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AN ALGORITHMIC COLLOCATION APPROACH FOR DIRECT SOLUTION OF SPECIAL AND GENERAL FOURTH - ORDER INITIAL VALUE PROBLEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT

This paper discusses a Multiderivative Collocation Method for direct solution of general initial value problems in ordinary differential equations of the form

$$y^{(n)}(x) = f(x, y, y', y'', \dots, y^{n-1}), y(a) = y_0 y^i(a) = y_i$$

i = 1,2,3. Our approach for the development of the method is based essentially on collocation of the differential system obtained from the basis function of degree n = 6 at the selected equidistant grid points. This is necessary in order to ensure symmetry in the method, which is essential for solving oscillatory initial value problems. Furthermore, a predictor for the calculation of the value of y_{n+k} for k = 4, and its derivatives that appear in the main method is developed. Taylor series expansion is used to calculate the values of $y_{n+,i}$ i = 1, 2, 3 and their derivatives which also appear in the main method. The interval of periodicity and the error constant of the method at $x = x_{n+4}$ are calculated. Evaluation of the proposed method at $x = x_{n+4}$ gives a particular discrete scheme as a special case of the method. Finally the efficiency of the method is tested on non-stiff initial value problems.

Keywords: Multiderivative Method, Numerical integrator, Collocation, Predictor, equidistant grid points, interval of periodicity, error constant
C. R. CATEGORIES: G 1.7

1 INTRODUCTION

Higher order ordinary differential equation of the form

$$y^{(n)} = f(x, y, y', \dots, y^{n-1}), y(a) = y_n, y^i(a) = y_i \quad i = 1, 2, \dots, n-1 \quad ()$$

for stepnumber k = 4 is considered in this paper. This class of problems h a lot of application in science and Engineering especially in Mechanical sy ns without dissipation and celestial mechanics. In practice the problem re reduced to first order system of equations and methods for first order are u to solve them (see for example Lambert (1973), Fatunla (1988), Sarafyan (0), Lambert (1991), Wright, et al (1991), Awoyemi (1993), Onumanyi et al

(1999), Lambert and Watson (1976). However, experience has shown in Awoyemi (1996), 1999, 2001) that this approach wastes a lot of computer time and human effort. Apart from the main program, separate sub programs are developed for the starting values and functions in the system of equations arising from (1.1). At this stage, many promising numerical integrators are wasted for lack of adequate knowledge by the author to produce a workable program to test his method.

In our last papers Awoyemi (1999, 2001) problem (1.1) for n = 2 in the form

$$y'' = f(x,,y,y'),\ y(a) = y_o,\ y'(a) = \eta \tag{1.2}$$

was solved directly without reducing it to a system of first order equations. This is possible because of the continuity of the coefficients of the method which allows derivatives of y with respect to x to be computed to any desired order. In the same paper the interval of periodicity was located on the negative x - axis (see Awoyemi (2001)) which is most desirable.

In the present work, the method Awoyemi (2001) is extended to handle directly (1.1) for n = 4 without reducing the problem to a system of first order equations. Thus equation (1.1) becomes

$$y^{(4)}(x) = f(x,y,y',y'',y'''),y(a) = y_o\ y^{(i)}(a) = y_i,\ i = 1,2,3;\ a \leq x \leq b,\ f,\ y \in R \tag{1.3}$$

It is, however, noted that unlike our previous method for equation (1.2) which has its interval of absolute stability on the negative x - axis, the present method has a larger interval of periodicity on the positive x - axis. This is in conformity with Multiderivative Methods for initial value problems (Fatunla (1988)). This class of methods is unique for their good accuracy and stability properties (see Wanner et al. (1978) Fatunla (1988) observed in Brown (1977), Jeltsch (1976), Jeltsh et al (1978), Lambert (1973), Enright (1974), and Twizell and Khaliq (1981, 1984) that Multiderivative Methods give high accuracy and possess good stability properties when used to solve first order initial value problems. Twizell and Khalid (1984) proposed a class of p-stable two - step higher derivative formulae for the special second order initial value problems of the form

$$y''(x) = f(x,y),\ y(a) = y_o,\ y'(a) = \eta, \tag{1.4}$$

The new method is also adopted to solve a special fourth order initial value problems of ordinary differential equation of the form.

$$y^{(4)} = f(x,y), y(a) = y_o,\ y^{(i)}(a) = y_i,\ i = 0, 1, 2, 3 \tag{1.5}$$

2. THE METHOD OF SOLUTION

In Awoyemi (1993, 1996, 1999, 2001). Canonical Polynomials were constructed as basis functions from the given initial value problems. Similarly for the present work, we construct Canonical Polynomial from problem (1.3) as follows

An operator L from equation (1.3) is defined as

$$L = 1 + \sum_{i=1}^{4} \frac{d^i}{dx^i} \tag{2.1}$$

From this definition, equation (1.3) is put as

$$L_y = \left(1 + \sum_{i=1}^{4} \frac{d^i}{dx^i}\right)y = f(x,y,y',y'',y''') + \left(1 + \sum_{i=1}^{3} \frac{d^i}{dx^i}\right)y \tag{2.2}$$

Also, the canonical polynomial used is defined as

$$L\psi_j(x) = x^j, \; j \geq 0 \tag{2.3}$$

which leads to the recurrence relation

$$\psi_j(x) = x^j - j\psi_{j-1}(x) - j(j-1)\,\psi_{j-2}(x) - j(j-1)(j-2)\,\psi_{j-3}(x) - j(j-1)(j-2)(j-3)\,\psi_{j-4}(x), j \geq 0 \tag{2.4}$$

Thus in this paper an approximate solution of the form

$$y_k(x) = \sum_{j=0}^{k+2} a_j\,\psi_j(x) \tag{2.5a}$$

whose higher derivatives are given as

$$y'(x) = \sum_{j=1}^{k+2} j a_j \psi_{j-1}(x) \tag{2.5b}$$

$$y''_k(x) = \sum_{j=2}^{k+2} j\,(j-1) a_j \psi_{j-2}(x) \tag{2.5c}$$

$$y'''_k(x) = \sum_{j-3}^{k+2} j\,(j-1)\,(j-2)\psi_{j-3}(x) \tag{2.5d}$$

$$y^{(4)}(x) = \sum_{j=4}^{k+2} j(j-1)(j-2)(j-3)\,\psi_{j-4}(x) \qquad (2.5e)$$

where $\psi_j(x)$ is the Canonical Polynomial and a_j, $j = 0, 1, 2, \ldots$ are the parameters to be determined. Hence on substituting equation (2.5 a,b,c,d,e) into equation (1.3), we get

$$\sum_{j=4}^{k+2} j(j-1)(j-2)(j-3)\psi_{j-4}(x) = f(x, y_k(x), y'_k(x), y''_k(x), y'''_k(x)) \qquad (2.6)$$

where, $y_k(x)$, $y'_k(x)$, $y''_k(x)$ and $y'''_k(x)$ have the values given in (2.5).
Thus collocating (2.6) at the grid points $x = x_{n+j}, (j = 0,2,4)$ and interpolating equation (2.5a) at the grid points $x = x_{n+j}$ $(j = 0, 1,2,3)$ for $k=4$, we have a system of equations

$$\sum_{j=4}^{k+2} j(j-1)(j-2)(j-3)\, a_j\, \psi_{j-4}(x_{n+i}) = f_{n+i,}\ i = 0,2,4 \qquad (2.7)$$

$$\sum_{j=0}^{k+2} a_j\, \psi_j(x_{n+i}) = y_{n+i,} \quad i = 0,1,2,3 \qquad (2.8)$$

which when solved either by matrix or Gaussian elimination method to obtain the values of the parameters a_j and then substituting them into (2.5a) give a scheme expressed in the form

$$y_k(x) = \sum_{j=0}^{k-1} \alpha_j(x)\, y_{n+j} + \sum_{j=0}^{k-2} \beta_{2j}(x)\, f_{n+2j} \qquad (2.9)$$

for (1.3) and (1.5) respectively,
where $f_{n+2j} = f(x_{n+2j}, y_{n+2}, y'_{n+2j}, y''_{n+2j}, y'''_{n+2j})$ in (1.3) and $f_{n+2j} = f(x_{n+2j}, y_{n+2j})$ in (1.5) respectively
If we now let

$$t = (x - x_{n+3})/h \qquad (2.10)$$

The following continuous coefficients in (2.9) are obtained as follows:

$$\alpha_0(t) = \frac{1}{6}\,(t^3 + 3t^2 + 2t),$$

$$\alpha_1(t) = \frac{1}{2}(t^3 + 4t^2 + 3t),$$

$$\alpha_2(t) = -\frac{1}{2}(t^3 + 5t^2 + 6t),$$

$$\alpha_3(t) = \frac{1}{6}(t^3 + 6t^2 + 11t + 6),$$

$$\beta_0(t) = \frac{h^4}{2880}(t^6 - 15t^4 + 74t^2 + 60t),$$

$$\beta_2(t) = \frac{h^4}{1440}(-t^6 - 6t^5 + 45t^4 + 330t^3 + 616t^2 + 336t)$$

$$\beta_4(t) = \frac{h^4}{2880}(t^6 + 12t^5 + 45t^4 + 60t^3 + 14t^2 - 12t). \qquad (2.11)$$

The first, second and third derivatives of (2.11) are given as follows noting that in (2.10), $\frac{dt}{dt} = \frac{1}{h}$

First Derivative:

$$\alpha'_0(t) = -\frac{1}{6h}(3t^2 + 6t + 2),$$

$$\alpha'_1(t) = \frac{1}{2h}(3t^2 + 8t + 3),$$

$$\alpha'_2(t) = \frac{1}{2h}(3t + 10t + 6),$$

$$\alpha'_3(t) = \frac{1}{6h}(3t^2 + 12t + 11),$$

$$\beta'_0(t) = \frac{h^3}{2880}(6t^5 - 60t^3 + 148t + 60),$$

$$\beta'_2(t) = \frac{h^3}{1440}(-6t^5 - 30t^4 + 180t^3 + 990t^2 + 1232t + 336),$$

$$\beta'_4(t) = \frac{h^3}{2880}(6t^5 + 60t^4 + 180t^3 + 180t^2 + 28t - 12). \qquad (2.12)$$

Second Derivative:

$$\alpha''_0(t) = -\frac{1}{6h^2}(6t + 6),$$

$$\alpha''_1(t) = \frac{1}{2h^2}(6t + 8),$$

$$\alpha''_2(t) = -\frac{1}{2h^2}(6t + 10),$$

$$\alpha''_3(t) = \frac{1}{6h^2}(6t + 12),$$

$$\beta''_0(t) = \frac{h^2}{2880}(30t^4 - 180t^2 + 148),$$

$$\beta''_2(t) = \frac{h^2}{2880}(-60t^4 - 240t^3 + 1080t^2 + 3960t + 2464),$$

$$\beta''_4(t) = \frac{h^2}{2880}(30t^4 + 240t^3 + 540t^2 + 360t + 28). \qquad (2.13)$$

Third Derivative:

$$\alpha'''_0(t) = -\frac{1}{h^3},$$

$$\alpha'''_1(t) = \frac{3}{h^3},$$

$$\alpha'''_2(t) = -\frac{3}{h^3},$$

$$\alpha'''_3(t) = \frac{1}{h^3},$$

$$\beta'''_0(t) \ \frac{h}{2880}(120t^3 - 360t),$$

$$\beta'''_2(t) \ \frac{h}{2880}(-240t^3 - 720t^2 + 2160t + 3960),$$

$$\beta'''_4(t) = \frac{h}{2880}(120t^3 + 720t^2 + 1080t + 360). \qquad (2.14)$$

The discrete schemes emerging from equations (2.11), (2.12), (2.13) and (2.14) for $t = 1 \Rightarrow x = x_{n+4}$ are as follows:

$$y_{n+4} - 4y_{n+3} + 6y_{n+2} - 4y_{n+1} + y_n = \frac{h^4}{24}[f_{n+4} + 22f_{n+2} + f_n], \quad (2.15)$$

order is P = 6, error constant $C_{p+2} = -0.0431$, and interval of periodicity $x(\theta) = (0, 16)$ (See Twizell and Khaliq 1984, Fatunla 1988 p. 235). Similarly

$$y'_{n+4} = \frac{1}{6h}[26y_{n+3} - 57y_{n+2} + 42y_{n+1} - 11y_n] + \frac{h^3}{2880}[442f_{n+4} + 5404f_{n+2} + 154f_n] \quad (2.16)$$

$$y''_{n+4} = \frac{1}{6h^2}[18y_{n+3} - 48y_{n+2} + 42y_{n+1} - 12y_n] + \frac{h^2}{2880}[1198f_{n+4} + 7204f_{n+2} - 2f_n] \quad (2.17)$$

$$y'''_{n+4} = \frac{1}{h^3}[y_{n+3} - 3y_{n+2} + 3y_{n+1} - y_n] + \frac{h}{24}[19f_{n+4} + 43f_{n+2} - 2f_n] \quad (2.18)$$

where

$$f_{n+i} = f(x_{n+i}, y_{n+i}, y'_{n+i}, y''_{n+i}, y'''_{n+i}),\ i = 0, 1, 2, 3, 4 \quad (2.19)$$

3. THE PREDICTORS

The same collocation approach employed in the development of the main method is adopted for the predictor to calculate y_{n+4}, y'_{n+4}, y''_{n+4}, y'''_{n+4} which appear in the main method except that collocation is not done at $x = x_{n+k}$, $k = 4$. The resulting method is given as

$$y_k(x) = \sum_{j=0}^{k-1} \alpha_j(x) y_{n+j} + \sum_{j=1}^{k-1} \beta_j(x) f_{n+j}, \quad (3.1)$$

with the coefficients listed as follows:

$$\alpha_0(t) = -\frac{1}{6}(t^3+3t^2+2t),$$

$$\alpha_1(t) = \frac{1}{2}(t^3 + 4t^2 + 3t),$$

$$\alpha_2(t) = -\frac{1}{2}(t^3 + 5t^2 + 6t),$$

$$\alpha_3(t) = \frac{1}{6}(t^3 + 6t^2 + 11t + 6),$$

$$\beta_1(t) = \frac{1}{720}[t^6 + 3t^5 + 15t^3 + 59t^2 + 42t].$$

$$\beta_2(t) = \frac{1}{360}[-t^6 - 6t^5 + 60t^3 + 121t^2 + 66t],$$

$$\beta_3(t) = \frac{1}{720}[t^6 + 9t^5 + 30t^4 + 45t^3 + 29t^2 + 6t]. \qquad (3.2)$$

Putting $t = 1 \rightarrow x = x_{n+4}$ in (3.2), we have a symmetric scheme given by

$$y_{n+4} - 4y_{n+3} + 6y_{n+2} - 4y_{n+1} + y_n = \frac{h^4}{6}[f_{n+3} + 4f_{n+2} + f_{n+1}] \qquad (3.3)$$

$P = 6$, $C_8 = \frac{1}{720}$, interval of periodicity $X(\theta) =$ (0,48) (Twizell and Khaliq 1984, Fatunla 1988 pp. 135.

Also using (3.2) the computed first, second and third order derivatives of equation (3.1) are given as follows:

$$y'_{n+4} = \frac{1}{6h}[26y_{n+3} - 57y_{n+2} + 42y_{n+1} - 11y_n] = \frac{h^3}{360}[185f_{n+3} + 452f_{n+2} + 113f_{n+1}] \qquad (3.4)$$

$$y''_{n+4} = \frac{1}{h^2}[3y_{n+3} . 8y_{n+2} + 7y_{n+1} - 2y_n] + \frac{h^2}{360}[449f_{n+3} + 452f_{n+2} + 149f_{n+2} \qquad (3.5)$$

$$y''' = \frac{1}{h^3}[y_{n+3} - 3y_{n+2} + 3y_{n+1} - y_n] + \frac{h}{24}[f_{n+3} - 8f_{n+2} + 13f_{n+1}] \qquad (3.6)$$

Taylor series expansion is also used to calculate the values of y_{n+1}, y_{n+2} and y_{n+3} and their first, second and third order derivatives at $x = x_n$ in (2.19) as follows:

$$y_{n+i} = y(x_n+ih) = y(x_n) + ihy'(x_n) + \frac{(ih)^2}{2!}y''(x_n) + \frac{(ih)^3}{3!}y'''(x_n) + \frac{(ih)^4}{4!}f_n + \frac{(ih)^5}{5!}f'_n + \frac{(ih)^6}{6!}f''_n + \ldots;$$

$$y'(x_n+ih) = y'(x_n) + ih\,y''(x_n) + \frac{(ih)^2}{2!}y'''(x_n) + \frac{(ih)^3}{3!}f_n + \frac{(ih)^4}{4!}f_n' + \frac{(ih)^5}{5!}f_n'' + \ldots;$$

$$y''(x_n+ih) = y''(x_n) + ih\, y'''(x_n) + \frac{(ih)^2}{2!} f_n + \frac{(ih)^3}{3!} f_n' + \frac{(ih)^4}{4!} f_n'',$$

$$y'''(x_n+ih) = y'''(x_n) + ih\, f_n + \frac{(ih)^2}{2!} f_n' + \frac{(ih)^3}{3!} f_n'', \quad i = 1, 2, 3$$

where $f_n = f(x_n, y_n, y'_n, y''_n, y'''_n)$

To ensure good accuracy of our method the expansion by Taylor series is extended to second total derivatives of f with respect to x, as shown above. Furthermore, the values of f'_n and f''_n are evaluated by the method of partial derivatives as follows:

We write (2.19) in the form
$f = f(x, y, y', y'' y''')$
(3.4)

$$\therefore f' = \frac{df}{dx} = \frac{\partial f}{\partial x} + y' \frac{\partial f}{\partial y} + y'' \frac{\partial f}{\partial y'} + y''' \frac{\partial f}{\partial y''} + f \frac{\partial f}{\partial y'''}, \quad y = \frac{d^i y}{dx^i}, \quad i = 1, 2, 3$$

(3.5)

Similarly

$$f'' = \frac{d^2 f}{dx^2} = 2\,(Ay' + By'' + Cy''' + Df) + E + F, \tag{3.6}$$

where,

$$A = \frac{\partial^2 f}{\partial x \partial y} + y'' \frac{\partial^2 f}{\partial y \partial y'} + y''' \frac{\partial^2 f}{\partial y y''} + f \frac{\partial^2 f}{\partial y \partial y'}$$

$$B = \frac{\partial^2 f}{\partial x \partial y'} + y \frac{\partial^2 f}{\partial y' \partial y''} + f \frac{\partial^2 f}{\partial y' \partial y'''}$$

$$C = \frac{\partial^2 f}{\partial x \partial y''} + f \frac{\partial^2 f}{\partial y'' \partial y'''}$$

$$D = \frac{\partial^2 f}{\partial x \partial y'''}$$

$$E = y'' \frac{\partial f}{dy} + y''' \frac{\partial f}{dy'} + f \frac{\partial f}{dy''} + f' \frac{\partial f}{dy'''}$$

$$F = \frac{\partial^2 f}{\partial x^2} + (y')^2 \frac{\partial^2 f}{\partial y^2} + (y'')^2 \frac{\partial^2 f}{\partial y'^2} + (y''')^2 \frac{\partial^2 f}{\partial''^2} + f^2 \frac{\partial^2 f}{\partial y'''^2}$$

4. TEST PROBLEMS

Five test problems, four linear and one non-linear are solved using a mesh size of $h = 1/320$ for $n = 10$, which automatically changes with different n to test the efficiency of this method. While the errors arising from problems 1-4 were not necessarily compared with the errors arising from another different method, the maximum errors for different n iterations, $n \geq 3$, are recorded at the values of x shown. However, the errors recorded by the new method for problems 5 were compared with the errors earlier recorded for the same problem by Wright et al (1991) for different n at Chebyshev points under the caption SINFLA and SINFL B.

Problems considered:

(i) $y^{iv} = (y')^2 - y\, y''' - 4x^2 + e^x(1+x^2-4x);\ 0 \leq x \leq 1$

$y(0) = 1,\ y'(0) = 1,\ y''(0) = 3,\ y'''(0) = 1$

Theoretical solution: $y(x) = x^2 + e^x$

(ii) $y^{iv} + y'' = 0 \quad 0 \leq x, \leq \pi/2$

$y(0) = 0,\ y'(0) = \dfrac{1.1}{72-50\Pi}\quad y''(0) = \dfrac{1}{144-100\Pi},\quad y''(0) = \dfrac{1.2}{144-100\Pi}$

Theoretical Solution: $y(x) = \dfrac{1-x-\cos x - 1.2\sin x}{144-100\Pi}$

(iii) $y^{(4)} + 2y'' + y = 3x + 4,\ y(0) = y'(0) = 0;\ y''(0) = y'''(0) = 1$

Theoretical solution $y(x) = (x-4)\cos x - (1.5x+4)\sin x + 3x + 4$

(iv) $y^{iv} = Q(x)/EI,\ 0 \leq x, \leq 1$

$y(0) = 0,\ y'(0) = \varphi(x)/48,\ y(0) = 0,\ y(0) = -3\varphi/8$

$\varphi(x) = 1,\ E = 1,\ I = 1$

Theoretical Solution: $y(x) = \dfrac{\varphi}{48}(2x^4 - 3x^3 + x)$

(v) $y^{iv} = \alpha x y''' - y + e^x(2+\alpha x),\ 0 \leq x \leq 1$

$y(0) = 1$, $y'(0) = 1$, $y''(0) = 1$, $y'''(0) = 1$, $\alpha = 1$ (see **Khaliel et al** 19

Theoretical solution: $y(x) = e^x$

NUMERICAL SOLUTION OF TEST PROBLEMS 1, 2, 3, 4, & 5

TABLE 1
PROBLEM 1

n	X	Y – EXACT	Y - COMPUTED	ERROR
3	1.057292	0.3996429979D+01	0.3996429479D+01	5.01D - 07
5	1.029167	0.3857916613D+01	0.38579165489D+01	2.42D - 08
6	1.017857	0.3803291730D+01	0.3803291721D+01	8.22D - 09
10	1.009375	0.376273441D+01	0.3762723441D+01	5.30D - 10
15	1.007552	0.3754049433D+01	0.3754049433D+01	5.39D - 11
17	1.006536	0.3749221178D+01	0.37499221178D+01	2.95D - 11

TABLE 2
PROBLEM 2

n	X	Y-EXACT	Y-COMPUTED	ERROR
3	1.065972	0.8783220719D+00	0.8783219648D+00	1.07D - 07
5	1.031250	0.8096679708D+00	0.8096679672D+00	3.57D-09
6	1.019097	0.7865068294D+00	0.7865068283D+00	1.10D-09
10	1.009375	0.7682966326D+00	0.7682966326D+00	3.38D-11
15	1.007639	0.7650744186D+00	0.7650744186D+00	1.35D-11
17	1.006596	0.7631431579D+00	0.7631431579D+00	1.03D-11

TABLE 3
PROBLEM 3

n	X	Y-EXACT	Y-COMPUTED	ERROR
3	1.057292	0.2251834352D-03	0.2250032620D-03	1.8D-07
5	1.029167	0.5578313129D-04	0.5577450806D-04	8.62D-09
6	1.017857	0.2052723393D-04	0.2052432017D-04	2.91D-09
10	1.009375	0.5579316616D-05	0.5579141643D-05	1.75D-10
15	1.007552	0.3609625416D-05	0.3609611353D-05	1.41D-11
17	1.006536	0.2699073341D-05	0.2699066405D-05	6.94-12

TABLE 4
PROBLEM 4

n	Tau Method	Finite difference method	Multi-derivative collocation method
3	———	———	0.157D - 07
4	———	———	0.233D - 08
5	———	———	0.516D - 09
6	———	———	0.151D - 09
7		———	0.499D -10
8	0.212 E -5	0.141 E -5	0.183D - 10
9	0.797 E - 8	———	0.636D - 11
10	0.129 E-8	0.330 E -6	0.191D - 11
11	0.412 E -11	0.725 E -7	0.444D - 13

TABLE 5
PROBLEM 5, $\alpha = 1$

n	SINFL A	SINFL B	NEW METHOD
3	3.00E - 8	3.00E – 8	5.03D - 7
5	3.69E - 8	3.69E – 8	2.43D - 8
6	-	1.66E –9	8.26D - 9
8	1.46E - 9	2.41E – 9	1.58D - 9
10	2.18E - 9	-	5.41D - 10
11	4.70E - 10	4.71E - 10	2.75E - 10
17	4.92E - 10		3.17E - 11

CONCLUSION

A collocation approach which produces a family of order six multi-derivative method with large interval of periodicity (see equations 2.15 and 3.3) has been proposed to solve problems (1.3) and (1.5) respectively. Five test examples, four linear and one non-linear were solved to test the efficiency of the method. In Tables 4 and 5, the errors arising from problems 4 and 5 using the new method were compared with those obtained by Onumanyi and Ortiz (1982) and Wright et al (1991) who earlier solved the same problem.

A close look at the results in Table 4 and 5 shows that the new method gives better results for these problems. Furthermore, apart from being symmetric and therefore useful for oscillatory initial value problems, the new method had only three functions to be evaluated per iteration and thus reduces computational burden and cost implication (see Awoyemi (1999, 2001)).

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