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COMPUTATION OF RESISTIVITY TRANSFORM DERIVATIVES IN GEOPHYSICAL SOUNDING

J.C. EGBAI
Department of Physics
Delta State University, Abraka, Nigeria.

ABSTRACT

The resistivity transform derivative deals with the computation of resistivity transform data for an assumed model of the earth. It can be calculated readily from theoretical models. Some of the difficulties in the interpretation process may be reduced if each field apparent resistivities curve is converted into an equivalent resitivity transform curve.

The theory based on an earth model of n layers were analysed. Interpretation which operate in the resistivity transform domain benefit from the use of forward linear filter to effect a quick transformation from the domain of measured apparent resistivities.

Data were collected from about 57 vertical soundings. These data were incorporated into a computer programme where theoretical as well as field curves were generated and analysed.

1. INTRODUCTION

The resistivity transform derivatives deals with the computation of resistivity transform data for an assumed model of the earth. As a basis for interpretation, resistivity transform function may be used. The letter has a significant advantage because it may be calculated more readily for theoretical models. Therefore, some of the difficulties in the interpretation process may be reduced if each field apparent resistivities curve is converted into an equivalent resistivity transform curve. The conversion procedure before 1971 was not simple. Koefoed (1965) decomposed the field curve into a sum of partial apparent resistivity functions and then obtained the equivalent transform curve as a sum of corresponding partial transform functions while others (Meinardus, 1970; Chan, 1970) preferred to calculate the resistivity transform directly by numerical integration.

Koefoed (1970) adopted the resistivity transform function as a supplement for the "raised kernel function". The function is as shown

$$T(\lambda) = 2\ell H^*(\lambda) \qquad (1)$$

where ℓ is resistivity, H is the thickness and λ is the anisotropy. This he defined recursively for an assumed earth model. The resistivity transform function in related directly to the Stefanesco and Slichter Kernels. It also depends only on the thickness and resistivities of subsurface layers and is independent of the electrode array used in the field.

This resistivity transform function utilized in this work is derived after Ghosh (1971). If the observed apparent resistivity is given by

$$\ell_a(L) = L^2 \int^{\infty} T(\lambda)\, J_1(\lambda L)\, d\lambda \qquad (2)$$

Then the function is given by Ghosh (1977) as:

$$T(\lambda) = \int \left(\frac{1}{r}\right) \ell_a(r) J_1(\lambda r) dr \qquad (3)$$

where J_1 is the first – order Bessel function of the first kind, and T (λ) is the transformed resistivity data.

Slichter (1933) and Langer (1933) derived the potential at the surface of a stratified earth in terms of the resistivity transform function as

$$V(r) = \frac{I\ell_1}{2\pi} \int_0^{\infty} T(\lambda) J_0(\lambda r)\, d\lambda \qquad (4)$$

where T (λ) is the Slichter Kernel, J_0 is the zero – order Bessel function of the first kind, r is distance and $\lambda = \left(\ell t/\ell i\right)^{1/2} = \left(\sigma/\sigma i\right) = arisotropy$

ℓ is resistivity while σ is conductivity.

The subscript ℓ means parallel to the layers while

The subscript t means tangential to the layers.

Hankel inversion of this integral gives the transform function explicitly in terms of the apparent resistivity functions:

$$T(\lambda) = \int_0^{\infty} L^{-1} \ell a(L) J_1(\lambda L) dL \qquad (5)$$

The above integrals for a (L) and T (λ) represent the inverse and forward cases respectively.

For a horizontally – stratified earth model consisting of n homogenous and isotropic layers, the resistivity transform is given by

$$Ti+1 = \frac{Ti + T^1_{n-i}}{1 + TiT^1_{n-1}/\ell^2_{n-1}} \qquad (6)$$

$$where T^1_{n-1} = \ell_{n-1} \left\{ \frac{1 - \exp(-2\lambda h_{n-i})}{1 + \exp(-2\lambda h_{n-i})} \right\} \qquad (7)$$

$$and T_2 = \ell_1 \left\{ \frac{1 + K_{12}\exp(-2\lambda h_1)}{1 - K_{12}\exp(-2\lambda h_1)} \right\} \quad \text{------(8)}$$

The transform is built up layer by layer starting from the surface. T_2 is the transform of the top row layers, T_3 of the top three layers and so on, down to a value T_n which is the transform of the whole section.

THEORY

An earth model of n layers is characterized by 2n – 1 parameters (ℓ_j : j = 1, 2, - - - n – 1, n, n + 1 - - - 2n – 1). The first n – 1 parameters are thickness (hi :i = 1, 2 - - -, n – 1) and the next n parameters are resistivities ((ℓ_i : i = 1, 2, - - - n). The resistivity transform function is defined by

$$T_{i+1} = \frac{T_i + T^1_{n-i}}{T + T_i T^1_{n-i} / \ell^2_{n-i}} \qquad i = 2, 3, \text{---}, n-1 \quad \text{------(9)}$$

where

$$T^1_{n-i} = \ell_{n-i}\left[\frac{1 - \exp(-2\lambda h_{n-i})}{1 + \exp(-2\lambda h_{n-i})}\right] = \ell_{n-i}\tanh(\lambda h_{n-i}) \quad \text{------(10)}$$

and

$$T_2 = \ell_1\left[\frac{1 + K_{12}\exp(-2\lambda h_1)}{1 - K_{12}\exp(-2\lambda h_1)}\right] \quad \text{------(11)}$$

The resistivity transform derivative with respect to any one model paramenter is defined by:

$$\left(\frac{\partial T_{i+1}}{\partial P_j}\right) = \frac{\left(1 + T_i T^1_{n-i}/\ell^2_{n-i}\right)\frac{\partial}{\partial P_j}\left(T_i + T^1_{n-i}\right) - \left(T_i + T^1_{n-i}\right)\frac{\partial}{\partial P_j}\left(1 + T_i T^1_{n-i}/\ell^2_{n-i}\right)}{\left(1 + T_i T^1_{n-i}/\ell^2_{n-i}\right)^2} \quad \text{----(12)}$$

where the partial derivatives on the right simplify as follows:

$$\frac{\partial}{\partial P_j}\left(T_i + T^1_{n-i}\right) = \left(\frac{\partial T_i}{\partial P_j}\right) + \left(\frac{\partial T^1_{n-i}}{\partial P_j}\right) \qquad (13)$$

$$\frac{\partial}{\partial P_j}\left(T_i T^1_{n-i}/\ell^2_{n-i}\right) = \left(T^1_{n-i}/\ell^2_{n-i}\right)\left(\frac{\partial T_i}{\partial P_j}\right) + \left(T_i/\ell^2_{n-i}\right)\left(\frac{\partial T^1_{n-i}}{\partial P_j}\right) - \left(2T_i T^1_{n-i}/\ell_{n-i}\right)\left(\frac{\partial \ell^1_{n-i}}{\partial P_j}\right) \qquad (14)$$

$$\left(\frac{\partial T^1_{n-i}}{\partial P_j}\right) = \tanh(\lambda h_{n-i})\left(\frac{\partial \ell^1_{n-i}}{\partial P_j}\right) + \ell_{n-i}\frac{\partial}{\partial P_j}\left(\tanh(\lambda h_{n-i})\right)$$

$$= \tanh(\lambda h_{n-i})\left(\frac{\partial \ell^1_{n-i}}{\partial P_j}\right) + \ell_{n-i}\, Sech^2(\lambda h_{n-i})\frac{\partial}{\partial P_j}(\lambda h_{n-i})$$

$$= \frac{1-\exp(-2\lambda h_{n-i})}{1+\exp(-2\lambda h_{n-i})}\left[\frac{\partial \ell_{n-i}}{\partial P_j}\right] + \frac{4\lambda \ell_{n-i}\exp(-2\lambda h_{n-i})}{\{1+\exp(-2\lambda h_{n-i})\}^2}\left(\frac{\partial h_{n-i}}{\partial P_j}\right) \qquad (15)$$

By using equations (13), (14) and (15) equation (12) becomes:

$$\frac{\partial T_{i+1}}{\partial P_j} = \left(\frac{\partial T_i}{\partial P_j}\right)\left\{1 + \left(T_i T^1_{n-i}/\ell^2_{n-i}\right) - \left(T_i T^1_{n-i}\right)T^1_{n-i}/\ell^2_{n-i}\right\}$$

$$+ \left(\frac{\partial h_{n-i}}{\partial P_j}\right)\left(\lambda \ell_{n-i}\right)Sech^2(\lambda h_{n-i})\left(1 + \left(T_i T^1_{n-i}/\ell^2_{n-i}\right) - \left(T_i + T^1_{n-i}\right)T_i/\ell^2_{n-i}\right)$$

$$+ \left(\frac{\partial h_{n-i}}{\partial P_j}\right)\left(\tanh(\lambda h_{n-i})\right)\left(1 + \left(T_i T^1_{n-i}/\ell^2_{n-i}\right) - \left(T_i + T^1_{n-i}\right)T_i/\ell^2_{n-i}\right)$$

$$+ 2\left(T_i + T^1_{n-i}\right)T_i T^1_{n-i}/\ell^3_{n-i}\Big) + \left(T_i + T^1_{n-i}\right)T_i/\ell^2_{n-i}\Big)^2 \qquad (16)$$

Finally, the transform derivative is given by:

$$\left(\frac{\partial T_{i+1}}{\partial P_j}\right)=\left(\frac{\partial T_i}{\partial P_j}\right)\left[1-\left(\frac{T^1_{n-i}}{\ell_{n-i}}\right)^2\right]+\left(\frac{\partial h_{n-i}}{\partial P_j}\right)\left\{\frac{4\lambda\ell_{n-i}\exp(-2\lambda h_{n-i})}{(1+\exp(-2\lambda h_{n-i}))^2}\left(1-\left(\frac{T_i}{\ell_{n-i}}\right)^2\right)\right\}$$

$$+\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right)\left(\frac{1-\exp(-2\lambda h_{n-i})}{1+\exp(-2\lambda h_{n-i})}\right)\left(1-\left(\frac{T_i}{\ell_{n-i}}\right)^2\right)+\frac{2T_iT^1_{n-i}(T_i+T^1_{n-i})}{\ell^3_{n-i}}\Bigg]$$

$$+\left(1+\frac{T_iT^1_{n-i}}{\ell^2_{n-i}}\right)^2 \quad \text{--(17)}$$

Note, in particular that:

$$\left(\frac{\partial h_{n-i}}{\partial P_j}\right)=\begin{vmatrix}0 & P_j\neq h_{n-i}\\ 1 & P_j=h_{n-i}\end{vmatrix} \quad \text{--(18)}$$

$$\left(\frac{\partial \ell_{n-i}}{\partial P_j}\right)=\begin{vmatrix}0 & P_j\neq \ell_{n-i}\\ 1 & P_j=\ell_{n-i}\end{vmatrix} \quad \text{--(19)}$$

In the particular case where $P_j \neq \ell_{n-i}, h_{n-i}$, the transform derivative reduces to:

$$\left(\frac{\partial T_{i+1}}{\partial \ell_j}\right)=\frac{\ell^2_{n-i}\left(\ell^2_{n-i}-T^1_{n-i}\right)}{\left(\ell^2_{n-i}+T_iT^1_{n-i}\right)^2}\left(\frac{\partial T_i}{\partial P_j}\right) \quad \text{--(20)}$$

Equation (9) is not a complete definition of the transform derivative because it does not include the terminal value (i = 1). This is found by differentiating equation (11)

$$\left(\frac{\partial T_2}{\partial P_i}\right)=\left(\frac{1+K_{12}\exp(-2\lambda h_1)}{1-K_{12}\exp(-2\lambda h_1)}\right)\left(\frac{\partial \ell_1}{\partial P_i}\right)+\ell_1\frac{\partial}{\partial P_i}\left(\frac{1+K_{12}\exp(-2\lambda h_1)}{1-K_{12}\exp(-2\lambda h_1)}\right) \quad (21)$$

The partial derivative on the far right of equation (21) reduces as follows:

$$=\left(\frac{1+K_{12}\exp(-2\lambda h_1)}{1-K_{12}\exp(-2\lambda h_1)}\right)+\frac{2\ell_1}{(1-K_{12}\exp(-2\lambda h_1))^2}\frac{\partial}{\partial P_i}-(1-K_{12}\exp(-2\lambda h_1))$$

$$\frac{\partial}{\partial P_i}(K_{12}\exp(-2\lambda h_1))=\exp(-2\lambda h_1)\left(\frac{\partial K_{12}}{\partial P_i}\right)+K_{12}\frac{\partial}{\partial P_i}(K_{12}\exp(-2\lambda h_1))$$

$$=\exp(-2\lambda h_1)\frac{\partial}{\partial P_i}\left(\frac{\ell_2-\ell_1}{\ell_2+\ell_1}\right)-2\lambda K_{12}\exp(-2\lambda h_1)\left(\frac{\partial h_1}{\partial P_i}\right) \quad (22)$$

$$\textit{where}\ \frac{\partial}{\partial P_j}\left(\frac{\ell_2-\ell_1}{\ell_2+\ell_1}\right)=(\ell_2+\ell_1)^{-2}\left\{(\ell_2+\ell_1)\frac{\partial}{\partial P_j}(\ell_2-\ell_1)-(\ell_2-\ell_1)\frac{\partial}{\partial P_j}(\ell_2+\ell_1)\right\}$$

$$=2(\ell_2+\ell_1)^{-2}\left\{\ell_1\left(\frac{\partial \ell_2}{\partial P_j}\right)-\ell_2\left(\frac{\partial \ell_1}{\partial P_j}\right)\right\} \quad (23)$$

Applying equations (22) and (23) to equation (21), the terminal derivative becomes:

$$\left(\frac{\partial T_2}{\partial P_i}\right)=\left(\frac{\partial h_1}{\partial P_i}\right)\left(\frac{-4\ell_1\lambda K_{12}\exp(-2\lambda h_1)}{\{1-K_{12}\exp(-2\lambda h_1)\}^2}\right)+\left(\frac{\partial \ell_2}{\partial P_i}\right)\left(\frac{2\ell_1\exp(-\lambda h_1)}{(\ell_1+\ell_2)\{1-K_{12}\exp(-2\lambda h_1)\}}\right)^2$$

$$+\left(\frac{\partial \ell_1}{\partial P_i}\right)\left(\frac{(\ell_1+\ell_2)^2\{1-K_{12}^2\exp(-4\lambda h_1)\}-4\ell_1\ell_2\exp(-2\lambda h_1)}{(\ell_1+\ell_2)^2\{1-K_{12}\exp(-2\lambda h_1)\}^2}\right) \quad (24)$$

The three partial derivatives on the right of equation are either zero or unity generalizing h_1, $_1$ and $_2$ by P_k.

$$\left(\frac{\partial P_k}{\partial P_i}\right)=\delta_{jk}=\begin{vmatrix}0 & j\neq k\\ 1 & j=k\end{vmatrix} \quad (25)$$

Applying the original recursion formula for the transform fun (as shown in equation 6), the transform derivatives may be expressed recur

$$\frac{\partial T_{i+1}}{\partial P_j} = \frac{1}{B}\left\{A_1\left[\frac{\partial T_n}{\partial P_j}\right] + A_2\left[\frac{\partial h_{n-1}}{\partial P_j}\right] + A_3\left[\frac{\partial \ell_{n-1}}{\partial P_j}\right]\right\} \text{----------} (26)$$

$$i = 2, 3, \text{-----------} n-1$$

$$j = 1, 2 \text{-----------} 2n-1$$

$$\frac{\partial T_2}{\partial P_j} = C_1\left[\frac{\partial h_i}{\partial P_j}\right] + C_2\left[\frac{\partial \ell_1}{\partial P_j}\right] + C_3\left[\frac{\partial \ell_2}{\delta P_j}\right]$$

$$\text{where } \left[\frac{\delta P_k}{\partial P_j}\right] = \delta_{jk} \text{ (The Kronecker delta)}$$

$$B = \left[1 + \frac{T_i T^1_{n-i}}{\ell^2}\right]^2$$

$$A_1 = 1 - \left[\frac{T^1_{n-i}}{\ell_{n-i}}\right]^2$$

$$A_2 = \lambda \ell_{n-i}\, Sech^2\left(\lambda h_{n-i}\right)\left\{1 - \left[\frac{T_i}{\ell_{n-i}}\right]^2\right\}$$

$$A_3 = \tanh\left(\lambda h_{n-i}\right)\left\{1 - \left[\frac{T_i}{\ell_{n-i}}\right]^2\right\} + \frac{2T_i T^1_{n-i}}{\ell^3_{n-i}}\left(T_i + T^1_{n-i}\right)$$

$$C_1 = \frac{-4\lambda \ell_1 K_{12} \exp(-2\lambda h h_1)}{\left[1 - K_{12}\exp(-2\lambda h_1)\right]^2}$$

$$C_2 = \frac{(\ell_1 + \ell_2)^2\left[1 - K^2_{12}\exp(-4\lambda h_1)\right] - 4\ell_1\ell_2\exp(-2\lambda h_1)}{(\ell_1 + \ell_2)^2\left[1 - K_{12}\exp(-2\lambda h_1)\right]^2}$$

$$C_3 = \left\{ \frac{2\ell_1 \exp(-2\lambda h_1)}{(\ell_1 + \ell_2)^2 [1 - K_{12} \exp(-2\lambda h_1)]^2} \right\}^2$$

EXPERIMENTAL WORK

The Schlumberger electrode configuration was utilized, Egbai and Asokhia (1998) as shown in fig. 1. The apparent resistivity value got for the field data was

$$\ell_3 = \frac{2\pi V}{I} \left\{ \frac{1}{\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{R_1} + \frac{1}{R_2}} \right\} \quad (27)$$

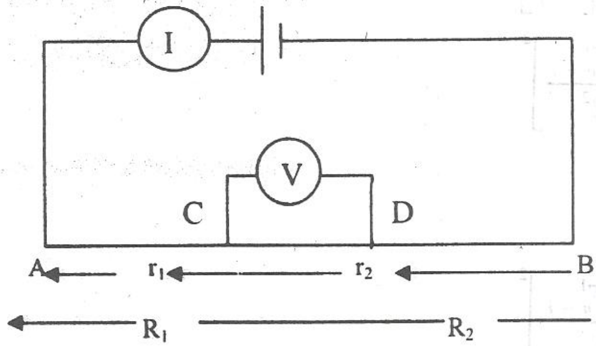

Fig. 1: Schlumberger electrode array. A and B are current electrodes while C and D are potential electrodes.

The result is independent of the positions of the electrode and is not affected when the current and potential electrodes are interchanged.

The research was carried out at Abraka P.O., Urhuoka and Erho all in Abraka Clan of Isiokolo L.G.A. of Delta State. Data were collected from about 57 vertical soundings.

By using equation (2) and (6) and incorporating them into a computer programme using eqn. 27, theoretical curves as well as field curves were generated.

RESULTS AND DISCUSSION

The behaviour of theoretical transform derivatives is illustrated in Figs. 2 to 7.

Figs. 2 and 3 show how the derivatives change as the depth of a two layer earth increases. Derivatives with respect to resistivity simply shift to the right as depth increases because the shape of the transform curves is unaffected by a change in depth. Derivastives with respect to depth decrease is magnitude as depth increases. The peaks in $\delta T/\delta h_1$ and $\delta T/\delta \ell$ show the transform curve is influenced most by the parameters h_1 *and* ℓ_1. It sho e observed that $\delta T/\delta \ell_2$ increases monotonically with λ^{-1}. This shows ℓ_2 has least influence at small L (small λ^{-1}) and most influence at large L (large λ^{-1}). For a conductive base, the transform derivatives are always confined between zero and unity.

The comparison between curves for a three layer earth is as shown in fig. 6. The transform derivative maxim indicate where each layer – parameter has the most effect on the observed data. At any position L, we can see which parameters control the transform curve. For example, h_1 *and* ℓ_1 have the most influence near L = 1, at L = 10, ℓ_1, h_1 *and* ℓ_1 have most effect on at L = 100, ℓ_3 is dominant but there is a small contribution from each parameter; at L = 1000, the transform curve is determined solely by ℓ_3.

Fig. 7 has similar explanation with fig. 6. The variations in ℓ_3 have great effect on the transform curve for L 30; however, variations in ℓ_4 have a negligible effect. At L = 1, ℓ_1, h_1 *and* h_2 have a significant effect; at L = 10, only h_2 *and* ℓ_3 are important. At L = 100, ℓ_3 is dominant but there are contributions from h_1 *and* h_2 *and* h_3 which add together to give the depth to bedrock. At L = 1000, ℓ_3 has an overwhelming effect but the contributions of h_1 *and* h_2 *and* h_3 have increased.

CONCLUSION

The resistivity transform function has been used as the basis for an analogy with electric filter theory which permits data to be exchanged readily between the apparent resistivity and transform domains.

Interpretation which operate in the resistivity transform domain benefit fiom the use of forward linear filter to effect a quick transformation from the domain of measured apparent resistivities. They benefit from the simple recursive formulation of the transform option and its derivatives in terms of the resistivities and thicknesses of the assumed model.

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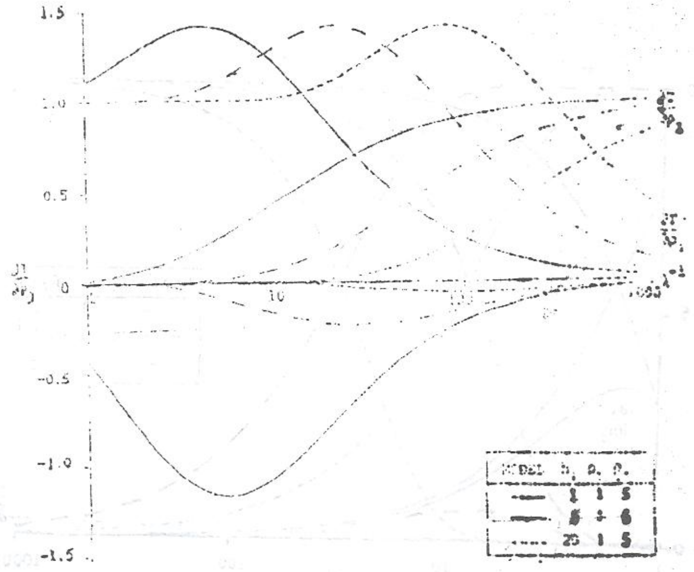

Fig. 2: Transform derivatives for a Two - layer Earth with variable Depth to Resistive Base

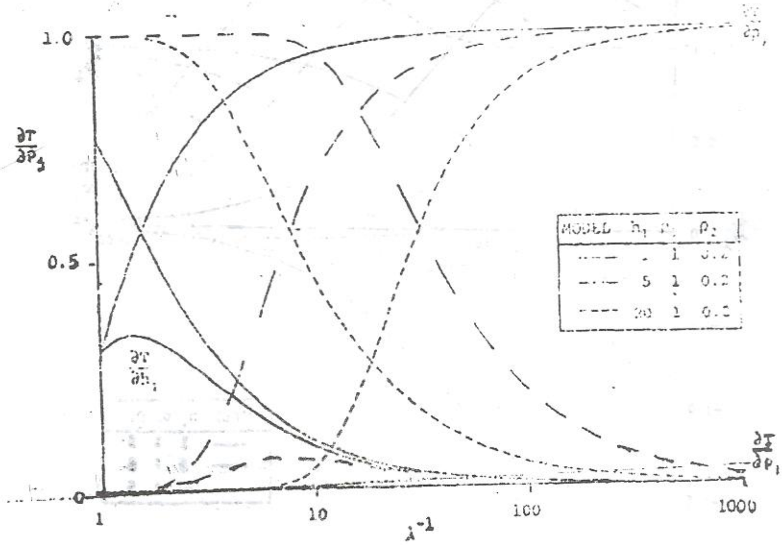

Fig. 3: Transform derivatives for a two-layer Earth with variable Depth to conductive Base

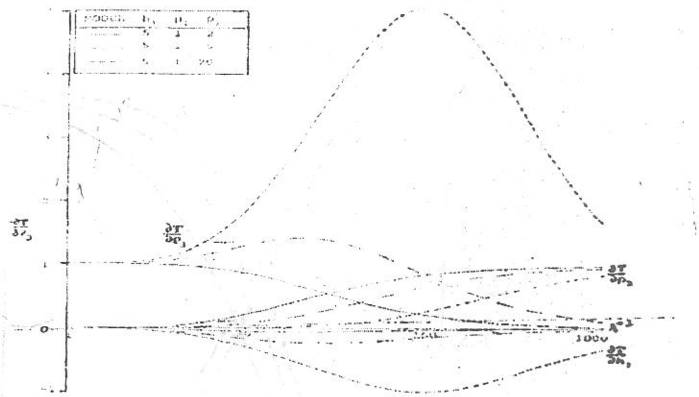

Fig. 4: Transform derivatives for a two-layer Earth with variable Resistively ($P_2 > P_1$)

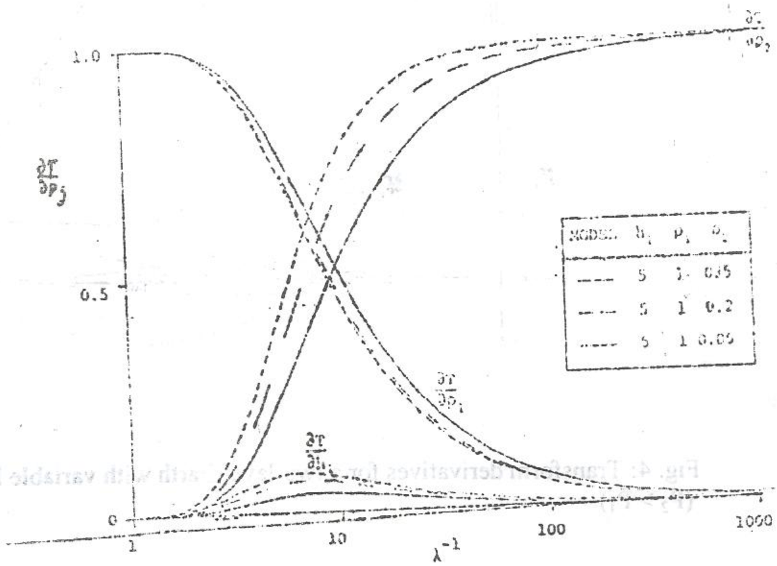

Fig. 5: Transform derivatives for a two-layer Earth with variable Bedrock Resistively ($P_2 > P_1$)

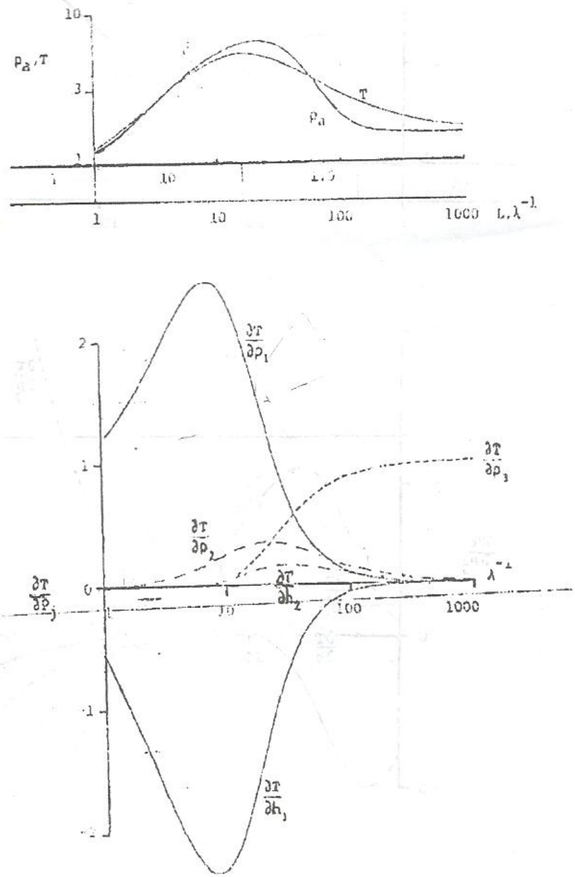

Fig. 6: Apparent Resistively, Resistively Transform and Transform **Derivatives** for a Three – layer Earth.

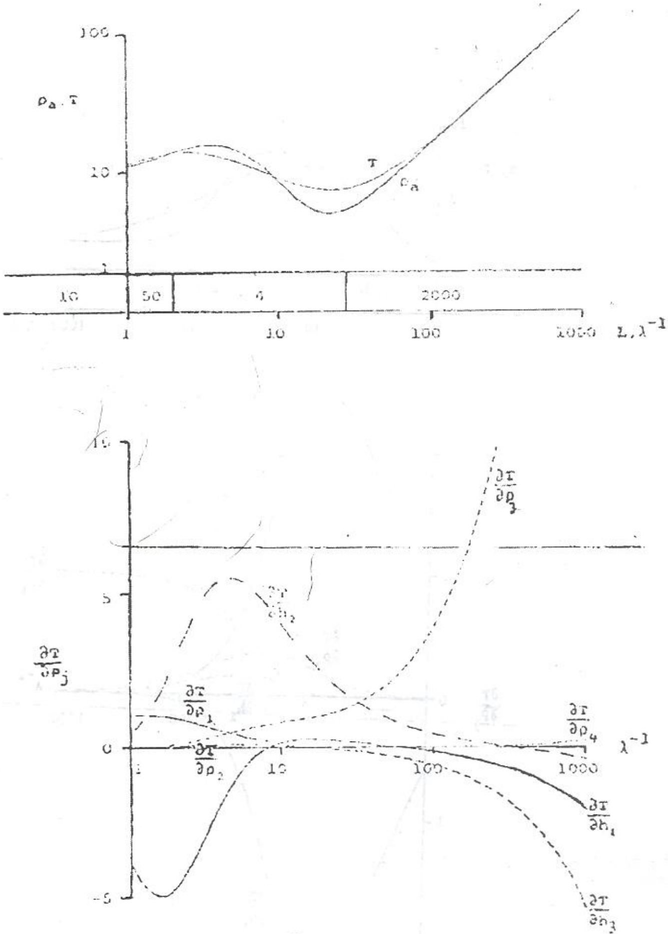

Fig. 7: Apparent Resistively, Resistively Transform and Transform Derivatives for a Four - layer