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REPLACEMENT MODEL USING SUBJECTIVE DATA

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ABSTRACT

In the absence of reliable information on the system failure-time history and defects arrivals, various authors have suggested use of subjective data; which can be obtained from expert opinions and judgments. In this short communication, we present a replacement model for a component using subjective information on the life history of the component in view.

Keywods: subjective information, replacement, renewal process, failure-time.

1. INTRODUTION

One of the major constraints in application of maintenance and replacement models is the lack of adequate information on the system's defect arrivals and subsequent failure. Statistical records on maintenance and replacement of units are hardly secured even in the major industries, particularly in developing countries. Because of the lack of reliable data various authors have suggested various methods for collecting data in a subjective manner, using expert judgments (Scarf, 1997; Christer and Waller, 1984; Van Noortwijk et al., 1992; Wang, 1997). If subjective data is properly collected and validated, then it is assumed to be a specimen for the information required on the current problem of interest. However, it is not easy to achieve that due to the numerous reasons pointed out by the authors. For instance, MacCrimmon, 1968; Tversky and Kahneman, 1974 pointed out that some people are willing to alter the information they provided after being told that they are inconsistent, this is because they want to be rational and be carried along with the majority of people. According to Hogarth, 1975, man is a selective, stepwise information processing system with limited capacity. Bias in human judgments is an inherent feature that can be reduced significantly if the procedure and method of eliciting person's opinion are carefully structured (Wang, 1997; Cooke, 1988). Christer and Waller, 1984a,b suggested a method for using subjective opinions of experts in estimation of the delay time distribution. Wang, 1997 indicated the possible improper use of a point estimate in the Christer-Waller method of delay time estimation, because of the various points he raised, some of which we mentioned above.

In this paper we present a replacement model using subjective opinions of experts. Throughout this paper we assume that

1. There is no reliable statistical record of the unit's failure-time history.
2. The subjective information provided by respondents is within the time interval $(0, t]$.
3. The respondents are capable of providing the desired information and provide it to the best of their ability.

4. Each respondent's opinion provided on the subject matter is independent of other respondents' opinions on the same subject matter.

2. NOTATIONS AND DEFINITIONS

Base on the above assumptions we can model a replacement policy by use of a subjective data. To do that, we consider the following definitions.

Y is the failure-time of the component according to the respondents.

$F(y)$ is the cumulative probability distribution of the random variable Y.

$N(t)$ is the number of failures in the interval $(0, t]$.

$n(t)$ is a copy of $N(t)$ as provided by the respondents for the interval $(0, t]$.

x is a policy adopted in selection of the respondents, e.g. years of experience of the respondents.

$P_i(x)$ is the probability that the ith respondent is correct on the information he provided. This probability is to be given by the respondent himself, as suggested by some authors.

$G(x)$ is the probability measure that the respondents are correct about $N(t)$ and it is assumed to be known. That is $G(x) = \prod p_i(x)$.

3. REPLACEMENT MODEL

The number of failures in the interval $(0, t]$ according to the respondents can be expressed as

$$n(t) = N(t)G(x) \quad (1)$$

where $G(x) > 0$ and $N(t)$ is unknown, since the exact distribution of the failure-time is unknown. Thus

$G(x) \rightarrow 1$ as $x \rightarrow \infty$; therefore $n(t) \xrightarrow{a.s.} N(t)$

and $G(x) \rightarrow 0$ as $x \rightarrow 0$; therefore $n(t)$ deviates largely away from $N(t)$.

Furthermore, let y_i be the failure-time according to the ith respondent. Then, $y_1, y_2, \ldots, y_m$ is a random variable that is continuous with probability distribution function $F(y)$. Y_i; $i = 1,2,\ldots,m$ are identically independently distributed random variables. The expected number of failures in the interval $(0, t]$ according to the respondents is therefore

$$E[n(t)] = F(t) + \int_0^t H(t-u)dF(u) = H(t),$$

the renewal function, Barlow and Proschan, 1965. Hence, from equation (1)

$$E[n(t)] = G(x)E[N(t)] \qquad (2)$$

Next, suppose we consider the following replacement policy; Cox, 1982.

"Make planned replacement at times kt; k = 1,2,... if failure occurs before t make service replacement."

The mean cost per unit time for this policy, assuming ordinary renewal process is according to Cox, 1982 given as

$$C(t) = \frac{c_p + c_s E[N(t)]}{t} \qquad c_p < c_s \qquad (3)$$

where c_p is the average cost of planned replacement and c_s is that of service replacement.

From equation (2), equation (3) becomes

$$C(t) = \frac{c_p G(x) + c_s E[n(t)]}{tG(x)} \qquad c_p < c_s \qquad (4)$$

Differentiating equation (4) with respect to t and equate to zero, we determine an optimal value of t. Thus, we can obtain optimal value of t such that

$$th(t) - H(t) = \frac{G(x)c_p}{c_s} \qquad (5)$$

where $h(t) = dH(t)/dt$ is the rate of occurrence of failure at time t, Lam, 1995.

Let $t = t^*$ be the optimal value, then

$$C(t) = \frac{c_p G(x) + c_s H(t^*)}{t^* G(x)}; \qquad c_p < c_s \qquad (6)$$

is the optimum mean replacement cost per unit of time in the interval $(0, t^*]$.

Theorem.

For a very large t, the mean cost per unit of the replacement policy mentioned above is approximately

$$\frac{2\mu c_p}{\mu^2 - \sigma^2} \qquad (7)$$

where μ and σ^2 are the mean and the variance of the failure-times; which can be estimated from the distribution of the subjective data.

Proof.

Consider the limiting value of $H(t)$, which according to Cox, 1982 can be expressed as

$$H(t) = \frac{t}{\mu} + \frac{\sigma^2 - \mu^2}{2\mu^2} \tag{8}$$

From equation (5) and (8) we obtain that

$$\frac{\mu^2 - \sigma^2}{2\mu^2} = \frac{G(x)c_p}{c_s}$$

Thus

$$G(x) = \left(\frac{\mu^2 - \sigma^2}{2\mu^2}\right)\frac{c_s}{c_p} \tag{9}$$

Hence, from equation (4), (8) and (9) the mean cost per unit time of replacement is

$$\lim_{t\to\infty} C(t) \cong \frac{2\mu c_p}{\mu^2 - \sigma^2} \tag{10}$$

Alternatively, if t is very large, then equation (4) becomes

$$\lim_{t\to\infty} C(t) = \lim_{t\to\infty}\left[\frac{c_p}{t} + \frac{c_s}{G(x)}\cdot\frac{H(t)}{t}\right]$$

$$= \frac{c_s}{G(x)}\lim_{t\to\infty}\frac{H(t)}{t} = \frac{c_s}{\mu G(x)}$$

since $\lim_{t\to\infty}\frac{H(t)}{t} = \frac{1}{\mu}$, from the elementary renewal theorem, Barlow and Proschan, 1965. Hence, using equation (9) the proof follows immediately.

Corollary. If the policy adopted in selection of the respondents is one dimensional, such as years of experience (say), then from equation (9) for a very large t

$$x \equiv G^{-1}\left[\left(\frac{\mu^2 - \sigma^2}{2\mu^2}\right)\frac{c_s}{c_p}\right] \tag{11}$$

provided the distribution of G is known.

4. PROBLEM REALIZATION

For realization of the problem, suppose that the failure-time of the unit is Erlang of order α, with probability mass function

$$f(y) = \frac{\beta(\beta y)^{\alpha-1}}{\Gamma(\alpha)} \ell^{-\beta y} \qquad y > 0 \tag{12}$$

Then, according to Barlow and Proschan, 1965 for $\alpha = 2$ the rate of occurrence of failure is

$$h(t) = \frac{\beta}{2}\left[1 - \ell^{-2\beta t}\right] \tag{13}$$

and the expected number of failures in $(0, t]$ is

$$H(t) = \frac{\beta t}{2} - \frac{1}{4}\left[1 - \ell^{-2\beta t}\right] \tag{14}$$

From equation (5)

$$(1 + 2\beta t)\ell^{-2\beta t} = 1 - \frac{4G(x)c_p}{c_s} \tag{15}$$

An optimal value of t is that which satisfies equation (15), to determine that value we suggest the use of numerical approximation in this case. Further, assume $G(x) = 0.95$, $c_s = 4$, $c_p = 1$ and $\beta = 1$, then from equation (15).

$$(1 + 2t)\ell^{-2t} = 0.05 \quad ; \quad t > 0$$

this give approximate value of t^* as 2.37. The expected number of failures in the interval (0, 2.37] is $H(2.37) \cong 1$ failure (since $H(t)$ is integral). Therefore, the mean replacement cost is $C(2.37) \cong 2.20$.

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