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Heat Transfer Modelling of Roofing Elements of Buildings in the Tropics

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Abstract

Mathematical models representing the behaviour of inside temperatures due to the response of roofing element of a building structure to outside environment variables have been developed for two tropical locations. Comparison between the measured data and simulated results from the models showed that the variations of the inner surface temperatures of the roofing element were very similar in trend to the actual measured data but differed in magnitude significantly throughout the period under study. Appropriate models have been recommended for the studied locations. The application of the models may find relevance in other locations that share similar climatic conditions with that of the studied locations in the evaluation of the predictive ability of the models with regards to roofing elements of building structures in the control of thermal fluxes within the tropical environment.

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1.0 Introduction

The control of thermal fluxes entering into storage structures in tropical countries has been a major subject of scientific discourse over the years. In such climates, overheating of the inside temperature occurs frequently. The need for the design of storage structures including buildings to control thermal fluxes into such systems through the composite elements cannot therefore, be over-emphasized.

In the last two decades or so, several passive solar building architectures have been developed to reduce building temperature swings in the hot arid climatic zones. Such design strategies included the use of layered walls to minimize the actual interior temperature fluctuations in order to achieve some degree of thermal comfort in building envelopes. One of the design strategies was developed and applied by the Indians of the American Southwest [5], which yielded a natural air-conditioning effect [2].

Ordinarily, a paramount contribution to the control of thermal fluxes entering into buildings should be the proper orientation of the building façade and the sizing and placement of windows for natural ventilation. Natural ventilation is seen as a necessary means of guaranteeing the healthy condition of air inside buildings.

In practice however, the control of thermal fluxes entering into storage facilities are generally realized through the application of insulating materials. Insulating materials such as rockwool fibres, asbestos fibres, polystyrene foam and plastics have inherent ability to prevent the passage of radiant heat into structures by reflecting back most of the incident radiation or preventing heat transfer by impeding the flow of conducted heat. Materials possessing good insulation properties thus modify significantly the energy balance in any built environment.

In the tropical environment, natural ventilation and reduction of thermal load from solar radiation form the basis on which design strategies for comfort purposes are evolved. In order to effect a reduction in the high ceiling temperature resulting from the radiant heat gain by the roof, the roof system must be designed to absorb as little solar radiation as possible and / or otherwise be designed to offer high impedance to heat energy flow from outside to the inner surface of the ceiling.

In this work, an attempt is made to characterize the thermal performance of roofing elements of building structures, effectively assessed through mathematical modeling of the element.

The predicted inside air temperatures due to the response of the roofing materials to outside environmental conditions have been estimated based on heat transfer characteristics of parameters calculated using independent variables.

2.0 Heat Transfer Model For Roof Element

To evaluate the thermal conditions within any building in the tropical climatic zone, the process of heat transfer by conduction is always considered the most important process. The three basic forms of heat transfer by conduction are as follows:

- (i) Transient or unsteady state, in which the rate of heat flow and temperature vary with time. This condition may be partially applicable to some parts of Nigeria.
- (ii) Steady state condition, in which the temperature and heat flow do not vary with time. Such situations hardly exist in practice.
- (iii) Periodic heat condition, in which the variation of temperature and heat flow through the elements of the building structure is sinusoidal. This is practically what exists in most typical climatic zones, such as the Nigerian environment.

The main objective of this work is to derive mathematical models to describe the heat transfer mechanisms of typical roofing elements of buildings in the tropical environment. The first part of this study considers the solution of the problem using the basic heat equation as adopted in an earlier work [13]. The basic equation is written in the form:

$$\frac{\partial T}{\partial t} = Q_T - Q_{NV} - Q_{cd} \tag{2.1}$$

where $\frac{\partial T}{\partial t}$ is the rate of change of temperature, Q_T is the rate of heat production, which for consistency, will also be referred to as heat of thermal radiation, Q_{NV} is the cooling rate and Q_{cd} is the rate of heat flow due to convection-conduction.

For most practical purposes, $\frac{\partial T}{\partial t}$ is assumed negligible, since the variations of temperature in the tropical environment, and hence, $\frac{\partial T}{\partial t}$ are relatively very small under quasi-equilibrium conditions. Again, for a moment, we assume the cooling rate is due to the contributions of mechanical sources (active devices) so that the contributory effect of this term is also ignored.

Equation (2.1) may now be put as

$$Q_T = Q_{cd} \tag{2.2}$$

Equation (2.2) will henceforth be referred to as Model 1. Assuming passive solar architecture is adopted, so that the inside of the roofing element is in contact with natural ventilation, and then Q_{NV} may be re-introduced. Hence, equation (2.2) becomes:

$$Q_T = Q_{NV} + Q_{cd} \tag{3.3}$$

This equation will be referred to as Model 2. The analytical form of the component terms in the two models will be treated in due course.

In the next part of the derivation of models to solve the stated problem, the procedure will be carried out along the lines used by Desmarais et al. [6]. The methodology is based on the unsteady state heat balance equation expressed in the form:

$$Q_I - Q_T - Q_V - Q_i \pm (Q_{cd}) = m_a c_p \frac{dT}{dt} \tag{2.4}$$

where Q_I is the solar heat gain, Q_V is heat of ventilation, Q_i is heat of infiltration, c_p is the heat capacity or specific heat capacity of air, $m_a (= \rho V)$ is the mass of air, ρ is air density and V is the volume of the roofing element.

In a typical tropical climatic setting, ventilation is caused by natural wind, particularly where there is no means of mechanical cooling, so that equation (2.4) is put in a more convenient form as:

$$Q_I - Q_T - [Q_V + Q_i] \pm Q_{cd} = m_a c_p \frac{dT}{dt} \tag{2.5}$$

The convenience of equation (2.5) will become apparent in the course of this work.

Solar radiation is always considered a prominent parameter in the description of the energy balance of buildings. On the average, about 80% of temperature rises in buildings are due to solar radiation and the remaining to conduction exchanges [9]. The discomfort experienced in most humid tropical environments as a result of increase in internal temperature is often due to poor architectural designs and irrelevant choice of building materials. Rockwool fibres, polystyrene and polyurethane are examples of insulation materials which can be used in building constructions in humid tropical climates to repel some amounts of radiant heat energy falling on the material surface.

For such radiant energy incident on any material surface exposed to the environment, the relevant variables are related according to the well-known equation [14]:

$$\eta + \alpha + \tau = 1 \tag{2.6}$$

where η is the reflectivity, α is the absorptivity and τ is the transmissivity of the material, so that equation (2.6) becomes:

$$\tau = (1 - \alpha) \tag{2.7}$$

where the reflectivity term, η, is considered irrelevant in this analysis, because its contributory effect on heat transfer as long as the roofing element is concerned, is insignificant. The solar heat gain Q_1, may be represented by the equation:

$$Q_I = \alpha H A_r \tag{2.8}$$

where H is the average daily monthly solar radiation intensity for the location under consideration (Wm^{-2}), A_r is the effective area of the roofing material, measured in m^2.

The effective thermal heating Q_T due to solar radiation is also considered an input to the inside energy balance (equation 2.4) and may be represented by:

$$Q_T = (1 - \alpha) q_{iw} \tag{2.9}$$

where q_{iw} is the long wave radiation heat transfer coefficient and can be represented approximately by the expression

$$q_{iw} = \sigma A_r \left(\varepsilon_r (T_r + 273)^4 - \varepsilon_0 (T_0 + 273)^4 \right) \tag{2.10}$$

where σ is the Stefan-Boltzmann constant = $5.670 \times 10^{-8} W/m^2 K^4$, ε_r is the emissivity of the building material, T_r is the temperature at which the emissivity of the building material is obtained and T_0 is the outside temperature.

In conformity with Kirchoff's law, condition of thermal equilibrium is often assumed and used in many practical applications.
In this case,

$$\alpha = \varepsilon_r \tag{2.11}$$

To simplify the analysis of heat energy transfer in a built environment, the apparent emissivity of the atmosphere, ε_0'(=0.7) is used in this study.

The heat transfer by natural ventilation $Q_{NI'}$ may be used to replace the quantities $[Q_{I'} + Q_i]$ in equation (2.5). $Q_{NI'}$ may then be represented by

$$Q_{NI'} = N_{I'} \rho V c_p (T_0 - T_{in}) \tag{2.12}$$

where N_V is the natural ventilation rate or air change per minute, $\rho = 1.24 Kg/m^3$ and $c_p = 1.01 KJ/KgK$ for dry air temperature between 0 and 30°C [14]. The definition of N_V in equation (2.12) is based on the consideration of natural ventilation flow through an opening [1,6,12].

N_V is a function of wind speed by definition and the temperature difference, which in this instance, is the temperature difference between the outer and inner surfaces of the roofing element.

For the purpose of modeling, the flow of air may be assumed uniform and hence the natural ventilation rate becomes applicable. The maximum value of N_V is taken as 0.5 air change per minute [6].

The convection-conduction heat loss of a building element, Q_{cd}, may be found using the equation given as [5]:

$$Q_{cd} = UA_r(T_0 - T_{in}) \tag{2.13}$$

where U is the global heat transfer coefficient, which depends on various factors and characteristics, such as the geometry of the system, heat transfer resistance of the materials used in the construction and air conditions. The air condition referred to in this case, is the wind speed prevailing within the locality and has direct impact on the thickness of the boundary layer of the building element.

Thermal conductivity, k, and the building material thickness, x, are usually quoted by manufacturers as property values of the materials. If these parameters are introduced into (2.13), then we have:

$$Q_{cd} = \frac{kA_r}{x}(T_0 - T_{in}) \tag{2.14}$$

where $U = \frac{k}{x}$. From the foregoing, Model 1 may be recast as follows:

$$(1-\alpha)q_{iw} - \frac{kA_r}{x}(T_0 - T_{in}) = 0 \tag{2.15}$$

In the same vein, Model 2 may be rewritten as:

$$(1-\alpha)q_{iw} - \left[N_V \rho V c_p + \frac{kA_r}{x}\right](T_0 - T_{in}) = 0 \tag{2.16}$$

Finally, the introduction of equations (2.8), (2.9), (2.12) and (2.14) into equation (2.5) gives the lumped parameter heat transfer model for a single layer roofing (or roof) element and is expressed as:

$$\tau H A_r - (1-\alpha)q_{iw} - \left[N_V \rho V c_p + \frac{kA_r}{x}\right](T_0 - T_{in}) = m_a c_p \frac{dT}{dt} \tag{2.17}$$

where $m_a c_p \frac{dT}{dt}$ may be described as the heat energy stored in the material medium. Equation (2.17) can finally be put in the form:

$$\tau H A_r - (1-\alpha)q_{iw} - \left[N_V \rho V c_p + \frac{kA_r}{x}\right](T_0 - T_{in}) = m_a c_p \frac{T_0 - T_{in}}{\Delta t} \tag{2.18}$$

where Δt is the time interval for successive days. Using the above derived models (i.e., equations 2.15, 2.16 and 2.18), a purely mathematical solution can be found for all building materials used for roofing, provided the correct material property values are determined and applied.

The complete storage structure, such as a building, is always in constant contact with environmental variables (i.e., both the roof and the walls) on its either side.

The effective temperature at the outer surface of the roof is dependent on the combined effects of outside air temperature and solar radiation, put into a single quantity called the sol-air temperature, T_{SA} and may be determined using the approximate expression:

$$T_{SA} = \frac{\alpha H_G}{h_{so}} + T_0 \tag{2.19}$$

where h_{so} is the convective heat transfer coefficient (W/m^2K).

Heat transfer by convection is the process of energy transport by the combined action of heat conduction, energy storage and mixed ratio, which results in conventional currents that are set up at the surfaces of the material. For surfaces without windscreens, the coefficient of convective heat transfer, h_{conv}, is in the first order a linear function of the localized wind speed, v, and is given by the equation [17, 18]:

$$h_{conv} = 3.1 + 4.1 \cdot v \; (Wm^{-2}K^{-1}) \tag{2.20}$$

The variations of temperature and heat flow are normally periodic in nature and result in periodic temperature distribution and heat flow through the roof and walls of a building structure. The relevant equation to represent the variation may be expressed as:

$$T_j = T_{j0}(x) + \sum_{n=1}^{\infty} T_{jn} e^{in\omega t} \tag{2.21}$$

where $\omega = \frac{2\pi}{24} h^{-1}$ and n is an integer.

In general, appropriate boundary conditions at the material interface may be applied in order to solve the problem. This approach will be considered in a later work using hourly measured values of temperature and other relevant meteorological parameters.

3.0 Data Acquisition

The reference building envelope used in this work is the model house (Figure 1) built by the Nigerian Building and Road Research Institute in Kano (lat. 12.1°N, long. 8.5°E, and altitude 472 m) Nigeria which has a total roof area of 245.904 m^2 with an open courtyard [15]. The dotted line represents the perimeter wall. In the actual building, a gable roof was used. However, the flat roof option was considered in the analysis, because of its prevalence in the studied station. The gable roof option is to be considered in a more elaborate work later. In consideration of the parameter values to be adopted in this analysis, three climatic variables, namely, the mean maximum temperature, T_0, the solar radiation intensity, H_G, and the mean maximum wind speed, v, were calculated using data obtained from the Ibadan Central Station of the International Institute of Tropical Agriculture (IITA) Ibadan, Nigeria for two tropical locations (in Nigeria), namely, Ibadan and Kano. Mean values of the measured input parameters are, $T_0 = 39\,^0\text{C}$, $H_G = 23.3\ MJ/m^2$ and $v = 2.1 m/s$ for Kano and $T_0 = 35\,^0\text{C}$, $H_G = 15.5\ MJ/m^2$ and $v = 1.9 m/s$ for Ibadan in March 1994, a typical hot period.

Most roofing tiles / sheet products are manufactured using raw materials mix consisting of Portland cement, coconut fibre, etc., and undergoes several production processes. The processes are controlled to ensure production of roofing sheets to desired thickness, strength and durability.

The building material such as Rockwool fibre blanket finds application in diverse sectors of the economy, including the manufacturing industry, petrochemical industry and the building and construction industry. Property values of the product chosen for this study are assumed to have the following:

- Thermal conductivity, $k = 0.040 W/mK$ obtained at 32 ^{0}C
- Absorptivity, $\alpha = 0.90$
- Material thickness, $x = 0.075m$

These values are consistent with those quoted in other sources [3,14,19,21].

4.0 Results and Discussion

The difference between the calculated sol-air temperature from available data and the measured ambient temperature is considered insignificant in this analysis.

The predicted inner surface temperature values of the roofing element were obtained using equations (2.15), (2.16) and (2.18). The simulated results are compared with the measured data as shown in Figures 2 and 3 for Ibadan and Kano, respectively. The measured data are indicated by curve (a), while curves (b), (c) and (d) represent simulated results for Models 1, 2 and 3 respectively.

It is observed from the figures that the variations of simulated results for both Ibadan and Kano are very similar in trend to the actual measured values but differ significantly in magnitude on daily basis. These observations invariably imply that the hourly variations of simulated results will also follow the same pattern.

The characteristics of the measured data and simulated results are summarized in Table 1, according to the following format:

- Average difference in temperature between measured values and simulated results (D1).
- Percentage deviations in temperature between measured values and simulated results (D2).

For Ibadan, it is seen from the table that Model 3 outperformed the other two models in terms of the average difference in temperatures and in the percentage deviations between the measured and the simulated values. Model 2 follows as the next best performed model, while Model 1 brings up the rear. Models 1 and 2 predict inner surface temperatures far below the 27 ^{0}C limit proposed by Garg and Gupta [11] as the acceptable limit for thermal comfort, for the built environment. Model 3 may thus be recommended for use in Ibadan and in any other locations in the tropical climatic zone that have similar climatic conditions.

In the case of Kano, Model 2 appears to be the most suitable for the location. This is followed very closely by Model 1, while Model 3 brings up the rear. Model 2 may, therefore, be recommended for use in Kano and other locations that share the same climatic conditions within the tropical region. Due to its simplicity in application, Model 1 may also be appropriate since its performance is very close to that of Model 2. The use of Model 3 will predict temperature swings extremely far away from acceptable limits. In other words, Model 3 will grossly underestimate temperature swings in this climatic zone and should not be used.

The simulated results in Kano using Model 3 gives credence to the proposal in Section 2.0 that the transient or unsteady state heat transfer mechanism may be partially applicable to some locations in Nigeria. The result also tends to suggest that the choice of models for specific sites may be latitude-dependent, for periodic variations.

The differences between the measured and simulated values of temperature in this work are significant because the models have been able to achieve the necessary thermal comfort conditions (temperature swings in the range between 24^{o}C and 27^{o}C) that are vital to any effective passive system for natural comfort conditioning of building envelopes in the tropical environment.

The experimental determination of the performances of these models is to be carried out in an ongoing project which would culminate in the testing of the efficacy of these derived models, including modified versions for a complete building wall structure within the Nigerian environment. The diurnal variations of the temperature swings will form the main focus of the work. The building materials whose property values are of particular interest, are some brands of stucco and oven baked roofing sheets and corrugated sheets manufactured from fibre, cellulose and Portland cement.

In the meantime, it can be safely concluded that the simple lumped parameter heat transfer models presented in this article and recommended for specified locations, can be used to predict the behavioural trend of the inner surface temperatures of building and other insulating materials, with some degree of confidence.

5.0 Conclusion

Three lumped parameter models representing the behaviour of inside surface temperatures of roofing elements as a result of their response to outside environmental variables have been developed for two tropical locations, namely, Ibadan and Kano, both in Nigeria. The models were developed from the analysis of the component terms in the basic heat equation and the heat balance equation in terms of the transient or unsteady state heat transfer mechanism.

Comparison between the measured data and the simulated results showed that the variations in their behavioural trend are similar but differed in magnitude throughout the period of investigation.

Appropriate models have been proposed for the two studied locations. The best suitable model in terms of providing thermal comfort conditions as a result of the control of thermal fluxes entering the roofing element of a typical building structure had a mean temperature difference of approximately 10 ^{0}C between the measured data and the predicted inside temperatures, with variations of inside temperature swings between 25 and 28% for Ibadan. In the case of Kano, the best model had a mean temperature difference of 11 ^{0}C between the measured data and the predicted inside surface temperature with the inside surface temperature swings varying between 28 and 31%.

The proposed models may also find relevance in locations in the tropics that have the same climatic conditions like those of the studied stations.

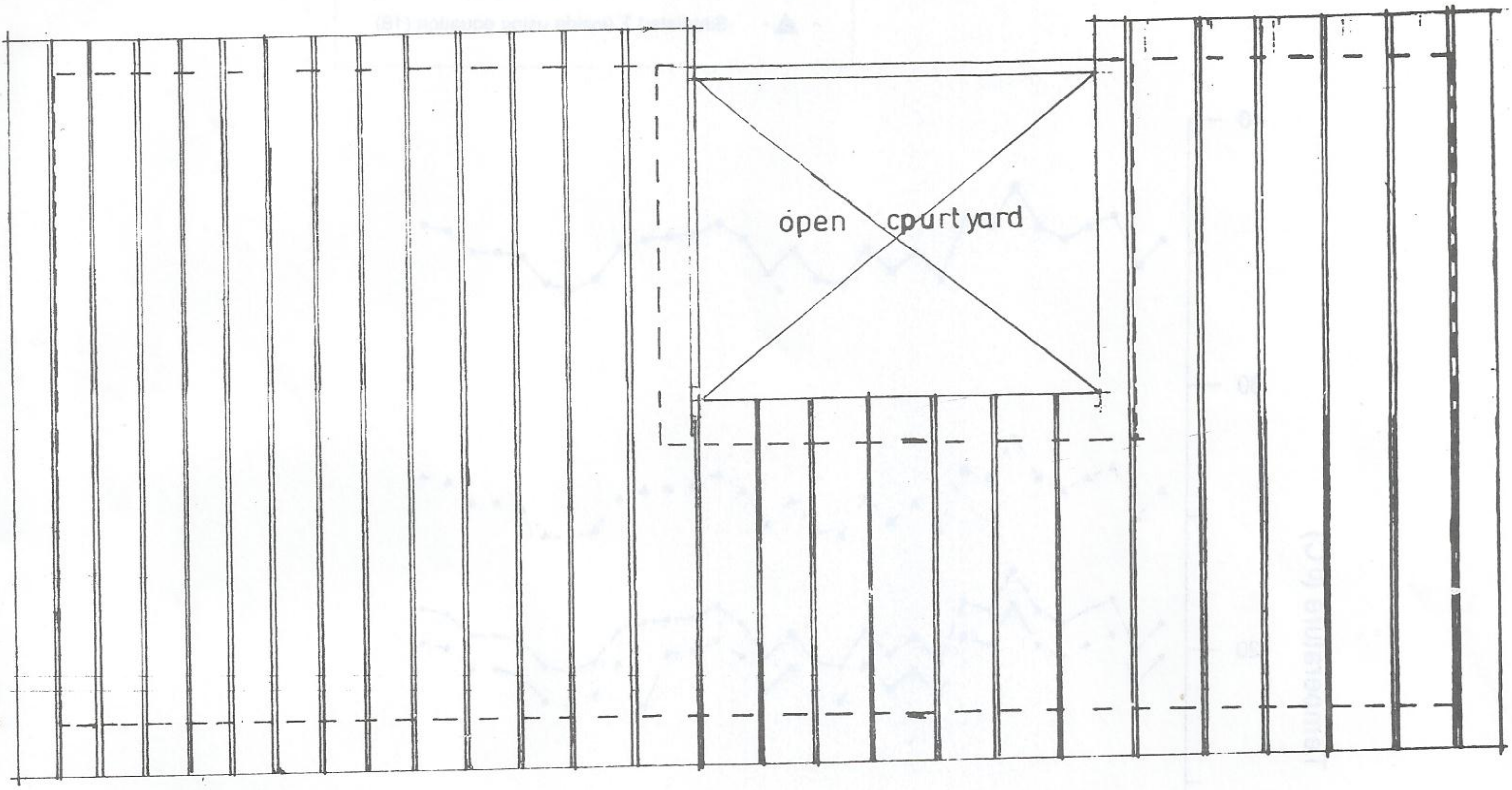

Figure 1: Roof plan of NBRRI model house in Kano [with a flat roof option]

Table 1: Summary of the results of Models' Performance

Location		Model 1	Model 2	Model 3
Ibadan	D1	15.8 ^{0}C	14.5 ^{0}C	9.5 ^{0}C
	D2	42-47%	39-43%	25-28%
Kano	D1	12.3 ^{0}C	11.3 ^{0}C	24.5 ^{0}C
	D2	30-34%	28-31%	60-67%

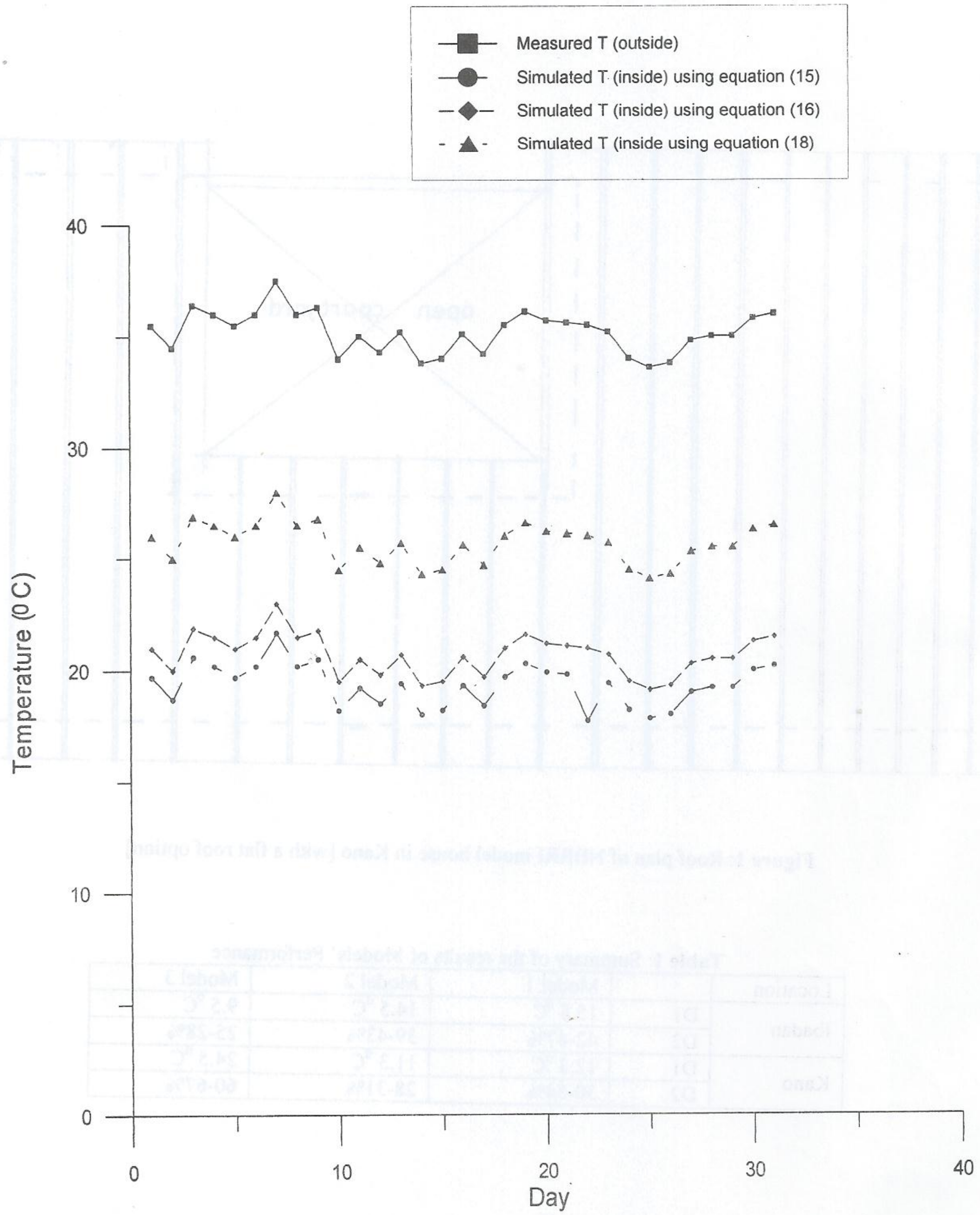

Figure 2: Inner surface temperature simulations compared to measured data for Ibadan

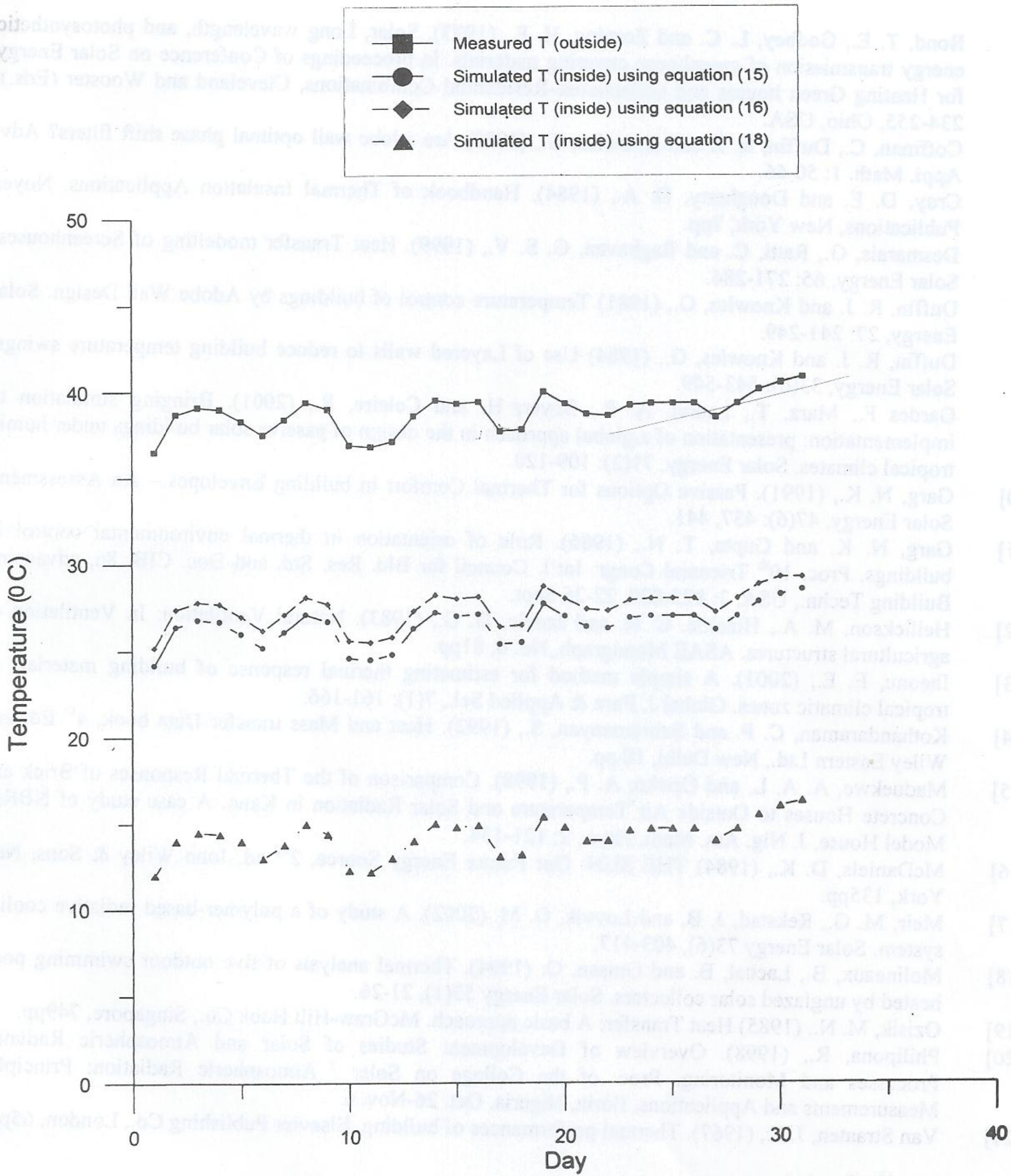

Figure 3: Inner surface temperature simulations compared to measured data for Kano

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