# Minimal Repair and Replacement Policy Based On Salvage Value.

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**Abstract.**

*In most of the complex systems failures occur that when repaired the system is returned to a condition similar to a working system of its kind and age. Repair or replacement of a component from a system does not significantly affect the condition of the whole system. In that case, conducting minimal repairs will prolong life of the system. Therefore, in this paper we present a minimal repair and replacement policy that is based on the salvage value of the system, at its retirement age. Expressions for optimal replacement date, number of repairs and average cost for the proposed policy are derived. Analytically, they do not provide explicit expressions; however, numerical solutions are always possible.*

**Keywords:** *Minimal repair, replacement, salvage value, failure rate.*

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## 1.0 Introduction

Various contributors discussed minimal repair and replacement date of complex systems extensively. Barlow and Hunter (1960) introduced for the first time in literature the idea of minimal repairs. They suggested (Policy 1) that minimal repairs up to a time $t$ should be allowed on a system when maintenance or replacement can return it to as good as new condition. They obtained expressions for expected cost per unit of time and an integral equation which could be used to find the system's optimal retirement date, $t$. Another policy proposed by Park (1979) is to perform minimal repairs for the first ($n$-1) failures and replace the whole system at the $n$th failure (Policy 2). Also, Muth (1977) suggested that minimal repairs should be conducted up to a time $t$ and the whole system be replaced at the first failure after $t$ (Policy 3).

Phelps (1981) studied the above three policies with Weibull distribution and found that Policy 1 performed better than policy 2 which is also better than Policy 3. He also provided conditions for which the average cost for Policy 3 can be obtained as an approximate value from the average costs of Policies 1 and 2.

In this paper we introduce and present a minimal repair and replacement policy (4) based on salvage value of the system. The policy has great potentiality of applications on many complex systems, such as automobiles. In modeling salvage value, Tapiero and Venezia (1979) considered in their paper variables such as deterioration due to usage, obsolescence rate, maintenance policy, operating revenues e.t.c., all being functions of age, $t$. However, we present an alternative expression for salvage value in its simplistic form, which we found to fit well the model proposed by Phelps (1981) when retirement age of the system is very large. The only parameter in the model, $\alpha$ is assumed to contain effects due to the variables stated by Tapiero and Venezia (1979), since the variables are all functions of time, $t$ and $\alpha$ is related to $t$ in the model.

## 2.0 Assumptions

Throughout this paper we assume that:

(a) The life-time $X$ of a system has increasing failure rate (IFR), which is defined as $r(x) = \dfrac{f(x)}{\overline{F}(x)}$

where $\overline{F}(x) = 1 - F(x)$ is the survivor function, $F(x)$ is the cumulative distribution function of $X$ and $f(x)$ is the probability mass function of $X$.

(b) Repair arises only when the system fails.

(c) The cost of replacement of the whole system, $S_0$ is greater than or equal to the average cost of minimal repairs $S_r$, that is $S_0 \geq S_r$. And $S_0$ is constant in the interval (0, *t*].

(d) The salvage value *S(t)* is a decreasing function of time *t*.

(e) The number of failures (or repairs) in the interval (0, *t*] is denoted by *N*(0, *t*].

(f) Salvage value is an age-deteriorating function.

## 3.0 Policy.

The policy we propose in this paper is to *perform minimal repairs up to n^th^ repair. After the n^th^ repair at time t, sell the system and replace by new one.*

Consider the above proposed policy, we find the expected cost of conducting minimal repairs in the interval (0, *t*) as $S_r E[N(0, t)]$, where $E[N(0, t)]$ is the expected number of failures in (0, *t*). Hence, the average cost per unit of time for the policy is given as

$$C(t) = \frac{S_r E[N(0,t)] + S_0 - S(t)}{t} \qquad (3.1)$$

where *S*(*t*) is the salvage value at time *t* and it can be simply modeled as

$$S(t) = S_0 \left[ 1 - \ell^{\frac{\alpha}{t}} \right]; \quad t \geq 0 \qquad (3.2)$$

where α is a positive constant (see figure1).

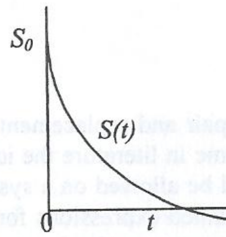


**Figure 1**: Behavior of salvage value.

Equation (3.1) can now be presented as

$$C(t) = \frac{S_r n(t) + S_0 \ell^{-\frac{\alpha}{t}}}{t} \qquad (3.3)$$

where $n(t) = E[N(0,t]] = \int_0^t r(x)dx$, (see Barlow and Proschan (1965)).

Now, we seek for the optimum replacement date and hence the optimum number of minimal repairs at time *t*, so that equation (3.3) is minimum. Differentiating the equation with respect to *t* and equating to zero, we get

$$\left[ tr(t) - \int_0^t r(x)dx \right] S_r + \left( \frac{\alpha}{t} - 1 \right) S_0 \ell^{-\frac{\alpha}{t}} = 0 \qquad (3.4)$$

An explicit expression for replacement date *t* will not be found easily from above equation (3.4) using analytical methods. However, numerical approximation can be used. A unique solution for *t* exist, since *r*(*t*) is an increasing function of $t > 0$ and $S_0 \geq S_r$ (Phelps (1981)) and $r' = f(t)$.

## 4.0 Problem Realization.

To make realization of the problem stated in this paper, we consider the following extreme value distribution, the Weibull distribution with shape and scale parameters β and λ , respectively.

$$F(t) = 1 - \ell^{-(\lambda t)^{\beta}} \quad ; \lambda > 0, \beta > 0 \tag{4.1}$$

Next, equation (3.4) becomes

$$(\lambda t)^{\beta} (\beta - 1) S_r + \left(\frac{\alpha}{t} - 1\right) S_0 \ell^{-\frac{\alpha}{t}} = 0 \tag{4.2}$$

This expression requires numerical approximation if it is to be solved. However, if the system's retirement age is very large and the state of the system attains level of cast-off and zero salvage value, then the upper bound for $t$ is approximately given by

$$t_u \cong \frac{1}{\lambda}\left[\frac{S_0}{(\beta-1)S_r}\right]^{\frac{1}{\beta}} \quad \beta > 1 \tag{4.3}$$

Which is the same with policy 1. Thus, the model we presented gives additional information on policy 1, which is no more than a cast-off policy.

The expected number of minimal repairs and average cost for policy (4) when $t = t_u$ are given below as equations (4.4) and (4.5), respectively.

$$n(t_u) = \int_0^{t_u} \lambda\beta(\lambda x)^{\beta-1} dx = (\lambda t_u)^{\beta}$$

$$\equiv \frac{S_0}{(\beta-1)S_r}; \beta > 1 \tag{4.4}$$

Equation (4.4) is not obtained by Phelps (1981) for policy 1.

$$C(t_u) = \frac{\lambda S_0}{\beta - 1}\left[\frac{(\beta-1)S_r}{S_0}\right]^{\frac{1}{\beta}} \left\{ 1 + (\beta - 1)\ell^{-\alpha\lambda\left[\frac{(\beta-1)S_r}{S_0}\right]^{\frac{1}{\beta}}} \right\} ; \beta > 1 \tag{4.5}$$

Because at time $t = t_u$ the salvage value is zero, then the parameter $\alpha$ can be considered to have attained zero value. Thus, equation (4.5) becomes

$$C(t_u) = \lambda\beta S_r \left[\frac{S_0}{S_r(\beta-1)}\right]^{1-\frac{1}{\beta}} \quad ; \beta > 1 \tag{4.6}$$

This result also confirms the average cost for policy 1 obtained by Phelps (1981).

Furthermore, explicit solution exists for $t$, when the lifetime distribution of the system is negative exponential, that is when $\beta = 1$ in equation (4.1). The optimum replacement date is thus

$$t^* = \alpha \quad ; \alpha > 0 \tag{4.7}$$

The expected number of minimal repairs is $\lambda\alpha$ and the expected cost for the policy (4) is approximately given by

$$\lambda S_r + \frac{S_0}{2.72\alpha} \tag{4.8}$$

## 5.0 Conclusion

In this paper we propose a replacement policy for a complex system based on its salvage value at retirement date. The salvage value model we presented in this paper assumes that the system deteriorates with time. An integral expression is obtained, which can be used to determine optimal replacement date for the system in view and hence its optimal number of minimal repairs. To solve the expression, numerical approximation is suggested in the case where analytical solution fails. Two-parameter Weibull distribution was considered for realization of the problem. That enabled us to confirm results obtained by Phelps (1981), in the case of policy 1, when the system is allowed to operate for a very long period of time. A situation when salvage value is reduced to zero. Also, for a negative exponential distribution of lifetime we showed that analytical solution is explicitly determined.

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