Resistivity Inversion – A Computer Iteration Technique for the Interpretation of Vertical Electrical Sounding

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Abstract

Resistivity inversion is a means of automatic interpretation of soundings in terms of the resistivity distribution in the earth. There exists problem of interpretation in geophysics of resistivity method which makes it difficult due to the mathematical complexities implicit in the derivatives of suitable theoretical models. The empirical procedures have not help matters in the test of field data. As a result of these mathematical complexities, the resistivity method has experienced some empirical procedures which have given rise to simple and quick solutions. This work attempts to fix the structure of the earth approximately with the simple mathematical model in the interpretation process. This procedure was applied in the Vertical Electrical Sounding at Ogwashi-Uku, Delta State using about seven iterations. The result shows that the iteration technique is very economical in that it saves cost, energy and time with a very low root mean percentage (RMS%) error between the theoretical and field curves. The high accuracy of the iteration method could be seen from the Driller's log which confirms very perfect agreement with result of the geophysical survey.

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1.0 Introduction

Resistivity inversion is a means of automatic interpretation of soundings in terms of the resistivity distribution in the earth. The earth model presently is restricted to horizontal layers which are homogeneous and electrically isotropic. The resistivity inversion method operates efficiently on mini-computers.

Inversion methods are classified according to their domains, which may be either the apparent resistivity or the resistivity transform domain. Apart from a change of notation, the theoretical basis of inversion is identical for the two domains.

A programme for inversion of Schlumberger soundings in the resistivity, transform domain has been written and analysed for the effects of interpretation, extrapolation, sampling interval, initial models, the number of layers, the number of data and data accuracy. The total storage requirement of the programme is formulated in terms of the number of field data and the number of layers represented by the data. The absence of computers before made numerical calculations very difficult.

The problems of interpretation in geophysics of resistivity method is very difficult despite the operational simplicity of the resistivity method (Frohlich, 1973). This is as a result of the mathematical complexities implicit in the derivatives of suitable theoretical models. As a result of these mathematical complexities, the resistivity method has experienced some empirical procedures which have given rise to simple and quick solutions, Barnes (1952), Narayan and Ramanujachary (1967). The empirical procedures have not help matters in the test of field data.

Vozoff (1960) was first to apply an iterative inversion technique to an earth model of horizontal layers. He applied Slichter's Kernel (Shichter, 1933) and stressed that it could be obtained either from field data by numerical integration of measured potentials or from an assumed model by recursion on the layer parameters. The initial model was varied iteratively to minimise

$$\sum_{i=1}^{m}\left\{\mathbf{K}^{*}(\lambda_i)-\mathbf{K}(\lambda_i)\right\}^2$$

where K^* and K were field and model Kernels respectively.

Meinardu (1970) extended Vozoffs work on the Slichter Kernel and developed an efficient method of automatic inversion for resistivity soundings on a horizontally-layered earth. The method of conversion of field apparent resistivity data to equivalent Kernel data by means of Gaussian integration was highly discussed and the principles of Gauss-Newton and steepest descent methods of iteration was highly elaborated. Based on the limitations of the two methods, he presented Marquardt's algorithm (Marquardt, 1963) as an alternative which guarantees convergence (of steepest descent) and achieves convergence quickly (of Gauss-Newton).

Zohdy (1973b), adopted a method of automatic interpretation for the Schlumberger array by n-point sounding curves by n-layer Dar Zarrouk curves or modified Dar Zarrouk curves (when the slope of the sounding curve is less than 1). The layer parameters of the Dar Zarrouk curve were regarded as a first-approximation solution and were used to generate a theoretical resitivity curve for comparison with field data. The anomalies between the two sets of apparent resistivities were used to modify the model until a close match was found. The number of layers in the detailed model may be reduced to a more realistic value by smoothing the final Dar Zarrouk curve.

Kunetz and Rocroi (1969, 1970) adopted a method for automatic interpretation based on an alternative formulation of the apparent resistivity function for horizontal layers given as

$$\ell_a(r) = \frac{2\ell_1 r^2}{\pi} \int_0^{\infty} \lambda \phi_n(\lambda) K_1(\lambda r) d\lambda \tag{1.1}$$

where $K_1(\lambda_r)$ is a modified Bessel function of the second kind

$$\phi_n(\lambda) = \frac{\ell_n}{\ell_1} \left\{ e_n^2(\lambda) + H_n^2(\lambda) \right\}^{-1} = 1 + 2\sum_{N=1}^{\infty} QN \cos(2N\lambda) \tag{1.2}$$

2.0 Theoretical Analysis

2.1 Resistivity Transform Domain

The resistivity domain permits the same programme core to be used for the inversion of data from any electrode array because it deals with only one formulation of the transform function and its derivative is required. The actual electrode array becomes relevant in the input and output links with the programme, core through appropriate forward and inverse linear filters. The flow chart in Fig. 1 shows a propose procedure for the automatic interpretation of data obtained by any electrode array.

3.0 Inversion Theory

The theory of inversion is the same for the apparent resistivity or the resistivity transform domain. The theoretical notation used below is based on inversion in the transform domain. Suppose there exist a set of m apparent resistivity observations such that $\ell_a^*(L) = \left\{\ell_a^*(L_i)\right\},\ i = 1,2,\cdots,m$ (3.1)

which due to an n-layer earth characterised by thickness $\underline{h}^* = \left(h_j^*\right)\ \{j = 1,2,\cdots,(n-1)\}$ (3.2)

and resistivities $\underline{\ell}^* = \ell_k^*,\ k = 1,2,\cdots,n$ (3.3)

If we designate the true layer parameters by $\underline{P}^* = \left(P_j^*\right),\ j = 1,2,\cdots,n-1,n,n+1,\cdots 2n-1$ (3.4)

where the first n-1 elements are layer-thicknesses and the last n elements are layer-resistivities. After conversion to the resistivity transform domain we have, in effect, m observations $\underline{T}^* = T_i,\ i = 1,2,\cdots,m$ (3.5)

over an *n*-layer earth defined by $\underline{P}^* = \left(P_j^*\right),\ (j = 1,2,\cdots,n-1)$. A trial earth model $\underline{P} = (P_j)$ will give theoretical transform data $T = (T_I)$. The aim of the inversion procedure is to find the model vector $\underline{P}$ which minimises the sum-of-squares. $\phi = \sum_{i=1}^{m} \left(T_i^* - T_i\right)^2$ if $\underline{\Delta P} = \underline{P}^* - \underline{P}$ and $\underline{\Delta T} = \underline{T}^* - \underline{T}$, (3.6)

the problem may be linearised by defining a matrix $\underline{A}$ which takes the parameter correction vector and maps it into the vector of differences between observed and calculated transform data:

$$\underline{A}\ \underline{\Delta P} = \underline{\Delta T} \tag{3.7}$$

This is equivalent to linearising the transform function by a Taylor series expansion about a trial model:

$$T_i(\underline{P}+\Delta P)=T_i(\underline{P})+\sum_{j=1}^{2n-1}\left[\frac{\delta T_1}{\delta P_j}\right]\Delta P_j \tag{3.8}$$

provided that we defined the element of matrix $\underline{A} = a_{ij}$ to be the transform derivatives with respect to each layer-parameters: $a_{ij} = \frac{\delta T_i}{\delta T_j}$

Because the problem is non-linear, there is no exact matrix inverse $\underline{A}^{-1}$ which gives $\Delta\mathbf{P}$ when $\Delta\mathbf{T}$ is known. A generalised matrix inverse $\underline{A}^{+}$ may be found such that, in a least-squares sense, $\underline{\Delta\mathbf{P}} = \underline{A}^{+}\Delta\mathbf{T}$. The generealised inverse may be found rigorously by an eigenvalue decomposition of matrix $\underline{\underline{A}}$ or it may be approximated by the Marquardt method. Marquardt (1963) showed that the sum-of-squares (ϕ) could be minimised by $\underline{\Delta\mathbf{P}}$ where $\underline{\Delta\mathbf{P}}$ also satisfied the equation.

$$\left(\underline{A}^{T}\underline{A}+\varepsilon\underline{I}\right)\underline{\Delta}\mathrm{P} = \underline{A}^{T}\Delta\mathrm{T} \tag{3.9}$$

where $\varepsilon > 0$ and $\underline{A}^{T}$ is the transpose of matrix $\underline{\underline{A}}$. The generalised inverse may be expressed as:

$$\underline{\underline{A}}^{+} = \varepsilon \xrightarrow{\lim} 0\left(\underline{A}^{T}\underline{A}+\varepsilon\underline{\underline{I}}\right)^{-1}\underline{A}^{T}$$

(3.10)

This should be approximated by using small values of ε. On the other hand, Marquardt (1963) proved that:

$$\sum_{j=1}^{2n-1}\left(\mathrm{P}_j^{*}-\mathrm{P}_j\right)^2 \to 0, \text{ as } \varepsilon \to \infty \tag{3.11}$$

which indicates that there will always be a larger value of ε which gives a better model representation of the earth. To hasten convergence from practical point of view, values of ε should be decreased quickly such that $\varepsilon \to \frac{\varepsilon}{u},\ (u \succ\succ 1)$ and should be increased slowly, when necessary, to reduce the sum-of-squares: $\varepsilon \to V\varepsilon\ (V \succ 1)$.

Golub (1965) developed an algorithm for solving linear least-squares problems by use of the genearlised inverse and a more efficient version is described by Jennings and Osborne (1970). The algorithm relies on the factorisation of the matrix $\underline{\underline{A}}$ into an orthogonal matric $\underline{O}^{T}$ and another matrix $\underline{\underline{\mathbf{R}}}$ which has all zero elements below the leading diagonal: $\underline{\underline{A}} = \underline{O}^{T}\underline{\mathbf{R}}$. An ε-appendage is applied to the factor matrix $\underline{\underline{\mathbf{R}}}$ rather than the original matrix in order to avoid refactorising $\underline{\underline{A}}$ for different trial values of ε. The matrix [$\underline{\mathbf{R}}$/ε I] undergoes orthogonal factorisation and the parameter correction vector $\underline{\underline{\Delta\mathbf{P}}}$ is found by back-substitution. The new model estimate is found from the previous estimate by:

$$\mathrm{P}^{(r+1)} = \mathrm{P}^{(r)} + \Delta\mathrm{P}^{(r)} \tag{3.12}$$

where r is an iteration index. Each step of the above algorithm is shown in Table 1.

The apparent resistivity and resistivity transform data are plotted on bi-logarithimic scale so that more appropriate observed functions would be $\log \ell^{*}_{a}$ and $\log \underline{T}^{*}$, otherwise, matching is biased toward resistivity layers. Hence minimizing

$$\phi = \sum_{i=1}^{m}\left(\log \mathrm{T}_j^{*} - \log \mathrm{T}_j\right)^2 \tag{3.13}$$

the elements of matrix $\underline{\underline{A}}$ becomes $a_{ij}^{1} = \dfrac{\delta(\log T_i)}{\delta P_j} = \dfrac{1}{T_i}\dfrac{\delta T_i}{\delta P_j} = \dfrac{a_{ij}}{T_i}$

It follows that inversion theory for matching in a logarithmic sense is identical to the linear case in its formulation provided that each row of matric $\underline{\underline{A}}$ is normalised to its corresponding transform value.

The flow chart in Figure 2 summarises the logic of automatic interpretation of resistivity soundings in the transform domain. Having obtained the sum-of-squares $\phi^{(0)}$ for an initial model estimate, a new model is generated for some initial value $\varepsilon^{(0)}$. If the sum-of-square is greater than the original value, then $\varepsilon^{(0)}$ is increased until it leads to a new model for which $\phi / \varepsilon^{(0)} / < \phi^{(0)}$. If the measure-of-fit is below some preassigned cut off, then the current model is accepted as the final solution. If not, the current model enters the first iteration with a sum-of-squares $\phi^{(1)}$ **and a value** $\varepsilon^{(1)} \leq \varepsilon^{(0)}$. New models are generated for $\varepsilon^{(1)}$ and increased values of $\varepsilon^{(1)}$ if necessary, until $\phi\left[\varepsilon^{(1)}\right] < \phi^{(1)}$. The current model either enters the second iteration or exists if the fit is satisfactory.

The measure of goodness-of-fit is termed the "root-mean-square percentage error" and is defined by

$$\%\mathbf{RMS} = 100\left\{\frac{1}{m}\sum_{i=1}^{m}\left[\frac{T_i^{*}-T_i}{T_i^{*}}\right]^2\right\}^{1/2} \tag{3.14}$$

4.0 Methodology

The experimental work was carried out at Ogwashi-Uku in Aniocha Local Government Area of Delta State. The town is about 15km to Asaba, the Headquarters of Delta State. It is situated around latitude 6^0 12^1N and longitude 6^0 32^1E. The Schlumbeager electrode configuration was adopted for the data acquisition, Egbai and Asokhai, (1998). The field procedure is to expand the current electrodes successively while the potential electrodes remain fixed. The process yields a rapidly decreasing potential difference across the potential electrodes which ultimately exceed the measuring capabilities of the instrument. At this point a new value for potential electrode separation is selected, typically 2 to 4 times larger than the preceding value and survey is continued. The distance between the potential electrodes must never exceed 2/5 of AB/2 where AB is the distance between current electrodes; that is CD $\leq$ AB/2. The Terrameter SAS 300AB model was used in the Vertical Electrical Sounding (VES).

5.0 Results and Discussion

The computer assisted interpretation used for this project is based on the algorithm which employs digital linear filters for the fast computation of the resistivity function for a given set of layer parameters (Egbai, 1997). The computer program used is shown in the Appendix.

The model parameters were used for iterative operation with the computer to interpret the data. The computer programme was based on the equations in the theory. The following stages are necessary for the iteration.

The input stage is to convert the field data to equivalent transform data. This is followed by chaining to the inversion stage. The coefficient of the forward, filter is then used to generate field transform data.

The next stage is the inversion which is to invert the transform data to give a solution model and then to chain to the final stage for output.

The output stage consists of the interpreted model following inversion and a digital comparison between field and model apparent resistivities. Theoretical transform data are computed for the interpreted model and are convolved with O'Neill's inverse filter coefficients to give theoretical apparent resistivity data.

The VES curves in Fig. 7-9 show the result of the theoretical curves and the field curves. The resistivity values were 590.00, 610.00 and 476.00 ohms-metres at the surface for the three stations investigated at Ogwashu-Uku. The maximum value rises to about 6800 ohm-m for current electrode separation of about 800m. The result shows a five-layer curve for the three stations. Table 2 shows summary of results. The result from the Driller's log Figure 4 to Figure 6 was compared with that of VES survey and a high positive correlation was established. About seven iterations were used to obtain the result and a total of 51 vertical soundings were obtained from the three locations all having KH curve i.e $\ell_1 < \ell_2 > \ell_3 < \ell_4 > \ell_5$.

6.0 Conclusion

Resistivity inversion is a means of automatic interpretation of soundings in terms of the resistivity distribution in the earth. The method operates efficiently on mini-computers. The iteration technique used in the interpretation of Vertical Electrical Sounding is very economical in that it saves cost, energy and time with a very low root mean percentage (RMS%) error between theoretical and field curves. The very high accuracy of the iteration method could be seen from the Driller's log which confirms very high correlation with the result of surface Vertical Electrical Sounding.

Appendix

Figure 1: Flow Chart for automatic inversion for any electrode array

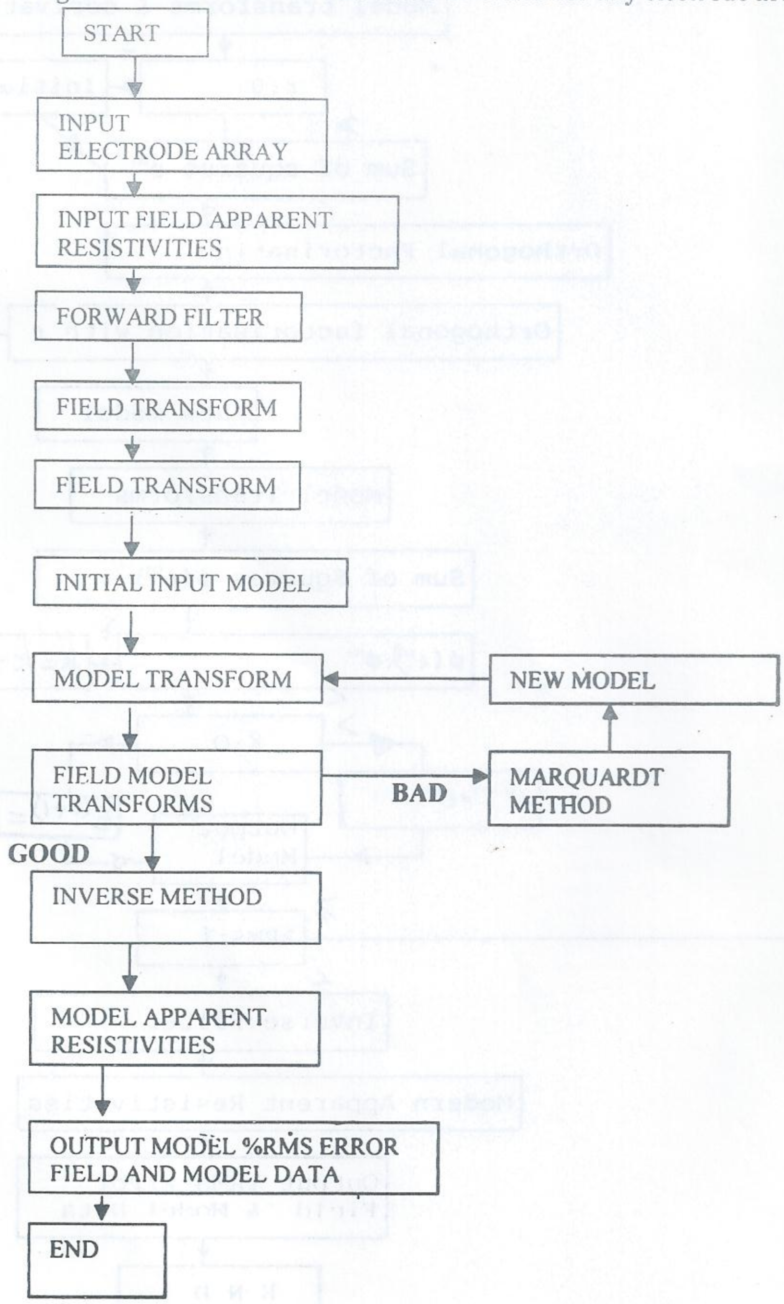


Figure 2: Flow Chart for Automatic Inversion by Marqquardt Method

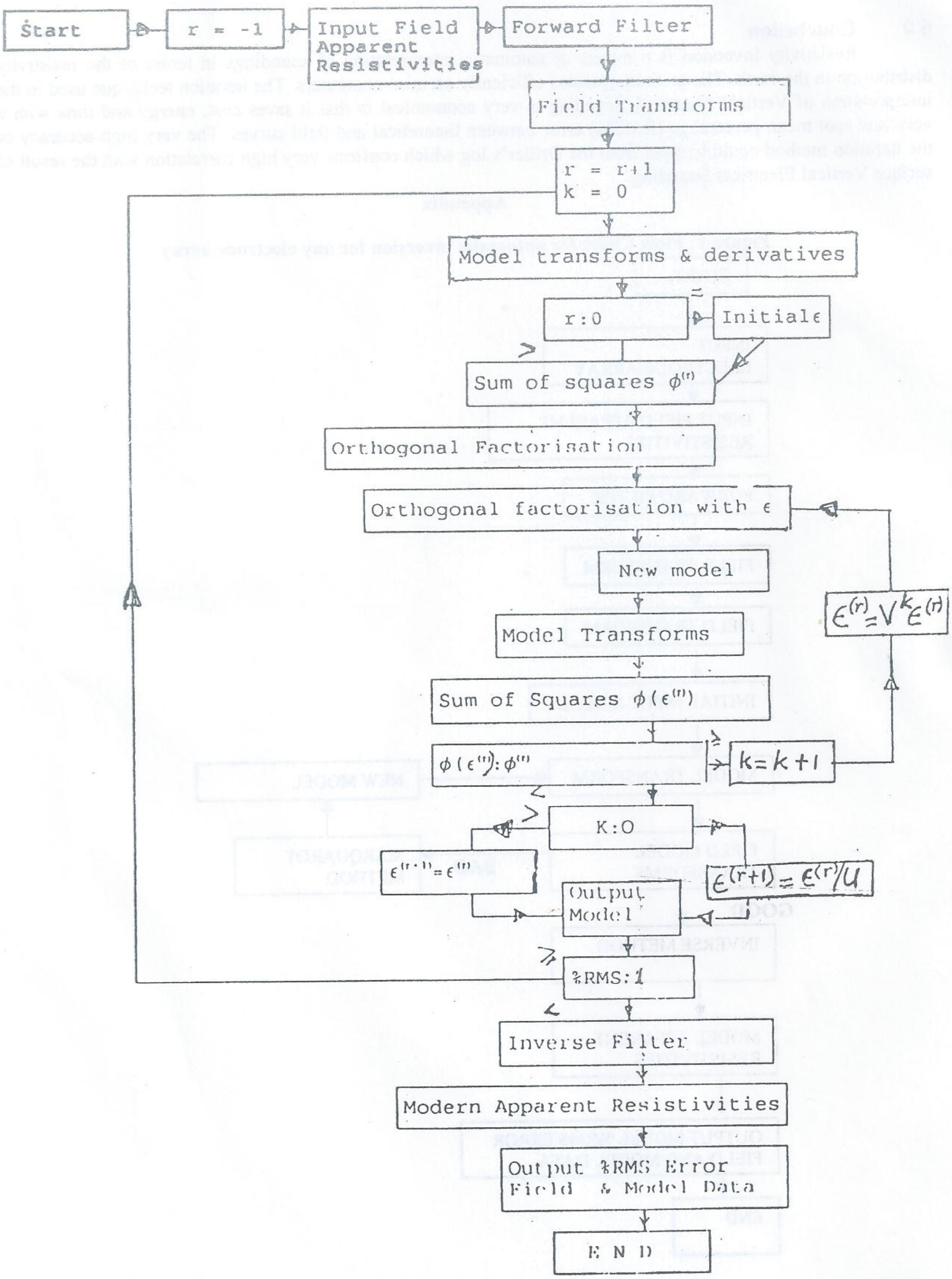


Table 1: Algorithm for an approximate generalized inverse of Matrix A by Orthogonal factorization

Orthogonal factorisation

Let $K = 2n-1$, $\underline{A} = A^{m \times k}$, $\Delta P = P^{k \times 1}$, $\Delta T = T^{m \times 1}$

For m data, k unknowns:

$$A^{m \times k} \quad \Delta P^{k \times 1} = \Delta T^{m \times 1}$$

Pre-multiply by orthogonal matrix, Q_1:

$$Q_1^{m \times m} \quad A^{m \times k} \quad \Delta P^{k \times 1} = Q_1^{m \times m} \, \Delta T^{m \times 1}$$

to give orthogonal factorisation of $\underline{A}$:

$$\begin{bmatrix} R^{k \times k} \\ \hline O^{(m-k) \times k} \end{bmatrix} \Delta P^{k \times 1} = \begin{bmatrix} C_1^{k \times 1} \\ \hline C_2^{(m-k) \times 1} \end{bmatrix}$$

where $\underline{R}$ is upper triangular.

Approximate by $\mathcal{E}$-appendage:

$$\begin{bmatrix} R^{k \times k} \\ \hline O^{(m-k) \times k} \\ \hline \mathcal{E} I^{k \times k} \end{bmatrix} \Delta P^{k \times 1} = \begin{bmatrix} C_1^{k \times 1} \\ \hline C_2^{(m-k) \times 1} \\ \hline O^{k \times 1} \end{bmatrix}$$

Pre-multiply by orthogonal matrix, Q_2:

$$Q_2^{(m+k) \times (m+k)} \begin{bmatrix} \underline{R} \\ \underline{O} \\ \mathcal{E} I \end{bmatrix}^{(m+k) \times k} \Delta P^{k \times 1} = Q_2^{(m+k) \times (m+k)} \begin{bmatrix} \underline{C}_1 \\ \underline{C}_2 \\ O \end{bmatrix}^{(m+k) \times 1}$$

to give orthogonal factorisation:

$$\begin{bmatrix} R_1^{k \times k} \\ \hline O^{m \times k} \end{bmatrix} \Delta P^{k \times 1} = \begin{bmatrix} \underline{C}_3^{k \times 1} \\ \underline{C}_2^{(m-k) \times 1} \\ C_4^{k \times 1} \end{bmatrix}$$

Where $\underline{R}_1$ is upper triangular.

Solve by back-substitution: $\Delta P^{k \times 1} = (R_1^{k \times k})^{-1} \, C_3^{k \times 1}$

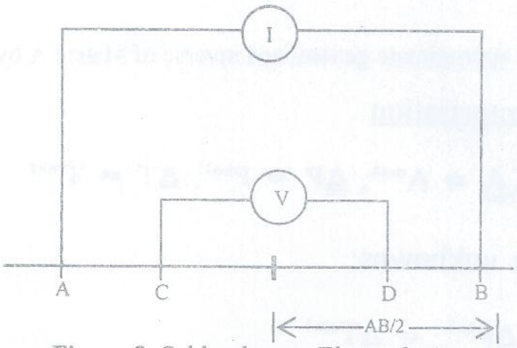


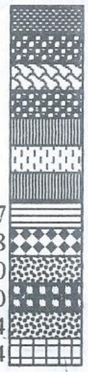
Figure 3: Schlumberger Electrode Array

I = current, V = Voltage
AB = Current Electrode
CD = Potential Electrode

Table 2: Summary of Results

STATION	A	B	C
1ST layer model ℓ_a	590.00Ωm	610.00Ωm	476.00Ωm
2ND layer model ℓ_a	1378.00Ωm	2341.00Ωm	2153.00Ωm
3RD layer model ℓ_a	933.00Ωm	1392.00Ωm	1126.00Ωm
4TH layer model ℓ_a	4686.00Ωm	5998.00Ωm	4826.00Ωm
5TH layer model ℓ_a	4927.00Ωm	6783.00Ωm	4392.00Ωm
1ST layer model depth	2.00m	2.00m	2.00m
2ND layer model depth	12.00m	12.00m	12.00m
3rd layer model depth	18.00m	18.00m	18.00m
4th layer model depth	45.00m	45.00m	45.00m
Approximate depth of aquifer	140.00m	149.00m	146.00m
Thickness of aquifer	12.00m	28.00m	25.00m
Percentage error between filed, data curve and theoretical curve RMS(%)	8.180283	5.1471808	8.561251

Location A: Fire Service Location – Isa – Ogwashi Road



Depth(m)	Lithology (Formations)
0 – 33.5	Laterite soil
33.5 – 42.7	Sandy clay
24.7 – 580	Coarse sand with clay
58.0 – 61.0	Sandy clay
61.0 – 76.2	Fine sand
76.2 – 94.5	Fine to medium sand
94.5 – 100.6	Fine sand
100.6 – 106.7	Clay
106.7 – 115.8	Fine-medium sand with clay
115.8 – 128.0	Coarse sand
128.0 – 134.0	Fine sand with clay
134.0 – 152.4	Coarse sand
152.4 – 155.4	Reddish clay

Figure 4: Driller's log Location A (Not drawn to scale)

Location B: Ase Primary School

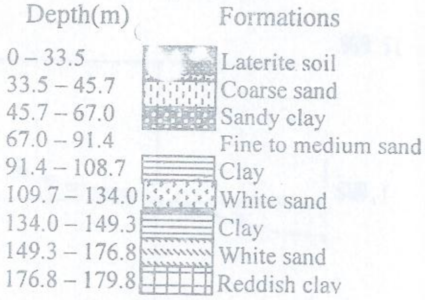


Figure 5: Driller's log for Location B (Not drawn to scale)

Location C: Asaba Road near Police Station

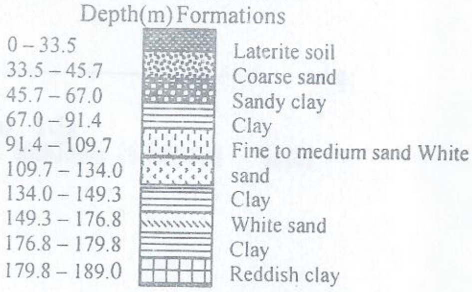


Figure 6: Driller's log for Location C (Not drawn to scale)

Table 3:

Project: Vertical Electrical Sounding (Ves)

Site: Ogwashi-Uku Location A.

Title: Resistivity Sounding Interpretation

Field and Theoretical Data

AB/2 Values (metres)	Observed Values (ohm-m)	Computed Values (ohm-m)
1.00	590.00	598.37
1.47	627.00	611.14
2.15	688.00	644.73
3.16	772.00	716.42
4.64	819.00	827.77
6.81	895.00	958.62
10.00	962.00	1082.43
14.70	1028.00	1178.25
21.50	1136.00	1325.89
31.60	1215.00	1284.40
46.40	1378.00	1405.09
68.10	1592.00	1675.29
100.00	3085.00	2096.21
150.00	4012.00	2637.98
200.00	4686.00	3042.00
300.00	4270.00	3589.20
400.00	3927.00	3933.66

Model Parameters

Geoelectric Layer	Specific Resistivity (ohm-m)	Thickness (Metres)	Culm. Thickness (metres)
1.00	590.00	2.00	2.00
2.00	1378.00	12.00	14.00
3.00	933.00	18.00	32.00
4.00	4686.00	45.00	77.00
5.00	4927.00	Infinity	Infinity

RMS Error (%) = 8.180283

Field measurements by Egbai, J.C. and Ekpekpo Arthur

Data interpreted by Egbai, J.C. and Ekpekpo, A

Figure 7: Field and theoretical curve for VES location A

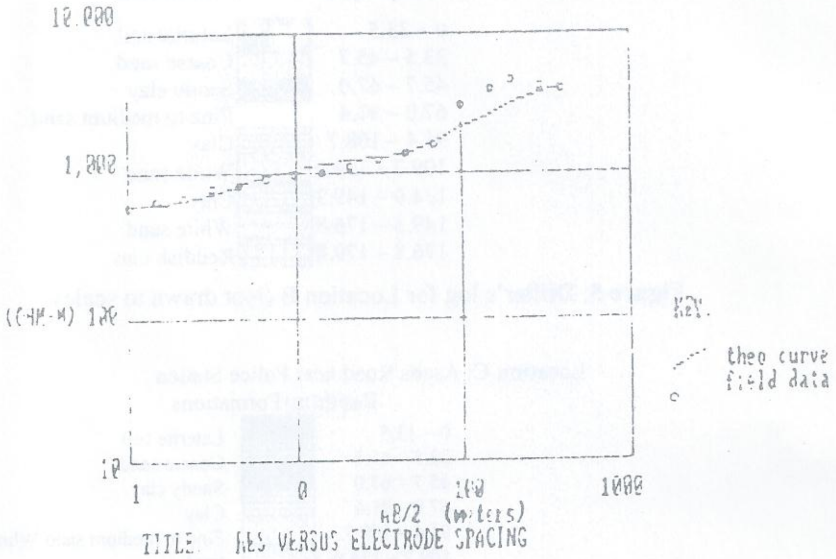


Table 4:
Project: Vertical Electrical Sounding (Ves)
Site: Ogwashi-Uku Location B.
Title: Resistivity Sounding Interpretation

Field and Theoretical Data

AB/2 Values (metres)	Observed Values (ohm-m)	Computed Values (ohm-m)
1.00	610.00	624.22
1.47	692.00	643.96
2.15	739.00	696.29
3.16	785.00	810.55
4.64	793.00	995.31
6.81	1125.00	1228.11
10.00	1356.00	1473.79
14.70	1559.00	1695.48
21.50	1855.00	1854.89
31.60	1892.00	1961.54
46.40	2341.00	2113.16
68.10	2600.00	2443.25
100.00	3031.00	2987.49
150.00	4249.00	3711.06
200.00	5998.00	4255.08
300.00	5221.00	4991.48
400.00	5783.00	5454.22

Model Parameters

Geoelectric Layer	Specific Resistivity (ohm-m)	Thickness (Metres)	Culm. Thickness (metres)
1.00	610.00	2.00	2.00
2.00	2341.00	12.00	14.00
3.00	1392.00	18.00	32.00
4.00	5998.00	45.00	77.00
5.00	6783.00	Infinity	Infinity

RMS Error (%) = 5.1471808
Field measurements by Egbai, J.C. & Ekpekpo Arthur
Data interpreted by Egbai, J.C. & Ekpekpo, A.

Figure 8: Field and theoretical cure for VES location C

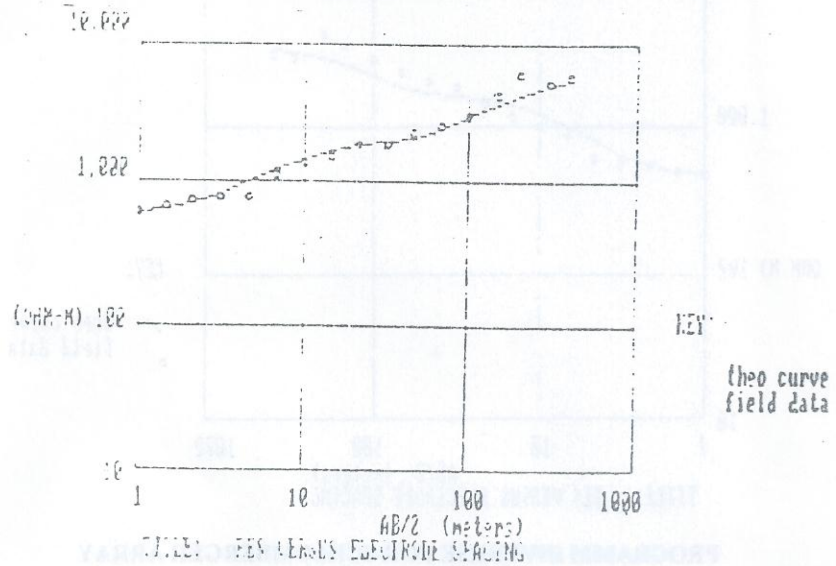


Table 5:
Project: Vertical Electrical Sounding (Ves)
Site: Ogwashi-Uku Location C.
Title: Resistivity Sounding Interpretation

Field and Theoretical Data

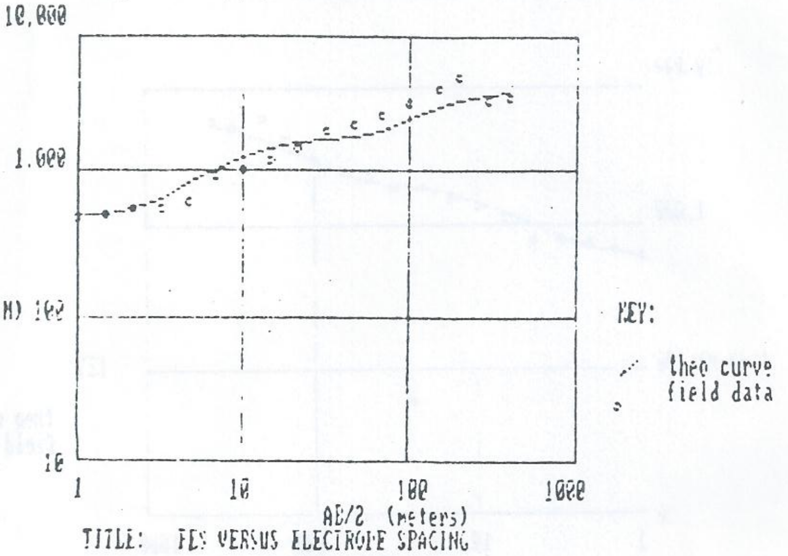
AB/2 Values (metres)	Observed Values (ohm-m)	Computed Values (ohm-m)
1.00	476.00	488.47
1.47	501.00	505.00
2.15	546.00	549.85
3.16	557.00	648.21
4.64	617.00	809.08
6.81	895.00	1015.22
10.00	1012.00	1238.24
14.70	1214.00	1445.55
21.50	1416.00	1596.24
31.60	1945.00	1684.12
46.40	2153.00	1782.85
68.10	2500.00	2011.06
100.00	3131.00	2401.42
150.00	3960.00	2901.94
200.00	4826.00	3248.33
300.00	3237.00	3665.38
400.00	3392.00	3894.59

Model Parameters

Geoelectric Layer	Specific Resistivity (ohm-m)	Thickness (Metres)	Culm. Thickness (metres)
1.00	610.00	2.00	2.00
2.00	2341.00	12.00	14.00
3.00	1392.00	18.00	32.00
4.00	5998.00	45.00	.77.00
5.00	6783.00	Infinity	Infinity

RMS Error (%) = 8.561251
Field measurements by Egbai, J.C. & Ekpekpo Arthur
Data interpreted by Egbai, J.C. & Ekpekpo, A

Figure 9: Field and theoretical curve for VES location C



PROGRAMM INVERRSE FOR SCHLUMBERGER ARRAY

```
      PROGRAM, RESIST(INPUT,OUTPUT,DATA1,TAPE1=INPUT, APE2-OUTPUT)
      INTEGER E
      COMMON/Z1/E,M,N/Z2/DELX,SPAC
      COMMON/ZA3/P(99)/ZA4/R(134)
      DIMENSION FLTR1(29), FLTR2(34),
      DIMENSION SN(30),R1(31)
      DATA(FLTRI(I),I=1,29).00046256,-.0010907,0017122,-0020687,1.0043048,-
      .0021236,.015995,.017065,.098105,.21918,64772,1.1415,2.471819,-3.515,2.7743,--1.201,.4544,-.19427,      .097364,-.054099,.0317293,-
      .019109,.011656,-0071544,.0044042,-002715,.0016749,-.0010335,4.00040124/
      DATA(FLRTA2(I),I=1,34)/.000233935,.00011557,.00017034,.000024935,1.00036665,.00053753,.00007896,0001154,.0017008,.0024959,.003664
      ,2.0053773,.007893,.011583,.01698,.024934,.036558,.053507,.078121,3.11319,.16192,.22363,.28821,.30276,.15523,-.32026,-.53557,.51787,4,-
      .196,.054394,-.015747,.0053941,-.0021446,.000665125/
C.
C CARD #1 ARRAYCHOICE, INPUT-
C         1—FOR SCHLUMBERGER,
C         2—FOR WENNER,
C         3—FOR BIPOLE-BIPOLE.
C CARD #2 SPAC,E,H  (FORMAT-FREE)
C         SPAC = CLOSEST A OR S SPACING (REAL)
C         E = NUMBER OF MODEL LAYERS (INTEGER)
C         M = NUMBER OF FIELD READINGS (INTEGER), 6/DEDCADE
C CARD #2A ENTER ONLY FOR BIPOLE-BIPOLE ARRAY, INPUT-
C         1—IF N-VALUES ARE VARIED
C         0—IF A-SAPCINGS ARE VARIED,
C CARD #2B ENTER ONLY FOR BIPOLE-BIPOLE, IF VALUE ENTERED IN 2A WAS-
C         1—INPUT N-VALUES (TOTAL, N) IN INCREASING ORDER (FORMAT-FREE)
C         0—INPUT ONE N-VALLUE. (N.NE.1).
C CARD #3 ENTER LAYER PARAMETERS. (TOTAL 2E-1, FORMAT-FREE)
C         ORDER—H(1),H(2),...H(E-1),R(1),R(2),...R(E)
C   *     *     *     *     *     *     *     *     *     *     *     *
C         REPEAT FOR ADDITONAL MODELS.
C
 1000 READ(1,*)INDEX
      IF(INDEX.EQ.0) STOP
      READ(1*)SPAC,E,H
      IF(INDEX-2) 40.40.5
      5 READ(1,*) IX
            IF (IX,EQ.1) GO TO 20
            J=1
            GO TO 35
      20 J = H
      35 READ(1,*) (SN(I), I = 1,J)
      40 N = 2*E-1
            SPACE=ALOG(SPAC,
            READ(1*) (P(I), I = 1,N)
            WRITE(2,42)
      42 FORMAT(//*APPARENT RESISTIVITY VALUES*)
      IF(INDEX-2)43,45,47
```

```
43 WRITE(2,44)
44 FORMAT(/* SCHLUMBERGER ARRAY*//)
        GO TO 50
45 WRITE(2,46)
46 FORMAT(/* WENNER ARRAY*//)
        GO TO 50
47 WRITE(2,48)
48 FORMAT(/* BIPOLE=BIPOLE ARRAY*//)
50 DELX = ALOG(10.)/6.
   IF (INDEX-2) 70,80,300
70 Y=SAPC-19.*DELX-0.13069
   DO 75, I = 1,N+2-8
        CALL TRANSFM(Y,I)
75      Y = Y + DELX
   CALL FILTER(FLTR1,29)
   GO TO 120
80 S=ALOG(2.)
   Y = SPAC-10.8792495*DELX
   DO 110, I = 1, N+33
        CALL TRANSFEM(Y,I)
        A = R(I)
        Y1 = Y + S
        CALL TRANSFM(Y1,I)
        R(I) = 2.*A-R(I)
110     Y=Y+DELX
   GO TO 119
300 M1=1
   IF(LX.NE.1) GO TO 111
   M1=H
   M=1
111 DO 117,I=1,N1
        Y=SPAC-10.8792495*DELX
        A=SN(I)
        A1=ABS(A-1.)
        S1=ALOG(A1)
        IF(A.LT.1.) Y=Y-ALOG(A)
        B=1. .
        IF(A.LT.1) B=A*A+A-1.
        S3=ALOG(A + 1)
        DO 116, J=1,N+33
            Y1=Y+S1
            CALL TRANSFM(YI,J)
            AA=R(J)/A1
            YI=Y+S2
            CALL TRANSFM(YI,J)
            AA=AA-2.*R(J)/A
            YI=Y+S3
            CALL TRANSFM(YI,J)
            R(J)=(AA+R(J)/(A+1.))*A*(A+1.)*A1/(2.*B)
116         Y=Y+DELX
        IF(IX.NE.1) GO TO 117
        CALL FILTER(FLTR2,34)
        R1(I)=R(I)
117     CONTINUE
   IF(M.NE.1) GO TO 119
   M=MI
   GO TO 120
119 CALL FILTER(FLTR2,34)
120 WRITE(2,125) E
125 FOMAT(//13* LAYER MODEL.*)
   WRITE(2,130)
130 FORMAT(/5X,*LAYER NO.*3X*THICKNESS*3X*RESISTIVITY*/)
   DO 140, I=1,E-1
        J=I
        WRITE(2,135) J,P(I),P(I+E-1)
135     FORMAT(9X,I2,5X,F8.3,7X,F8.3)
140     CONTINUE
   WRITE(2,145) E,P(N)
145 FORMAT(9X,I2,20X,F8.3)
   IF(INDEX-2) 205,205,150
150 IF(IX.NE.1) GO TO 190
   SP=EXP(SPAC)
   WRITE(2,160) SP
160 FORMAT(/* BIPOLE A-SPACING =*,F6.2)
   WRITE(2,170)
170 FORMAT(//10/*E*9X*RHO*/)
   DO 185, I=1,M
        WRITE(2,180) SN(I),R1(I)
180     FORMAT(7X,F7.2,3X,F9.3)
185 CONTINUE
```

```
     GO TO 240
190 WRITE(2,200) SN(1)
200 FORMAT(/*BIPOLE N-SPACING = *F6.2)
205 WRITE(2,210)
210 FORMAT(//7X*SPACING*7X*RH0*/)
     X-SPAC
     DO 230,I=1,H
          A=EXP(X)
          WRITE(2,220)A,R(I)
220       FORMAT(6X,F7.2,3X,F9.3)
230       X=X+DELX
240 GO TO 1000
     END

     SUBROUTINE TRANSFH(Y,I)
     INTEGER E
     COMMON/ZI/E.M,N
     COMMON/ZA3/P(99)/ZA4/R(134)
     DIMENSION T(50)
     U=1./EXP(Y)
     T(1)=P(N)
     DO 30, J=2,E
          A=EXP(-2.*U*P(E+I-J)
          B=(1.-A)/(1.+A)
     RS=P(N+1-J)
     TPR=RS*B
     T(J)=(TPR+T(J-1))/(1.TPR*T(J-1)/(RS*RS))
30        CONTINUED
     R(I)=T(E)
     RETURN
     END

     SUBROUTINE FILTER(FLTR,K)
     INTEGER E
     COMMON/ZI/E.M,N
     COMMON/ZA4/R(134)
     DIMENSION RES(31),FLTR(K)
     DO 20,I=1,M
          RE=0
          DO 10,J=1,K
               B=FLTR(J)*R(I+K-J)
10             RE=RE+B
20        RES(I)-RE
     DO 30, I=1,M
30        R(I)=RES(I)D
     RETURN
     END
```

References

[1] Barnes, H., 1952; Soil investigations employing a method of layer value determination for earth resistivity interpretation. Highway Research Board Bull. 65, pp. 20 – 36.

[2] Egbai. J.C. 1997, Correlation between resistivity survey and well logging in Delta State, Ph.D. thesis submitted to Edo State University, Ekpoma.

[3] Egbai, J.C. and Asokhai, M.B; 1998 Correlation between resistivity survey and well logging J.Nig. Ass. Math. Phys. Vol. 2, pp. 163 – 175

[4] Frohlich, R.K. 1973; Detection of fresh water aquifers in the glacial deposits of North and Western Missouri by geoelectrical methods. Wat. Res. Bull; Vol. 9, No. 4, pp. 723 – 734.

[5] Kunetz, G. and Rocroi, J.P; 1969; Automatic processing of electrical soundings. Paper presented at 31st E.A.E.G. meeting. Venice, May 21 – 23, 27 p.

[6] Kunetz, G. and Rocroi, J.P; 1970; Traitement automatique des sondages electriques. Geophys. Prosp. Vol. 18, No. 2 pp. 157 – 198

[7] Marquardt, D.W; 1963; An algorithm for least-squares estimation of non-linear parameters. J.Soc. Indust. Appl. Math; Vol. 11, pp. 431 – 441.

[8] Meinardus, H.A. 1970; Numerical interpretation of resistivity soundings over horizontal beds. Geophys. Prosp; Vol. 18, No. 3, pp. 415 – 433.

[9] Narayan, P.V.S. and Ramanujachary, K.R. 1967; An inverse slope method of determining absolute resistivity. Geophysics, Vol. 32, No. 6, pp. 1036 – 1040.

[10] Osborne M.R. and Jennings, L.S. 1970; Applications of orthogonal matrix transformations to the solution of systems of linear equations. Aust. Nat. Uni. Computer Centre, Canberra, Tech. Report No, 37, 45p.

[11] Shichter, L.B. 1933; The interpretation of the resistivity prospecting method for horizontal structures. Physics, Vol. 4, pp 307 – 322.

[12] Vozoff, K., 1960; Numerical resitivity interpretation, General inhomogeneity. Geophysics, Vol. 25, No. 6, pp. 1184 - 1191.

[13] Zohdy, A.A.R., 1973b; Automatic interpretation of resistivity sounding curves using modified Dar Zarrouk functions. Geophysics Vol. 38, No. 1, pp. 196 – 197 (Abstract.)