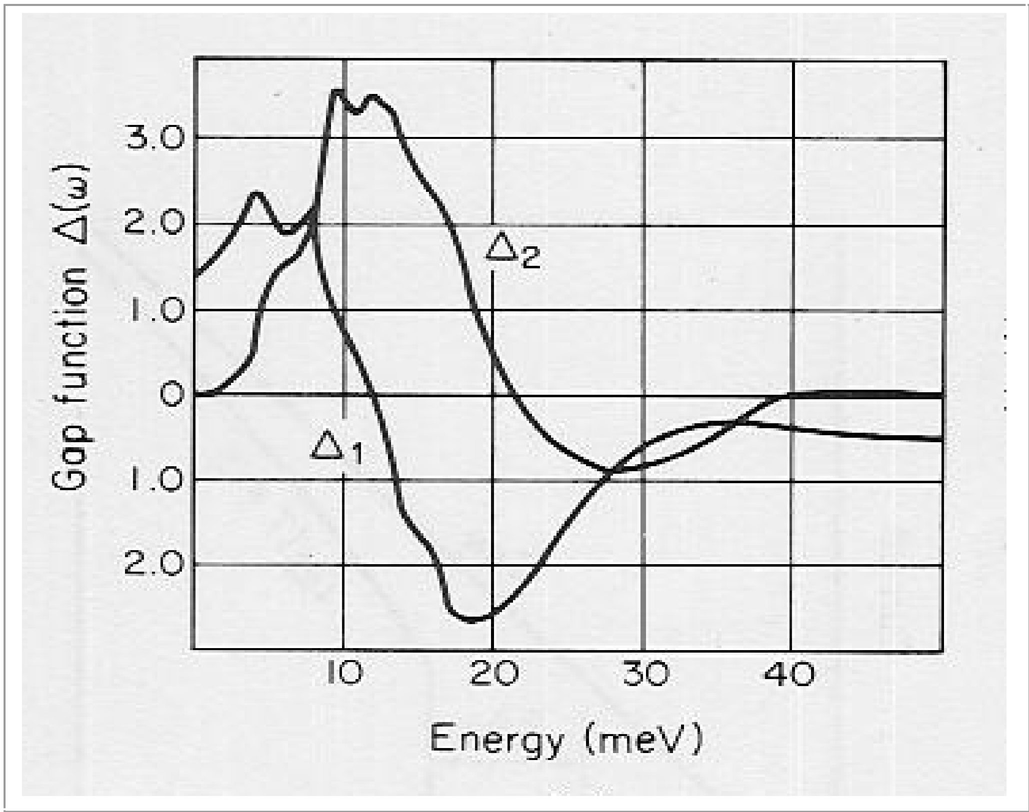


Figure 16: Complex Energy gap as function of energy for Pb

As the energy levels of the original nucleus (A,Z) is higher than that of the final nucleus (A,Z+1), this reaction is highly energetic, thus producing a possible new source of kinetic energy (from that of the electrons) as well as of electricity (because of the production of continuous current depending on the flow of the $\hat{\gamma}$). In this process the nucleus is just a receptacle for a reaction occurring inside individual neutrons, so that the energy is of subnuclear origin. Moreover, if experimentally verified, the "hadronic" energy would be clean inasmuch as it uses neutral, stable, light elements and the only products of the reaction are the electrons which are trapped via simple metallic shields to use their energy and the neutrinos are not harmful to the human body and to the environment.

6.0 Discussion and Conclusions

In this paper, I have used the occasion of Einstein's centenary celebration in the 2005 World Year of Physics by the Nigerian Association of Mathematical Physicists to distinguish Einstein's special/general relativity theories from "extended" relativity theory as well as to review Einstein's unsuccessful proposal for unification of his general relativity theory (of gravitation) and electromagnetism. The problem, identified in 1978 by Santilli as the lack of symbiosis between general relativity and conventional quantum mechanics among other problems, in particular the representation of particles as point-like objects interacting via action-at-a-distance forces mediated by gauge boson exchange, prompted the introduction of a new mathematical framework, known as isomathematics and isomechanics (whose quantum image is called "hadronic" mechanics), in which particles are represented as extended deformable objects whose mutual contact/overlap activate non-local integral forces (the so-called fifth force), besides the four basic (electromagnetic, weak, strong and gravitational) forces in nature. From the analogy of the standard (unified) gauge models of elec-

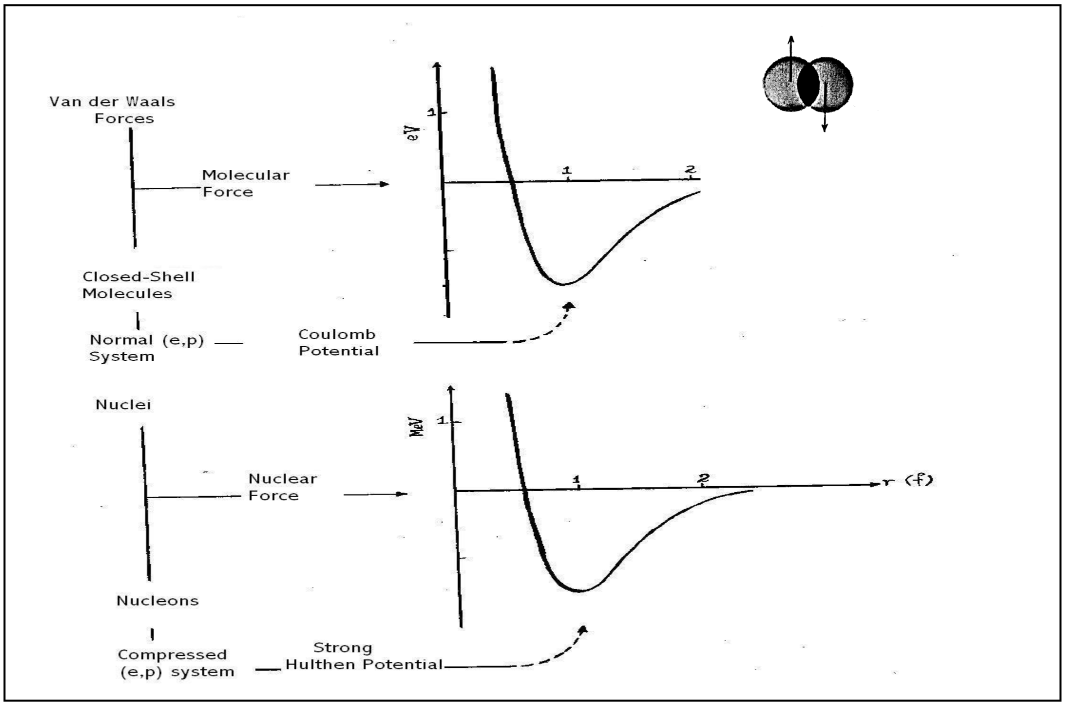


Figure 17: Analogy between Molecular Force (in conventional chemistry) and Nuclear Binding Force (in "hadronic" chemistry).

troweak forces with "hadronic" mechanics, I abstracted the model, $pe\ \bar{\nu}_e \to (W^-, p)_{HM} = n$, which we shall explore in detail in a subsequent paper. In this way I exhibited the difference between two soluble models, namely the conventional quantum mechanics model of the normal H-atom and perturbation treatment of neutron decay on one hand, and the "hadronic" mechanics model of the neutron as a compressed H-atom, and its exact solution and comparison with experimental data, on the other hand. Apart from establishing the generalization envisaged in "hadronic" mechanics, the structure model of the neutron as implies a representation of n-n nuclear binding forces in nuclei analogous to the molecular H-H binding forces in molecular systems, in terms of strong interaction ("hadronic") chemistry, with application to development of new clean "hadronic" energy of subnuclear origin.

We conclude, therefore, that the study of the "hadronic" model(s) of the neutron, especially $n = (W^-, p)_{HM}$, should be intensified from both theoretical and experimental points of view in order to reap the potential benefit of having an exactly soluble "hadronic" mechanics model of the internal structure of strongly interacting particles (hadrons) for testing various contending unification schemes for the basic models in nature (including gravity and the fifth force).

Acknowledgement

This research report is dedicated to the evergreen memory of my late son, Engr. (Dr.) Charles N. Animalu, who has tramped and toiled with me, and is gone!

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