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Equatorial Ionospheric E-Region Critical Frequencies

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Abstract

Studies of ionospheric E-region critical frequency indices have produced conflicting results. We examine diurnal variation of the penetrating frequency at Ibadan, Nigeria (geog. lat. 7.4^{0}N) using the empirical law. Appreciable departure was found when our data are compared with the experimentally observed hourly mean and the median of critical frequencies for the same month. Diurnal variation of E-layer critical frequency of the equilibrium curve for Chapman layer and the plot obtained by empirical expression at Cambridge, England (geog. lat. 51^0 N), is given for comparison. These results can be explained by a rapid recombination of electrons with positive ions.

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1.0 Introduction

Critical frequency of ionospheric E-layer, f_0E, has been measured and recorded for many years in all the ionospheric stations around the world. f_oE is therefore a very important variable in modelling of ionospheric and it is necessary to have a complete database.

However, the identification of the true penetration frequency is sometimes very difficult (Robinson, 1960), on the ionogram. Also, disturbances of the h'(*f*) curve, such as are caused by sporadic E-layer, E_s, made it impossible to deduce the N(h) curve with adequate accuracy from the records. The difficulties of identifying the penetration leaves some doubts as to how great are the day-to-day variation of the maximum electron density of the normal E-layer. To investigate this point, empirical law of the same order as that of the one predicted in Chapman's theory was used to calculate the critical frequency of E-layer.

For the equilibrium approximation to Chapman (1931) theory of layer formation, the variation of the critical frequency (ordinary ray) is described by:

$$f_o = \left[\left(\frac{q_o e^4}{\alpha \pi^2 m_e^2}\right) Cos\chi\right]^{\frac{1}{4}}, \tag{1.1}$$

where, *e* and *m* are the electronic charge and mass, respectively, χ the sun's zenith angle, α the recombination coefficient, assume constant, q_0 is the maximum rate of electron production for $\chi = 0$. Equation (1.1) can be written in the generalized empirical form: for E layer as:

$$f_o E = A Cos^n \chi, \tag{1.2}$$

with

$$A = \left(\frac{q_o e^4}{\alpha \pi^2 m_e^2}\right)^{0.25} \quad [Ckg^{-2}], \tag{1.3}$$

According to the Chapman theory, n = 0.25 in Equation (1.2), but a great number of workers have shown (Robinson, 1959 and references therein) that the monthly median values of penetration frequency published by the ionospheric observatories gives the value of n = 0.33 while the monthly mean values give the value of n = 0.29. In the light of these discrepancies between theory and experimental results, it is quite obvious that this problem should be revisited. The layout of the present paper is as follows. In Section 2, we highlight the theory. In Section 3, we present the results and discussions. Conclusions are in Section 4.

2.0 Theory

2.1 The Critical Frequency

The critical frequency is obtained from a theory of wave propagation through the ionosphere (e.g., Ratcliffe and Weekes, 1960; Stix, 1962). The refractive index, μ, of a wave of angular frequency ω, travelling through plasma containing free charges is given by:

$$\mu^2 = 1 - \frac{4\pi}{\varepsilon_o \omega^2} \sum_j \frac{N_j e_j^2}{m_j}, \tag{2.1}$$

where N_j, e_j, and m_j represent the number density, charge, and mass of each type of charged particle. In the ionosphere the electrons, of small mass, are considerably more important for this purpose than the massive ions, and the refractive index may be written as:

$$\mu^2 = 1 - \frac{4\pi N_e e^2}{\varepsilon_o m_e \omega^2}, \tag{2.2}$$

where N_e, m_e, and e refer to electrons. It is however particularly to be observed that the departure of μ, from its free-space value of unity is inversely proportional to the square of the wave frequency.

If the ionosphere is horizontally stratified (e.g., Jackson, 1969 Oyekola and Iheonu, 2002,), with N_e increasing upward, a plane wave incident at an angle, i, will be refracted until it is traveling horizontally at a level where N_e is sufficiently great to reduce the refractive index to a value given by the relation:

$$\mu = \text{Sin } i \tag{2.3}$$

and after that it will return to the ground. If the wave is incident vertically, with i = o, it will be returned from a, level where μ = o, and therefore where

$$N_e = \left(\frac{\varepsilon_o m_e}{4\pi e^2}\right)\omega^2 = 1.24x10^{-8} f^2, \tag{2.4}$$

where f is the wave frequency. This frequency is also the 'plasma resonance frequency' appropriate to a plasma of election density, N_e, so that a wave is reflected at vertical incidence, from that level in the ionosphere where the ordinary wave frequency is equal to the free electron plasma frequency, f_N. Equation (2.4) can be written as:

$$N_e(cm^{-3}) = 1.24x10^4 f_N{}^2 \text{ [MHz]}. \tag{2.5}$$

If there is a layer of electrons having a plasma frequency, f_o at the peak, the frequencies less than f_o will be influenced, but greater frequencies will not. The frequency, f_o, is called the critical frequently which has been much used in ionospheric research. If the layer is an equilibrium α Chapman layer, then the peak electron density, corresponding to critical frequency, f_o, is given by:

$$N_m \cong \left[\left(\frac{q_o}{\alpha}\right) Cos\chi\right]^{\frac{1}{2}}, \tag{2.6}$$

where q_o represents maximum electron production for overhead sun, χ is the zenith angle of the sun, α is recombination coefficient. If we combine Equations (2.5) and(2.6) we obtain the functional relation between critical frequency of E-region, $f_o E$, and the sun's zenith distance, χ, written as:

$$f_oE = 9x10^3\left[\left(\frac{q_o}{\alpha}\right)Cos\chi\right]^{\frac{1}{4}}. \qquad \textbf{(2.7)}$$

Equation (2.7) can be written in generalized form as:

$$f_oE \propto Cos^n\chi, \qquad \textbf{(2.8)}$$

where n is an index.

3.0 Empirical Formulae

The empirical expressions for calculating penetrating frequency of E-layer of ionosphere over Ibadan, geographic latitude, 7.4^{0}N, can be written as follows:

I. $\chi \le 73^o$:

$$(f_oE)^4 = (143.22)[Cos(7.4-\delta)]^{-0.0214}(Cos\chi)^{1.31} \qquad \textbf{(3.1)}$$

II. $73^o < \chi < 90^o$:

$$(f_oE)^4 = (143.22)[Cos(7.4-\delta)]^{-0.0214}\{Cos[\chi - 6.27\times10^{-13}(\chi-50)^8]\}^{1.31} \qquad \textbf{(3.2)}$$

III. $\chi \ge 90^o$:

$$(f_oE)^4 = (4.569)[Cos(7.4-\delta)]^{-0.0214}(e^{25.2-0.28\chi}) \qquad \textbf{(3.3)}$$

where $\delta = 2.22^o$,the solar declination of the sun for the month of September. We calculated the zenith distance,χ, by Chapman (1931) expression given by:

$$\cos\chi = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\theta, \qquad \textbf{(3.4)}$$

where ϕ is the latitude of the place in question,χ is the sun's declination angle, ωt is the hour of the sun given by he relation:

$$\omega t = (\text{Hour of the day } x\frac{360}{24} - 180)\ [\text{degree}]. \qquad \textbf{(3.5)}$$

4.0. Results and Discussion

The plot of the empirical relation given in Equations (3.1) – (3.5) for Ibadan during September 15 and the experimentally observed hourly mean and median values of critical frequency during September 1974 is shown in Figure 1. The main results are as follows:

1. There is no significant difference in both magnitude and trend in the two sets of indices. That is, mean, n = 0.29 and median, n = 0.33.
2. The diurnal behaviour of the curve obtained by empirical expression, n = 0.25 is in good agreement with Chapman theory. The height of the maximum follows the changes in the elevation of the sun during the daytime.
3. It is found that E-layer critical frequency is practically nonexistent during the nighttime by our empirical law. This indicates that no reflection of radio wave will occur during these periods.
4. There is comparable high degree of symmetry about middy between the measured and the calculated data. The symmetry implies a rapid recombination so that the ionisation follows the production function with little time lag.
5. There is a larger departure between the calculated and experimental curves during the period 0600-1200 LT than during 1200-1800 hours.

Nonetheless, in order to compare our data with previous study, Figure 2 gives the results of diurnal variation of E-layer penetration frequency at Cambridge, England, given by Robinson (1960). A close look at Figures 1 and 2 show a clear reasonable degree of symmetry about noon. This confirms the regular behaviour of the E-layer. The average peak critical frequency is 3.8 MHz for both Cambridge and Ibadan.

The maximum penetration frequency for our empirical expressions is 3.5 MHz. A difference of about 8%, from the two stations, which insignificant.

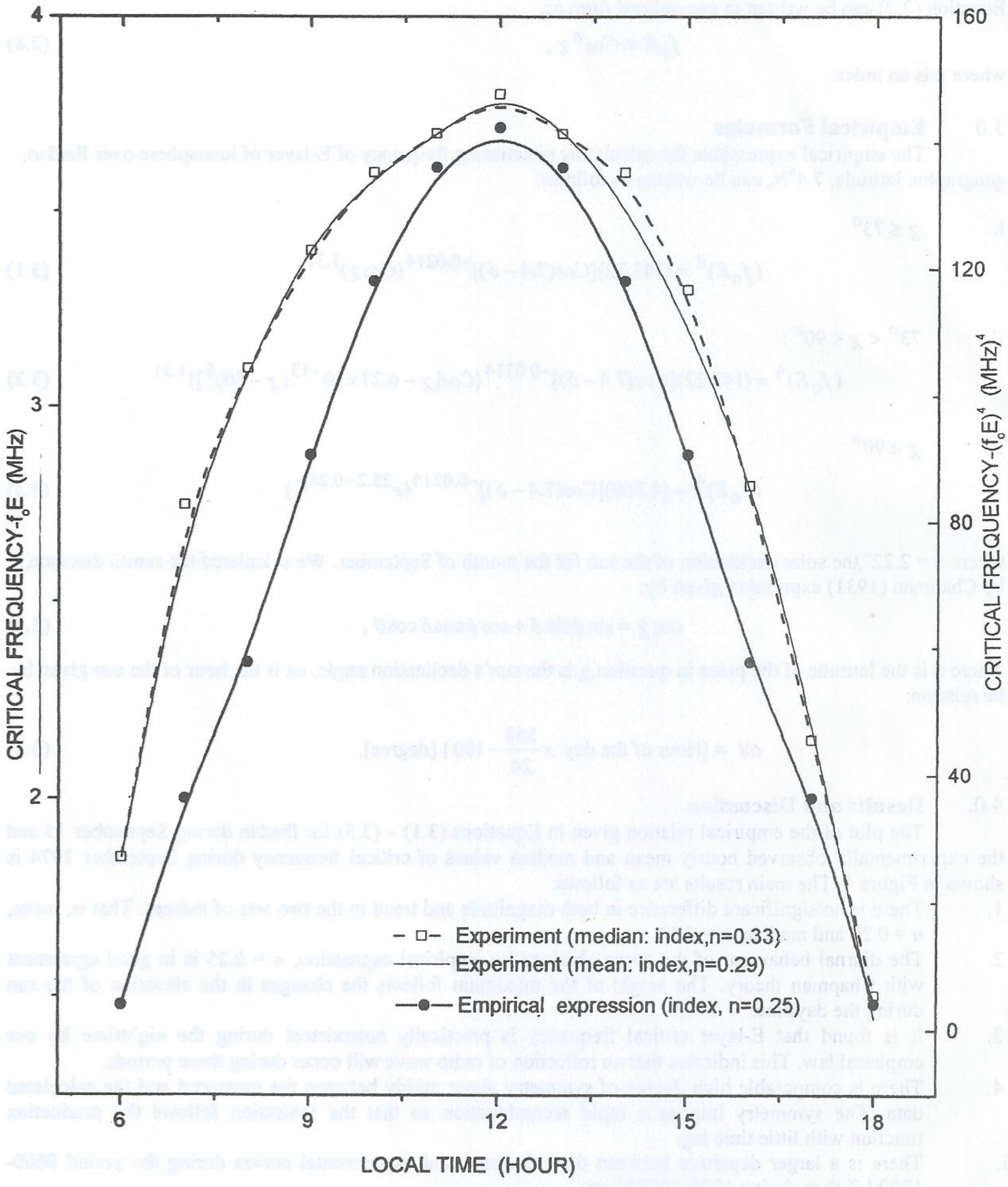

Figure 1 The comparison of E region critical frequency between observations during September 1974(right vertical axis) and the emperical formula(left vertical axis) for September 15 at Ibadan.

5.0 Conclusions

Critical frequency of E-layer calculated from the empirical expression is in agreement with Chapman theory. The formula can be used to predict the penetration frequency of E-region for any day of the year and for any station around the world. Also, there is no significant different in the indices representing the experimentally observed mean and the median values of critical frequency at equatorial E-region ionosphere

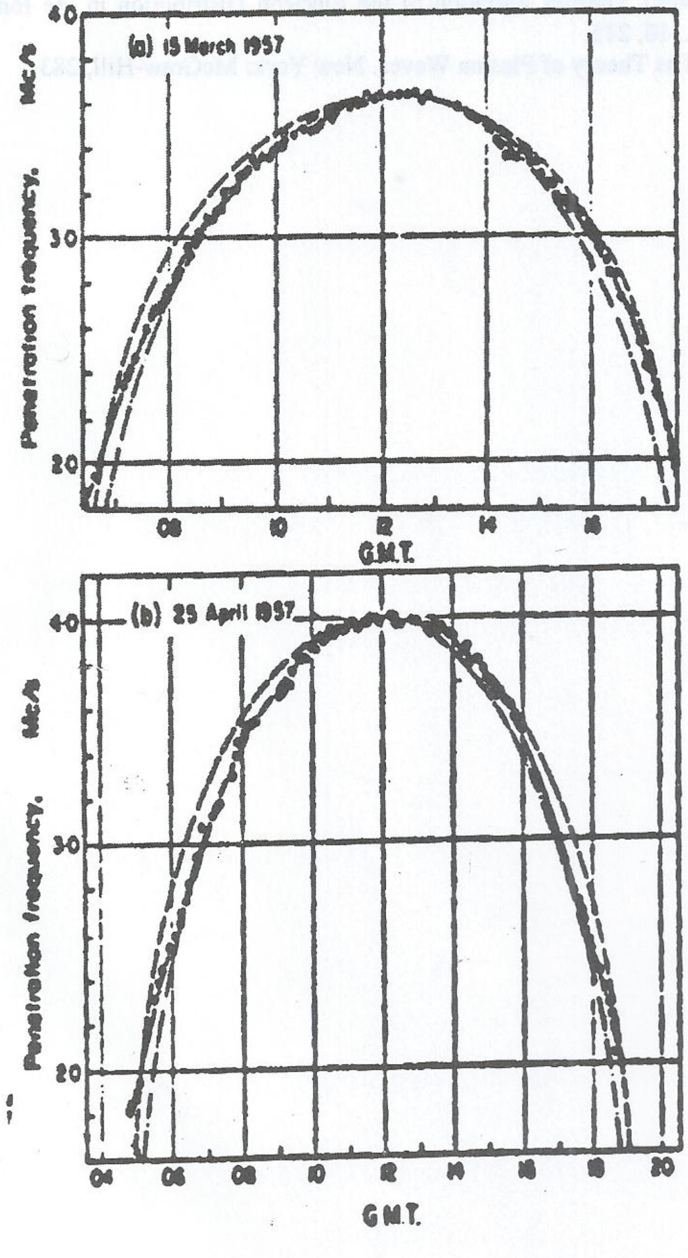

Figure 2: Diurnal variation of E-layer penetration frequency at Cambridge: dashed line denotes the equilibrium curve for Chapman layer, $f_o \propto Cos^{\frac{1}{4}}\chi$, dashed dot denotes empirical expression $f_o \propto Cos^{\frac{1}{3}}\chi$. After Robinson B.J. *J. Atmos. Terr. Phys.*, 18, 215-233, 1960.

References

[1] Chapman, S. (1931), The Absorption and Dissociative or Ionising Effect of Monochromatic Radiation in an Atmosphere on a Rotating earth, *Proc. Phys. Soc.*, 43, 26.

[2] Jackson, J.E. (1969), The Reduction of Topside Ionograms to Electron Density Profiles, Proc. IEEE, 57, 960.

[3] Oyekola, O.S. and Iheonu, E.E. (2002), Ionospheric Investigation Using Analog Bottomside Sounder Ionograms, J. Nig. Ass. Math. Phys., 6, 223.

[4] Ratcliffe, J.A. and Weeks, K. (1960), *The Upper Ionosphere in the Physics of the Upper Atmosphere,* New York: Academic Press, 397

[5] Robinson, B.J. (1960), Diurnal Variation of the Electron Distribution in the Ionospheric E-Layer, J. Atmos. Terr. Phys., 18, 215.

[6] Stix, T.H. (1962) The Theory of Plasma Waves. New York: McGraw-Hill, 283