Journal of the Nigerian Association of Mathematical Physics
Volume 7(2003)

Analytic curves for plume model of dispersion of air pollutants

Awele Maduemezia, F.A.S.
P. O. Box 9598, University of Ibadan Post Office, Ibadan, Nigeria
Telephone: +234 - (0)803-301-8979
e-mail: awele@alum.mit.edu

Abstract

In this paper the results of a detailed curve-fitting computer code for the determination of the Pasquill-Gifford parameters used for the specification of air dispersion in the plume model, are presented. It is shown that the vertical and horizontal variance parameters may both be written as

$$\sigma = \exp\left\{\alpha + \beta(\ln x) + \gamma(\ln x)^2\right\} \textit{ (in metres)}$$

where x is the distance from the source in kilometres, and the variance parameter, σ *is either σy (= sy) (horizontal) or σz (= sz) (vertical) and the curve-fitting coefficients* α, β *and* γ *are given once and for all in a Table.*

pp 173 – 176

1.0 Introduction

Dispersion is the process by which contaminants move through the atmosphere and a plume spreads over a large area, thus reducing the concentration of the pollutants it contains. The plume spreads both horizontally and vertically.

The most commonly used model for the dispersion of gaseous air pollutants is the Gaussian, developed by Pasquill. In this model, gases dispersed in the atmosphere are assumed to exhibit the ideal gas behaviour. The horizontal and vertical dispersion are measured in terms of variance parameters σy and σz, respectively.

In the preparation of Environmental Impact Assessments, it is imperative that a reliable mathematical model be built to predict the impact of air pollutants on the environment of an incinerator stack, for instance around a gas flaring outfit in the Delta region of Nigeria. For this purpose, the values of these parameters have to be read off published curves, called Pasquill-Gifford curves. This makes analytic manipulation of these parameters during the modeling process clumsy, and sometimes impracticable, depending on the sophistication of the modeling process. It would be useful to have analytic formulas for these variance parameters. This is what this paper proceeds to provide.

2.0 Dispersion of air-borne pollutants

We use the plume model (Kiely, 1998, Peirce *et al*, 1998, Calvert *et al*, 1984). The ***plume model***, is a Gaussian distribution represented by the equation

$$C(x,y,z) = \frac{Q}{2\pi\sigma_y\sigma_z U}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\left\{\exp\left[-\frac{1}{2}\left(\frac{z-H}{\sigma_z}\right)^2\right] + \exp\left[-\frac{1}{2}\left(\frac{z+H}{\sigma_z}\right)^2\right]\right\}. \qquad (2.1)$$

where

C = concentration at the point (x,y,z) in g/m^3
Q = source emission rate in g/s
U = wind speed in m/s
σ_y = lateral spread of the plume (a function of downwind distance x) in m
σ_z = vertical spread of the plume (a function also of downwind distance x) in m
H = effective stack height in m
x,y,z = position co-ordinates of the receptor point in m

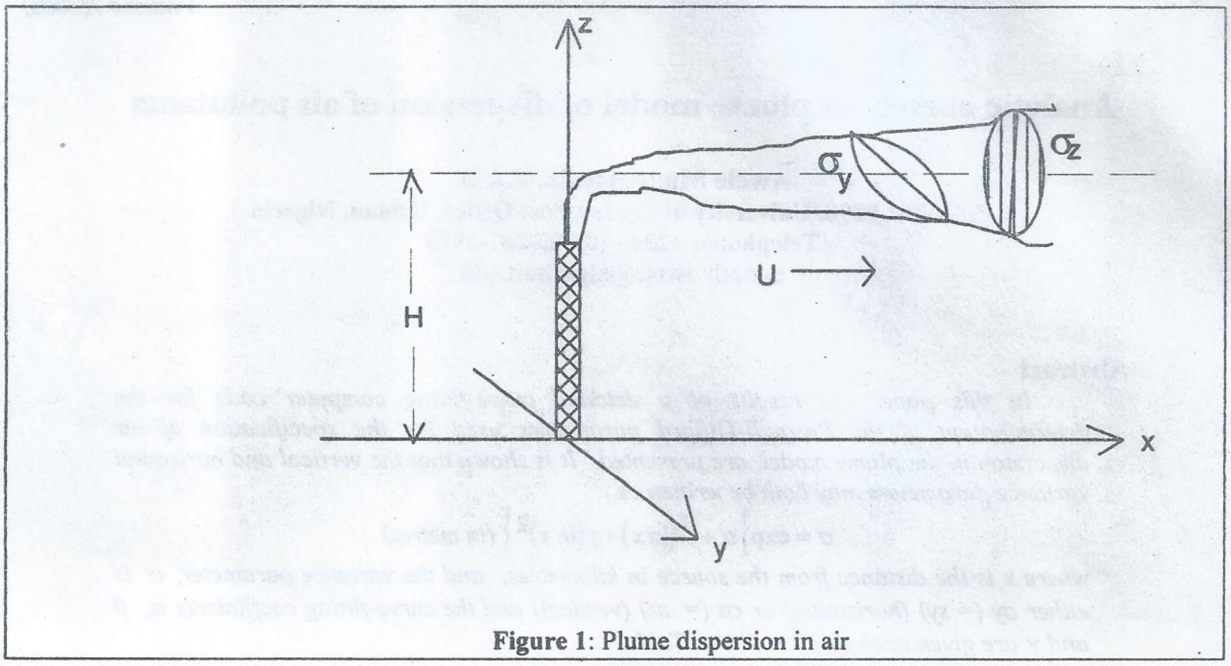

Figure 1: Plume dispersion in air

σ y (also written *sy,* for convenience) and σz (also written *sz* for convenience) are known as Pasquill-Gifford parameters and given graphically in Figures 2 and 3.

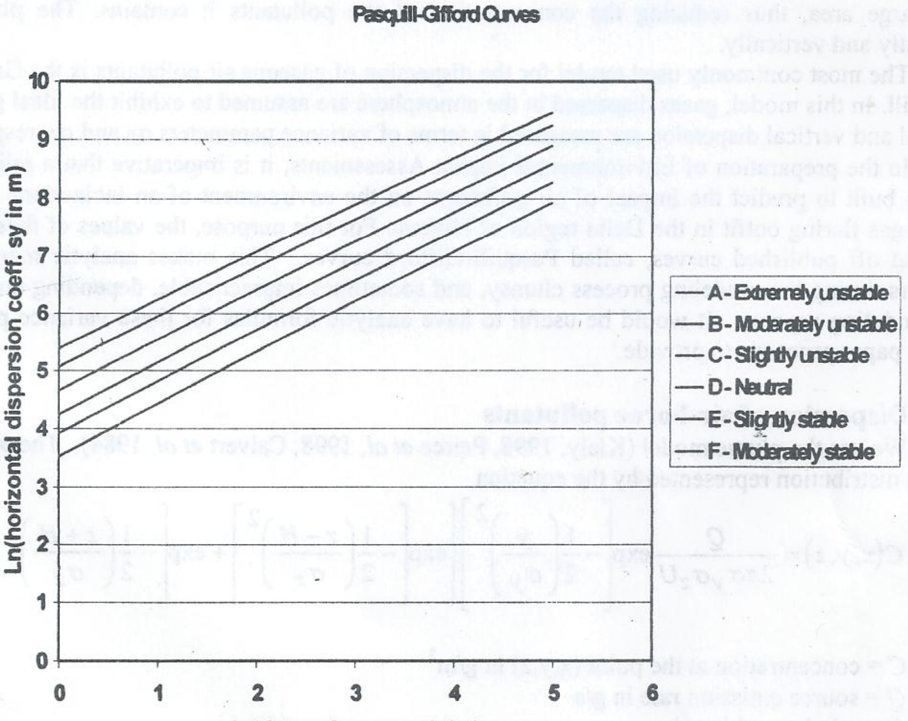

Figure 2: Correlations sy (= σy) based on the Pasquill stability classes A-F (Gifford, 1961)
(The curves are labeled A – F from top to bottom)

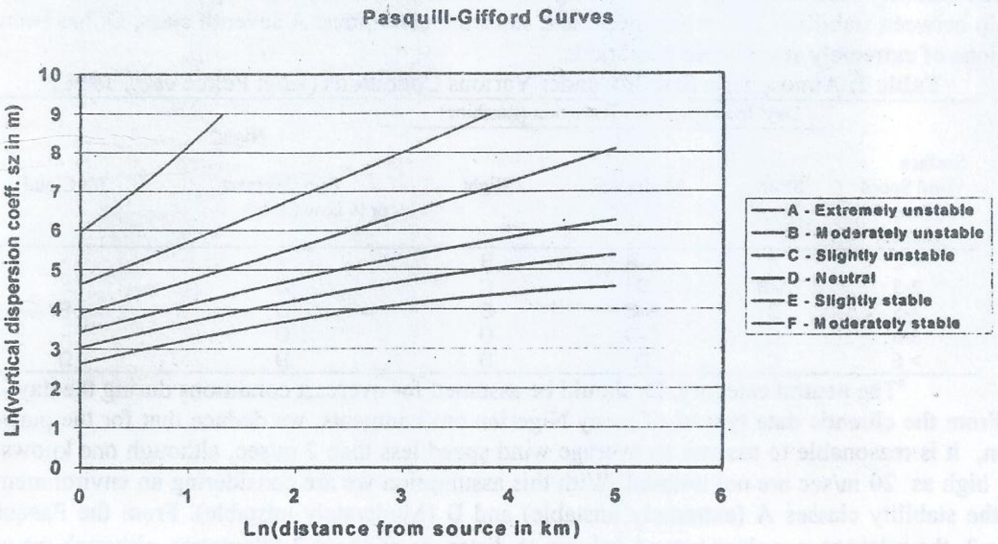

Figure 3: Correlations sz (= σz) based on the Pasquill stability classes A-F(Gifford, 1961)
(The curves are labeled A – F from top to bottom)

The environmental impacts of interest occur at the ground level. Thus we are interested only in ground level concentrations C(*x,y,z* = 0), given by

$$C(x,y,0)=\frac{Q}{\pi\sigma_y\sigma_z U}\exp\left[-\frac{1}{2}\left(\frac{y}{\sigma_y}\right)^2\right]\exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \tag{2.2}$$

Using the "worst-case" scenario, we note that the maximum ground level concentration occurs in the x-z plane, passing as it does through the plume centre-line, at *y* = 0. Thus this equation reduces further to

$$C(x,0,0)=\frac{Q}{\pi\sigma_y\sigma_z U}\exp\left[-\frac{1}{2}\left(\frac{H}{\sigma_z}\right)^2\right] \tag{2.3}$$

From computer codes developed at Enville Environmental Consultants, Ikeja, Lagos, it has been found that these curves can be fitted quite accurately with analytic functions, and can now both be written:

$$\sigma=\exp\left\{\alpha+\beta(\ln x)+\gamma(\ln x)^2\right\}\text{ (in metres)} \tag{2.4}$$

where *x* is the distance from the source in kilometres, σ is either σy (= sy) or σz (= sz) and the coefficients α, β and γ are given tin Table 1.

Table 1: Coefficientsα, β and γ for the analytic forms of the Pasquill-Gifford Curves

σ	Stability Classes	α	β	γ
σy	A	5.379192	0.8781496	-1.209921E-02
	B	5.06328	0.9007135	-1.249536E-02
	C	4.631164	0.8908263	-2.198692-03
	D	4.240321	0.9152816	-8.356369E-03
	E	3.921404	0.9077426	-6.759753E-03
	F	3.52402	0.9187907	-6.960382E-03
σz	A	5.952412	1.983428	0.2375019
	B	4.636487	1.051275	2.250874E-02
	C	3.899883	0.8192229	4.068976E-03
	D	3.42711	0.7274671	-3.232511E-02
	E	3.043375	0.6748435	-4.146981E-02
	F	2.631126	0.6596854	-5.446991E-02

Using these curves the Gaussian distribution C(*x*,0,0) can easily be evaluated for distances of up to 5 kilometres from the location of the incinerator stack and beyond.

The Pasquill-Gifford parameters σ_y and σ_z are just standard deviations, and as such are measures of the plume spread in the crosswind (lateral) and vertical directions. They depend on atmospheric stability and on

distance from the source. Atmospheric stability is classified in categories A through F, called stability classes.

The stability classes depend on incoming solar radiation and on wind speed. In Table 2 we show the relationship between stability class, wind speed, and sunshine conditions. A seventh class, G, has been proposed for conditions of extremely severe cold conditions.

Table 2: Atmospheric Stability under Various Conditions (*vide*: Peirce *et al.* 1998)

Surface Wind Speed at 10 m) (m/sec)	Day[a] Incoming Solar Radiation (sunshine)			Night[a]	
	Strong	Moderate	Slight	Thin Overcast or ½ Low Cloud	3/8 Cloud
< 2	A	A-B	B	-	-
2-3	A-B	B	C	E	F
3-5	B	B-C	C	D	E
5-6	C	C-D	D	D	-
> 6	C	D	D	D	D

[a] The neutral category, D, should be assumed for overcast conditions during the day.

From the climatic data typical of many Nigerian environments, we deduce that for the purposes of an illustration, it is reasonable to assume an average wind speed less than 2 m/sec, although one knows that wind speeds as high as 20 m/sec are not unusual. With this assumption we are considering an environment that falls between the stability classes A (extremely unstable) and B (Moderately unstable). From the Pasquill-Gifford curves Fig.3, the relevant σ_z-values extend only up to distances of about 3 kilometres, although we can use the analytical form to extend them much further.

For this calculation we assume an effective stack height of 50 metres, a slow wind speed of 2 m/s, a source emission rate Q of 5000 g/s. Fig. 4 gives the results for the two relevant stability classes A and B.

The modeling calculations show (Fig. 4, at the peaks of both curves) that the maximum concentration of emissions from the incinerator stack would be observed between 200m and 300 m from the stack. Assuming an initial emission concentration of 5000 g/s, which is typical of oil flare stacks, we find that the maximum observable concentrations of air pollutants would be 320mg/m^3. **This implies that monitoring stations should be established 200 to 300 metres from the incinerator stack along the prevailing wind direction, this being the position of greatest potential impact.**

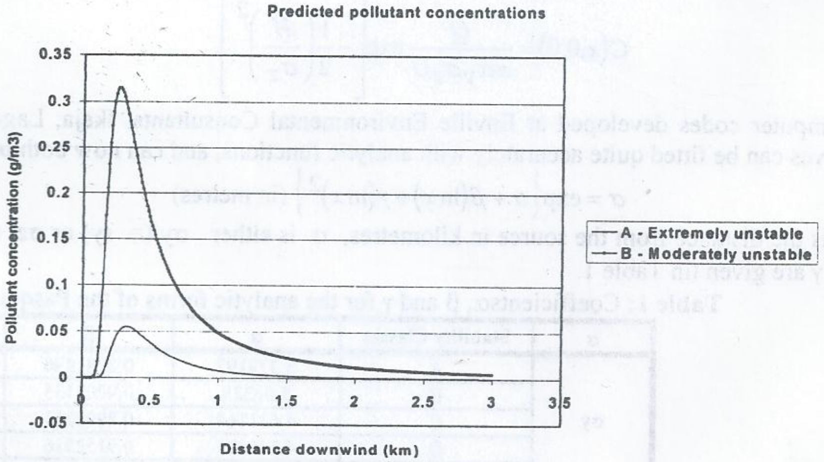

Figure 4: Predicted dispersion of air pollutants around the incinerator stack (Upper curve is B; lower curve, A)

References

[1] Calvert, S., & England, H. M. (Editors), (1984), *Handbook of Air Pollution Technology*, (John Wiley & Sons, N.Y.) (discusses Pasquill-Gifford Stability Classes according to solar radiation and surface wind speeds)
[2] Carpenter, R. A., (1995), *Risk Assessment*, in *International Association for Impact Assessment (IAIA) Journal* **13**, 153-187.
[3] Gifford, F. A. (1961), *Use of routine meteorological observations for estimating atmospheric dispersion*, Nuclear Safety, 2, 47-51
[4] Kiely, Gerard, (1998*), Environmental Engineering*, (Irwin McGraw-Hill, Boston)
[5] Peirce, J. Jeffrey; Weiner, Ruth F.; Vesilind, P. Aarne, (1998) Environmental *Pollution and Control,* (Butterworth-Heinemann, Boston)