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Projectile Motion in a Resistant Medium under the Influence of an Electric Field

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Abstract.

The motion of a particle which is projected into a resistant medium and subjected to a uniform gravitational field and electric field is considered. The drag force that acts upon the particle within the medium is proportional to the speed raised to power n. The problem is formulated in terms of particle – speed and local-path-angle variables, and the equations of motion that result into non-linear and coupled. Of particular interest is the effect of the magnetic field on the velocity.

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1.0 Introduction

In a recent paper Hayen [2] investigated projectile motion in a resistant medium. The particle is subjected to a uniform gravitational field. The drag force that acts upon the particle within the medium is proportional to the particle speed squared. Although the physical problem considered in [2] has been previously solved, the particular solution technique presented and certain solution properties uncovered are original.

In another paper Vajravelu and Rivera [3] investigated the hydro-magnetic flow at an oscillating plate and at a moving plate. They discussed the effect of the magnetic parameter on the flow.

In this paper we extend the model of Hayen [1] to include an electric field and general power n which is new and we study the resulting unsteady problem.

2.0 Mathematical Formulation

The set of coupled non-linear equations are

$$\frac{dv}{dt} = -g\sin\phi + \frac{\sigma}{\rho}B_0^2 v - \frac{k}{m}v^n, \quad v(0) = v_0$$
$$v\frac{d\phi}{dt} = -g\cos\phi, \phi(0) = \phi_0 \tag{2.1}$$

where v is he velocity, ϕ the local path angle, g the acceleration due to gravity, m the mass of the particle and k is a constant. Also, ρ is the fluid density, σ the electric conductivity, B_0 the imposed magnetic field, n a positive constant, x and t variables respectively.

3.0 Method of Solution

Consider the system of equation

$$\begin{aligned} x_1' &= f_1(x_1,\cdots,x_n t), x_1(t_0) = x_{10} \\ x_2' &= f_2(x_1,\cdots,x_n t), x_2(t_0) = x_{20} \\ &\vdots \\ x_n' &= f_n(x_1,\cdots,x_n t), x_n(t_0) = x_{n0} \end{aligned} \tag{3.1}$$

Theorem ([2] p. 484) let D denote the region

$$|t\text{-}t_0| \le a,\ \|\underline{x} - \underline{x}_0\| \le b,\ \underline{x} = (x_1,\cdots,x_n),\ \underline{x}_0 = (x_{10}\cdots,n_{n0}) \tag{3.2}$$

and suppose that $f(\underline{x},t)$ satisfies the Lipskhitz condition

$$\|f(\underline{x}_1,t) - f(\underline{x}_2,t)\| \le k\|\underline{x}_1 - \underline{x}_2\| \tag{3.3}$$

whenever the pairs $(\underline{x}_1,t)$, $(\underline{x}_2,t)$ belong to D, where k is a positive constant. Then there is a constant $\delta \succ 0$, such that there exist a unique vector solution $\underline{x}(t)$ of the system (3.1) in the interval $|t - t_0| \le \delta$. It is important to note ([2] p.485) that condition (3.3) is satisfied by requirement that $\frac{\partial f_i}{\partial x_i}\ i,j = 1,2,\cdots,n$ be continuous and bounded in D.

We now use the above theorem to prove the following theorem.

Theorem 3.1

Let $n \ge 1,\ 0 \prec v_0 \prec \infty,\ \frac{\pi}{2} \le \phi_0 \le \frac{\pi}{2}$. *Then* $\exists$ *a* $\delta \succ 0$ *such that there is a unique vector solution* (v,ϕ) *of the system* (1.1) *in the interval* $0 \le t \le \delta$

Proof

$$\frac{d}{dv}\left(-g\sin\phi + \frac{\sigma}{\rho}B_o^2 v - \frac{k}{m}v^n\right) = \frac{\sigma B_o^2}{\rho} - \frac{nk}{m}v^{n-1} \tag{3.4}$$

$$\frac{d}{d\phi}\left(-g\sin\phi + \frac{\sigma}{\rho}B_o^2 v - \frac{k}{m}v^n\right) = -g\cos\phi \tag{3.5}$$

also

$$\frac{d}{dv}\left(-\frac{g\sin\phi}{v}\right) = \frac{g\cos\phi}{v^2},\quad \frac{\partial}{\partial\phi}\left(\frac{-g\cos\phi}{v}\right) = \frac{g\sin\phi}{v} \tag{3.6}$$

Clearly (3.4) – (3.6) are continuous and bounded. Hence by the previous theorem, there exists a unique solution of the system.

3.1 Exact Solutions

Let $L = \frac{v_o^2}{g}$, $T = \frac{v_o}{g}$, v_o be units of length time, and velocity, then the non-dimensional equations are

$$\left.\begin{aligned} &\frac{dv}{dt} = -\sin\phi + \beta v - \alpha v^2, \quad v(0)=1 \\ &v\frac{d\phi}{dt} = -\cos\phi, \quad \phi(0) = \phi_o \\ &\text{where} \quad \alpha = \sqrt{\frac{kv_o^2}{mg}}, \quad \beta = \frac{\alpha B_o^2 v_o}{g\rho} \end{aligned}\right\} \tag{3.7}$$

Consider vertical motion: $\phi_o = \frac{\pi}{2}$.

In this special case, the particle is simply projected vertically upward. Let the time of ascent be t_a. Ascent phase $0 \prec t \prec t_a$. The initial and terminal conditions of this phase are $v(0)=1,\ x \equiv 0,\ v(t_a)=0$, and $\phi(0) = \frac{\pi}{2},\ y(0)=0,\ y(t_a)=h$, then

$$\frac{dv}{dt} = -\left(1 - \beta v + \alpha^2 v^2\right), \quad v\frac{dv}{dy} = -\left(1 - \beta v + \alpha^2 v^2\right) \tag{3.8}$$

Thus, if (i) $4\alpha^2 \succ \beta^2$, then

$$v = \frac{B}{2\alpha^2} + \sqrt{\frac{4\alpha^2 - \beta^2}{2\alpha^2}}\left\{\tan\left[\frac{\sqrt{4\alpha^2 - \beta^2}\left(t_a - t\right)}{2} + \tan^{-1}\left(\frac{-\beta}{\sqrt{4\alpha^2 - \beta^2}}\right)\right]\right\}$$

(ii) $4\alpha^2 = \beta^2$, then $v = \frac{1}{\alpha} - \left\{\alpha^2\left(t_a - t + \frac{1}{\alpha}\right)\right\}^{-1}$

(iii) $4\alpha^2 \prec \beta^2$ then

$$\frac{\alpha^2}{\sqrt{\beta^2 - 4\alpha^2}}\left\{\log_e\left|\frac{v - \frac{\beta}{2\alpha^2} - \frac{\sqrt{\beta^2 - 4\alpha^2}}{2\alpha^2}}{v - \frac{\beta}{2\alpha^2} + \frac{\sqrt{\beta^2 - 4\alpha^2}}{2\alpha^2}}\right| - \log_e\left|\frac{\frac{\beta}{2\alpha^2} + \frac{\sqrt{\beta^2 - 4\alpha^2}}{2\alpha^2}}{\frac{\sqrt{\beta^2 - 4\alpha^2}}{2\alpha^2} - \frac{\beta}{2\alpha^2}}\right|\right\} = \alpha^2\left(t_a - t\right)$$

4.0 Conclusion of the Results

When $\beta = 0,\ t_a = \frac{1}{\alpha}\tan^{-1}(\alpha)$ as obtained by Hayen [1]. When $\alpha = 1,\ t_a = 2$, where as when $\beta = 0,\ \alpha = 1,\ t_a = 1$. When $\beta^2 = 5\alpha^2,\ \alpha = 1,\ t_a = 0\cdot 98$. Also, when $\alpha = 1,\ \beta = 2\alpha = 2$, the particle cannot take off. So flight time increases only when $4\alpha^2 \succ \beta^2$ In other cases the electric field is a hindrance to the flight.

References

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