Pressure Drop Profile of an inverted Five-Spot Oil Production Pattern with a Horizontal Well Water Injector

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Abstract

To a large extent the success of an inverted five-spot pattern with four vertical oil producers and one water injector, depends on correct sizing of surface injection facility and proper understanding of both reservoir and wellbore characteristics. With vertical water injectors this pattern had been applied successfully in the past. The advent of the nascent horizontal well technology therefore arouses high curiosity about the applicability as fluid injector in such patterns. This paper derives a sand face pressure drop distribution in an inverted 5-spot pattern with a centrally located horizontal well injector. Source functions for a bounded reservoir containing a centrally located horizontal well injector are utilized. An analysis of the dimensionless pressure distributions shows that large patterns will yield more clean oil production than small patterns. If the producers produce oil at the same rate, a square pattern will yield equal volume of clean oil for the same time while a rectangular pattern will produce early water in nearby producers than the further ones.

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1.0 Introduction

An inverted five-spot oil production pattern consists of four peripherals oil producing ells (producers) and one fluid injection well (injector) placed in the centre. The pattern formed by the well arrangement could be rectangular or square shaped. The number of patterns desired for a reservoir depends on the fluid properties and reservoir, size, anticipated oil production rate (oil demand), available capital and technology (well-type and surface production facilities.

The main objectives of an inverted five-spot project are usually (1) for pressure maintenance and (2) enhanced oil recovery. As injection takes place, pressure distributions are developed between the injector and producers. The pressure gradient along the shortest streamline is the highest. Hence, injected fluid reaches the producers along these streamlines, which are found along the lines of symmetry.

One of the major determinants of the success of an inverted five-spot project is the proper selection of surface facilities such as injection pump facilities and flow lines. These facilities can only be correctly selected if the overall project life and corresponding pressure drop histories are known.

In this paper, the pressure profile of an inverted five-spot oil production pattern is derived for a horizontal well water injector, under the condition; of unit mobility ratio, that is, no water breakthrough. The profile is obtained by deriving the dimensionless pressure distribution for a rectangular drainage reservoir with four producers (either another horizontal or vertical well), situated along the pattern lines of symmetry.

The suitability of horizontal or lateral wells for production and or injection in oil recovery programmes has been studied and reported in the literature. [1], [2], [3], [4], and [5]. Their applicability has also been demonstrated in the field. [6], [7], [8] and [9]. In particular [6] shows that horizontal well injectors require only a tenth of the pressure drop in vertical wells for oil to be delivered even at appreciable rates.

Other possible applications of the dimensionless pressure expressions derived here are in (1) the estimation of the cumulative and individual well production at abandonment pressure, (2) the search for optimum wellbore conditions for more oil production and (3) the estimation of reservoir properties using type curves. The superposition theorem is used to calculate the interwell dimensionless pressure drops for known oil production rates.

2.0 Pattern, Wellbore and Reservoir Descriptions

The water injection and oil production pattern consists of 4-peripheral oil producers (of any angle) and one horizontal well water injector at the centre of a rectangular pattern. The well arrangement in the pattern is shown in Figure 1. The reservoir selected for the flood is bounded, anisotropic and contains oil of small but constant compressibility. Initial reservoir pressure is p_i. The only source of energy for the reservoir oil to flow is derived from water injection as the reservoir pressure is further increased to continually keep the dissolved gas in solution. The horizontal injector has length L (along the x-axis) width y (along z-axis.) It is located at z_w away from the bottom of the pay zone. If the horizontal well injects water at a rate $\left(\frac{-qbbl}{D}\right)$ the wellbore pressure distribution will be derived for the injector as a function of wellbore parameters and time (less than that required for the injected water to reach the producers.)

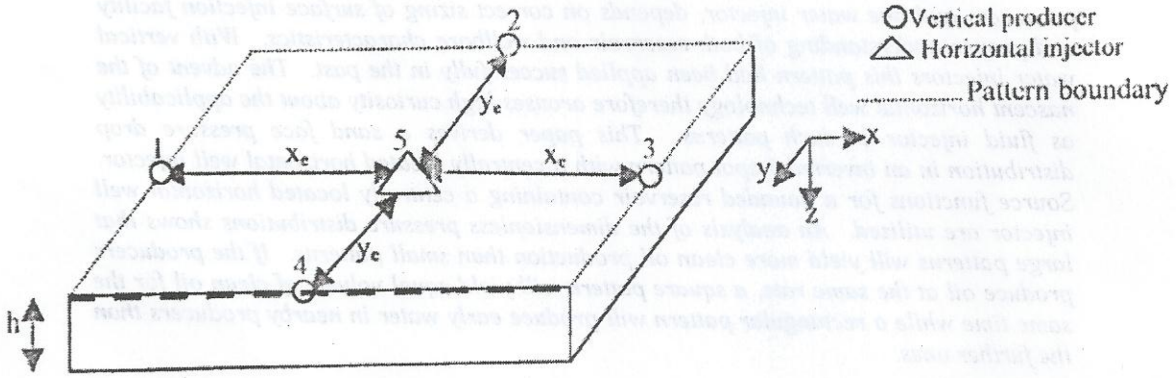

Figure 1: Inverted 5-spot flood pattern configuration

The external dimensions of the reservoir, where the producers are located, are x_e (along the x-axis) and y_e (along the y-axis.) Reservoir pressure distribution as a function of reservoir properties and time will also be derived from time less than that required for min(x_e, y_e) to be reached by injection pressure transient. If the respective production rates of the producers are q_1, q_2, q_3, and q_4, pressure drops in well 5 injecting at $-qbbl/D$ owing to q_1, q_2, q_3, and q_4 are calculated here for conditions of unit mobility ratio.

3.0 Mathematical Description

The horizontal well injecting water is a point source located in an infinite reservoir. The well is located at the centre of the x-axis of the reservoir. The source function that constitute the flow strength for the injector are therefore an infinite slab source in an infinite slab reservoir along the x-axis; an infinite plane source in an infinite slab reservoir along the x-axis; and an infinite plane source in an infinite slab reservoir along the z-axis. [10] and [11] provide these functions as

x-direction

$$s(x,t) = \frac{\frac{L}{2}}{x_e}\left[1 + \frac{4x_e}{\pi L}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{n^2\eta_x\pi^2 t}{x_e^2}\right)\sin n\pi\frac{L}{2x_e}\cos n\pi\frac{x_w}{x_e}\cos n\pi\frac{x}{x_e}\right] \qquad (3.1)$$

y-direction

$$s(y,t) = \frac{1}{y_e}\left[1 + 2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\eta_y\pi^2 t}{y_e^2}\right)\cos m\pi\frac{y_w}{y_e}\cos m\pi\frac{y}{y_e}\right] \qquad (3.2)$$

z-direction

$$s(z,t)=\frac{1}{h}\left[1+2\sum_{l=1}^{\infty}\exp\left(-\frac{l^2\eta_z\pi^2 t}{h^2}\right)\cos l\pi\frac{z_w}{h}\cos l\pi\frac{z}{h}\right] \tag{3.3}$$

The pressure drop in the wellbore for a constant production rate q per unit half-length of the well is given as follows using Newman's product method. [10].

$$\Delta p(x,y,z,\tau)=\frac{q}{\phi c}\int_0^t s(x,\tau)s(y,\tau)s(z,\tau)d\tau \tag{3.4}$$

The following dimensionless parameters are defined

$$i_D=\frac{2i}{L}\sqrt{\frac{k}{k_i}},\quad i=x,y,z \tag{3.5}$$

$$P_D(x_D,y_D,z_D,\tau_D)=\frac{2\pi kh\Delta p}{q\mu} \tag{3.6}$$

$$h_D=\frac{2h}{L}\sqrt{\frac{k}{k_z}} \tag{3.7}$$

$$L_D=\frac{L}{2h}\sqrt{\frac{k}{k_x}} \tag{3.8}$$

$$t_D=\frac{2kt}{\phi\mu c_t L^2} \tag{3.9}$$

The different pressure distributions are derived as follows

3.1 Injector

The injector dimensionless wellbore pressure PwD5, experiences infinite sources along the x-and y-axes. This is attributable to the fact that since only near wellbore environment is the only contributor to wellbore pressure, other points outside the wellbore are mathematically infinite. Therefore, the equivalent source functions to equations (3.1) and (3.2) are

$$s(x_D,t_D)=\frac{1}{4}\left[erf\frac{(1+x_D)}{2\sqrt{\tau}}+\frac{(1-x_D)}{2\sqrt{\tau}}\right] \tag{3.10}$$

and

$$s(y_D,t_D)=\frac{1}{2\sqrt{\pi t_D}}e^{-\frac{(y_D-y_{wD})^2}{4t_D}} \tag{3.11}$$

Substituting equations (3.3), (3.10) and (3.11) into equation (3.9) we have

$$P_D(x_D,y_D,z_D,\tau_D)=\frac{\sqrt{\pi}}{8}\int_0^{t_D}\left[erf\frac{(1+x_D)}{2\sqrt{\tau}}+\frac{(1-x_D)}{2\sqrt{\tau}}\right].\frac{e^{-\left(\frac{(y_D-y_{wD})^2}{4\tau}\right)}}{\sqrt{\tau}} \tag{3.12}$$

$$\left[1+2\sum_{m=1}^{\infty}\exp\left(-\frac{m^2\pi^2 t_D}{4h_D^2}\right)\cos m\pi\frac{z_{wD}}{h_D}\cos m\pi\frac{z_D}{h_D}\right]d\tau$$

3.2 Injector/Producer Streamline Pressure Drops

Using the superposition theorem, if the injector is active, then the pressure drop in the injector due to oil production in the 4-producers is written generally from equation (3.8) as

$$\Delta p=\frac{1}{2\pi kh}\left[\sum_{p=1}^{4}\left(q_p,\mu_p,B_p-q_i,\mu_i,B_i\right)P_{Dpi}-q_i,\mu_i,B_i P_{wDpii}\right] \tag{3.13}$$

for only stable displacement condition; (that is, no water breakthrough.) Here i stands for injection and p for production. The convention that q_p is positive and q_i is negative is adopted. The dimensionless pressures in equation (3.13) are derived below and area along the streamlines of the flood pattern. This means that all other points outside these streamlines are mathematically infinite from the injector, $P_{wDii} = P_{wD55}$ = dimensionless pressure drop in the injector due to injection, as is given by equation (3.12). The major components of P_{Dpi} are P_{D52} and P_{D54} along the y-axis streamline and P_{D51} and P_{D53} along the y-axis streamline. These dimensionless pressure drops are those monitored in the injector owing to oil production from the respective producers. Additional pressure drop owing to a rate change either in the injector or producer can be obtained using equation (3.13).

3.3. **P_{D52} and P_{D54}**

Along the axis of wells 2 and 4 $s(x_D, t_D)$ is an infinite plane source in an infinite reservoir while the $s(y_D, t_D)$ is a slab of thickness $y_{wD} = 0$ to y_D. The source $s(z_D, t_D)$ remains as given by equation (3.3). Hence, if each producer is y_{eD} from the injector then

$$P_{D5k} = \frac{\sqrt{\pi}}{y_{eD}} \int_0^{t_D} \frac{e^{-\frac{(y_D - y_{wD})^2}{4\tau}}}{\sqrt{\tau}} \left[1 + 2\sum_{m=1}^{\infty} \exp\left(-\frac{m^2\pi^2 t_D}{h_D^2}\right) \cos m\pi \frac{z_D}{h_D} \cos m\pi \frac{z_{wD}}{h_D}\right] \cdot \left[1 + 2\sum_{n=1}^{\infty} \exp\left(-\frac{n^2\pi^2 t_D}{y_{eD}^2}\right) \cos n\pi \frac{y_D}{y_{eD}} \cos n\pi \frac{y_{wD}}{y_{eD}}\right] d\tau \tag{3.14}$$

where k = well 2 or 4.

3.4 **P_{D51} and P_{D53}**

Along the axis of wells 1 and 3, $s(x_D, t_D)$ is given by equation (3.1); $s(y_D, t_D)$ is given by equation (3.11). $s(x_D, t_D)$ remains as given by equation (3.3). Therefore equation (3.9), assuming that wells 1 and 3 q_{re} each x_{eD} away from the injector

$$P_{D5j} = \frac{\sqrt{\pi}}{X_{eD}} \int_0^{t_D} \frac{e^{-\frac{(x_D - x_{wD})^2}{4\tau}}}{\sqrt{\tau}} \left[1 + \frac{2x_{eD}}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \exp\left(-\frac{n^2\pi^2 t_D}{h_D^2}\right) \sin n\pi \frac{\pi}{x_{eD}} \cos n\pi \frac{x_D}{x_{eD}} \cos n\pi \frac{x_{wD}}{x_{eD}}\right] \cdot \left[1 + 2\sum_{m=1}^{\infty} \exp\left(-\frac{m^2\pi^2\tau}{4h_{eD}^2}\right) \cos m\pi \frac{z_D}{h_D} \cos m\pi \frac{z_{wD}}{h_D}\right] d\tau \tag{3.15}$$

where j = well 1 or 3.

4.0 Computation of Dimensionless Pressures

A major flow period common to all horizontal well fluid flow is he early radial (infinite acting) flow period. During this period the well behaves as a fully penetrating vertical well of thickness L. Hence, both $s(y_D, t_D)$ and $s(z_D, t_D)$ are given by a form similar to equation (3.11) while $s(x_D, t_D)$ is given by equation (3.10). In all the integrals in equations (3.12), (3.14), and (3.15) this period lasts between $t_D = 0$ to $t_D = t_{De}$; (that is, t_{De} is the dimensionless time marking the end of the early radial flow.) In our computations t_{De} was obtained by solving for t_D at which the argument of the error function is approximately 1.8; this gives the error function of unity. The dimensionless pressure between $t_D = 0$ and t_{De} created by a rate history is then superposed on the p_D beyond t_{De}, nothing that the original rate history still persists in the wellbore even as new other boundaries are felt. In this case, equation (3.12) can be written as

$$P_{D55}=\frac{h_D}{8}\int_0^{t_D} e^{-\frac{\left[\frac{(z_D-z_{wD})^2+(y_D-y_{wD})^2}{4\tau}\right]}{\tau}}\left[erf\left(\frac{1+x_D}{2\sqrt{\tau}}\right)+erf\left(\frac{1-x_D}{2\sqrt{\tau}}\right)\right]d\tau+\frac{\sqrt{\pi}}{4}\cdot\int_{De}^{t_D} e^{-\frac{\left[\frac{(y_D-y_{wD})^2}{4\tau}\right]}{\sqrt{\tau}}}\cdot \left[erf\left(\frac{1+x_D}{2\sqrt{\tau}}\right)+erf\left(\frac{1-x_D}{2\sqrt{\tau}}\right)\right]\left[1+2\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2\tau}{4h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]d\tau \quad (4.1)$$

or

$$P_{D55}=-\frac{\alpha h_D}{8}E_i-\left[\frac{(z_D-z_{wD})^2+(y_D-y_{wD})^2}{4t_D}\right]+\frac{\sqrt{\pi}}{4}\int_{De}^{t_D} e^{-\frac{\left[\frac{(y_D-y_{wD})^2}{4\tau}\right]}{\sqrt{\tau}}}\left[erf\left(\frac{1+x_D}{2\sqrt{\tau}}\right)+erf\left(\frac{1-x_D}{2\sqrt{\tau}}\right)\right]\cdot\left[1+2\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2\tau}{4h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]d\tau \quad (4.2)$$

where $\alpha=2$ if $x_D \prec 1, 0$ if $x_D \succ 1$ and 1 if $x_D=1$.

Similarly equation (3.14) can be rewritten as

$$P_{D5k}=-\frac{\beta h_D}{8}E_i\left[-\frac{(z_D-z_{wD})^2+(y_D-y_{wD})^2}{4t_D}\right]+\frac{\sqrt{\pi}}{4}\cdot\int_{De}^{t_{Dz}} e^{-\frac{\left[\frac{(y_D-y_{wD})^2}{4\tau}\right]}{\sqrt{\tau}}}\left[erf\left(\frac{y_D}{2\sqrt{\tau}}\right)\right]\cdot\left[1+2\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2\tau}{4h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]d\tau+\frac{\sqrt{\pi}}{y_{eD}}\int_{Dz}^{t_D}\left[1+2\sum_{l=1}^{\infty}\exp\left(-\frac{l^2\pi^2\tau}{4h_D^2}\right)\cos l\pi\frac{z_{wD}}{h_D}\cos l\pi\frac{z_D}{h_D}\right]\cdot\left[1+2\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2\tau}{4h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]e^{-\frac{\left[\frac{(y_D-y_{wD})^2}{4\tau}\right]}{\sqrt{\tau}}}d\tau \quad (4.3)$$

where $\beta=1$ for early radial flow. In equation (4.3), the y and z boundaries are finite while the x-boundaries are considered infinite. The equation is valid for t_D equal to that when y_{eD} is just felt. For stable displacement this dimensionless time may be less. The period $t_{Dz}-t_{De}$ corresponds to the period between end of early time and end of the first linear flow but t_{De} in equation (4.3) is a very small quantity which means hat the nearest boundary after infinite flow is felt in a very short time. This is because y_D is also very small quantity for a line source well. Equation (3.15) just as equation (3.14) has three major components; the farthest boundary this time is along the y-axis. Considering the flow boundaries Equation (3.15) can be rewritten as follows

$$P_{D5j}=-\frac{\beta h_D}{8}E_i\left[-\frac{(y_D-y_{wD})^2+(y_D-y_{wD})^2}{4t_D}\right]+\frac{\sqrt{\pi}}{4}\int_{De}^{t_{Dz}} e^{-\frac{\left[\frac{(y_D-y_{wD})^2}{4\tau}\right]}{\sqrt{\tau}}}\cdot\left[erf\left(\frac{1+x_D}{2\sqrt{\tau}}\right)+erf\left(\frac{1-x_D}{2\sqrt{\tau}}\right)\right]\left[1+2\sum_{n=1}^{\infty}\exp\left(-\frac{n^2\pi^2\tau}{4h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]d\tau$$

$$+\frac{\sqrt{\pi}}{x_{eD}}\int_{Dz}^{tD}\left[1+\frac{2x_{eD}}{\pi}\sum_{l=1}^{\infty}\frac{1}{m}\exp\left(-\frac{m^2\pi^2\tau}{4h_D^2}\right)\sin m\frac{\pi}{x_{eD}}\cos m\pi\frac{x_{wD}}{x_{eD}}\cos m\pi\frac{x_D}{x_{eD}}\right]\cdot$$

$$\left[1+2\sum_{l=1}^{\infty}\exp\left(-\frac{l^2\pi^2\tau}{4h_D^2}\right)\cos l\pi\frac{z_{wD}}{h_D}\cos l\pi\frac{z_D}{h_D}\right]e^{-\left[\frac{(y_D-y_{wD})^2}{4\tau}\right]}\frac{d\tau}{\sqrt{\tau}} \qquad (4.4)$$

where α is described above. The integrals in equation (4.2), (4.3) and (4.4) are solved numerically. [12].

5.0 Results and Discussion

Results are obtained for a reservoir and wellbore with the following properties $h_D = 1\cdot 0$, $x_D = x_{wD} = 0\cdot 732$, $x_{eD} = 10$, $y_e = 100$, $y_{wD} = 5\times 10^{-4}$, $z_{wD} = 0\cdot 5$, $z_D = z_{wD} + (r_{wD} = y_{wD})$, $L_D = 1$, $k_x = k_y = k_z = 10md$ (isotropic reservoir). Other pertinent data are $B_0 = 1\cdot 25 bbl/STB$, oil viscosity, $\mu_0 = 0\cdot 5cp$. Production rates are $q_1 = q_2 = q_3 = q_4 = 300STB/D$, horizontal well injection rate, $q_5 = 500STB/D$, it is intended to put the project on stream for $t_D = 1, 10, 100, 1000$, respectively. A forecast of the pressure drop profile is required to enable injection facility (pump, surface lines, etc) to be selected for the purpose of supplying water in adequate volumes and rates. Table 1 shows the results of all the dimensionless pressures for the given data. Results obtained are in close agreement with those in [13], especially during the early radial period.

Table 1: Dimensionless Pressures for Example problem

Dimensionless Time t_D	P_{D51}	P_{D52}	P_{D53}	P_{D54}	D_{WD55}
10^{-6}	0.56225	0.28265	0.56225	0.28265	0.56225
10^{-5}	1.13625	0.57348	1.13625	0.57348	1.13625
10^{-4}	1.70015	0.86755	1.70015	0.86755	1.70015
10^{-3}	2.27579	1.19369	2.27579	1.19369	2.27579
10^{-2}	2.85144	1./42572	2.85144	1.42572	2.85144
10^{-1}	3.42708	1.72121	3.42708	1.72121	3.42708
1	4,50933	2.01732	4.50933	2.01732	4.50933
10	5.78608	2.34247	5.78608	2.32327	5.74608
10^2	7.05232	2.75248	7.05232	2.75248	7.05232
10^3	8.26227	3.4356	8.26227	3.42356	7.19123
10^4	8.83792		8.83792		7.63429

Application of the results in Table 1 to equation (3.13) gives the results in Table 2 below for the given dimensionless times. The dimensionless pressure drops due to the producers are calculated, assuming that the producers are equally spaced from the injectors along the same axis of symmetry.

Table 2: Pressure Drops for Wells in Example problem

t_D	P_{D51}	P_{D52}	P_{D53}	P_{D54}	D_{WD55}	$\Delta p(psi)$
1	4.50933	2.01732	4.50933	2.01732	4.50933	6.3
10	5.78608	2.34347	5.78608	2.34347	5.78608	9.4
100	7.05232	2.75248	7.05232	2.75248	7,05232	11.2
1000	8.26247	3.42356	8.26247	3.42356	7.19123	14.5

For good injection performance. $p_{wf} = p_1 + \Delta_p$, that is, the wellbore injection pressure p_{wf} is normally higher than the original reservoir pressure, p_i. From equation (3.13) and Table 1, therefore, the most contributors to p_{wf} are the p_{D51} and P_{D53}. That is the wells that receive the largest amount of injected water are the wells along the shorter streamlines. Therefore, while wells 1 and 3 are water out earlier, wells 2 and 4 may still be producing clean oil, for a given injection time. For a square pattern, given the same a real isotropy, injection rate and volume, a centrally located injector would cause water breakthrough at the producers at the same time. This means that the farther the producer from the injector the higher the likelihood of producing clean oil for a longer tie. Obviously, prolonged injection would require larger pumps as shown in Table. Injection facilities are selected assuming that the designed injection pressures as shown in Table 2 are delivered into the well across the sand face only. Additional Δ_p around the wellbore due to skin should be added to p_{wD55}. Repeated well test can reveal Δ_p due to skin. A plot of the dimensionless pressures in equations (3.12), (3.14) and (3.15) against dimensionless time can be used to obtain a match with observed pressure-time data for wellbore and reservoir characterization. This will, however, be advisable only when there is no water breakthrough yet. [14]. Other factors that directly affect the Δ_p are the oil viscosity and oil formation volume factor. Light oil will require low Δ_p to be displaced while heavy oils will require larger Δ_p to move them. The oil formation volume factor effects are in terms of the amount of the dissolved gas. Reservoir oil with high gas saturation would affect pressure drops as light oil. Oil with low gas saturation will be difficult to displace because of its low total compressibility. For heavy crude, water impregnated with polymers will ensure satisfactory sweep but require large Δ_p to achieve efficient oil displacement.\

6.0 Conclusion

A comprehensive pressure drop profile has been derived for an inverted 5-spot waterflood pattern with a horizontal well injector. It has shown that the profile is affected by (1) pattern geometry (2) duration of injection (3) injection and production rates, (4) oil properties and (5) injection water properties. The effects of reservoir anisotropy and dip wellbore and variable rates are not considered, however, the following conclusions are drawn from the study:

(1) the larger the pattern size the longer the period of clean oil production but larger pressure drop requirement for centrally placed injector. Square patterns with central horizontal injectors offer water breakthrough at the same time in all the producers provided that the producers have the same production rate. On the other hand, at same production rate rectangular patterns cause an earlier water breakthrough in nearby producers.

(2) Only sandface pressure drops can be calculated using our models

(3) Heavy crude require large pressure drops for oil displacement by water.

Nomenclature

B	formation volume factor *bbl/STB*
c	compressibility, *1/psi*
E.O.R.	enhanced oil recovery
k	permeability, *md*
h	pay thickness, *ft*
i	distance either in *x, y, or z* directions, *ft*
p	pressure, *psi*
q	production/injection rate, *STB/D*
bbl	reservoir barrel
r	radius, *ft*
t	time, *hours*
s	*source*
STB	stock tank barrel
L	well length, *ft*
ϕ	porosity, *fraction*
μ	viscosity, *cp*
τ	dummy integration variable

Subscripts

D	dimensionless
e	external
i	initial
t	total
w	wellbore

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