J. Nig. Ass. Math. Phys.
Vol. 6 (2002)

ASYMPTOTIC ANALYSIS OF IMPERFECTION SENSITIVITY OF TOROIDAL SHELL SEGMENT WITH MODAL IMPERFECTION.

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ABSTRACT:
We consider a multimode analysis of imperfection sensitivity of the toroidal shell segment with modal imperfection. We derive an asymptotic formula for the loss in buckling strength. We compare our results with known analysis in the literature.

INTRODUCTION

The imperfection sensitivity of structures has occupied many analyses on buckling since the classic work of Koiter [1]. Budiansky and Hutchinson [2] gave a reworked version of this work. Budiansky and Amazigo [3] applied this version and derived an asymptotic formula for the buckling load of externally pressurized cylinders. Oyesanya [4] used the same procedure to derive an asymptotic formula for cylindrical shell of finite length for simultaneous buckling mode under a multimode analysis. Amazigo and Oyesanya [5] derived the formula for a column resting on a nonlinear elastic foundation.
In this work we consider an imperfect toroidal shell segment subjected to lateral and or hydrostatic pressure under simple support conditions. We derive an asymptotic formula for the buckling load for simultaneous buckling mode case and derive the simple mode case as a special case. We found that our results agree with results of Hutchinson [6] for the simple mode case. We also found that all our results reduce to results for the cylindrical shell when $r = 0$.

FORMULATION AND ANALYSIS

For the imperfect structure the governing equation that we consider is the Karman-Donnell shell theory equations, which in non-dimensional form are given by

$$\overline{\nabla}^4 w - K(\xi)(f_{xx} + \xi r f_{yy}) + \lambda\left[\tfrac{1}{2}a(w+\overline{w})_{xx} + \xi(1-\tfrac{1}{2}ar)(w+\overline{w})_{yy}\right] = -K(\xi)HS(f, w+\overline{w}) \quad (1)$$

$$\overline{\nabla}^4 f - (1+\xi\overline{n}^2)^2(w_{xx} + \xi r w_{yy}) = -\tfrac{1}{2}H(1+\xi\overline{n}^2)^2 S(w+\overline{w}, w) \quad (2)$$

$$w = w_{xx} = f = f_{xx} = 0 \quad on \quad x = 0, \pi \quad (3)$$

where

$$\overline{\nabla}^4 = \left(\frac{\partial^2}{\partial x^2} + \xi\frac{\partial^2}{\partial y^2}\right)^2 \quad and \quad S(p,q) = p_{xx}q_{yy} + p_{yy}q_{xx} - 2p_{xy}q_{xy} \tag{4}$$

w and f are the non dimensional displacement and stress function respectively and the non dimensional quantities are defined by

$$x = \frac{\pi X}{L}, y = \frac{Y}{r}, r = \frac{a}{b}, \xi = \left(\frac{L}{\pi r}\right)^2, \lambda = \frac{P_e a L^2}{D\pi^2}, H = \frac{1}{r}, K(\xi) = \frac{L^2}{\pi^2 aD}, (1+\xi\bar{n}^2)^2 = \frac{EhL^2}{\pi^2 a} \tag{5}$$

where X is the axial coordinate, Y the circumferential coordinate, r the ratio of the inner and outer radii,L the length, h the thickness, Pe the external lateral pressure, D the flexural rigidity and E the Young's modulus.

The classical problem is the linearised problem

$$\overline{\nabla}^4 w - K(\xi)\left(f_{xx} + \xi r f_{yy}\right) + \lambda\left[\tfrac{1}{2}a w_{xx} + \xi\left(1 - \tfrac{1}{2}ar\right)w_{yy}\right] = 0 \tag{6}$$

$$\overline{\nabla}^4 f - \left(1+\xi\bar{n}^2\right)^2\left(w_{xx} + \xi r w_{yy}\right) = 0 \tag{7}$$

$$w = w_{xx} = f = f_{xx} = 0 \quad on \quad x = 0,\pi \tag{8}$$

for which the classical buckling load is given by

$$\lambda_c = \left(1+\xi\bar{n}^2\right)^{-2}\left[\frac{1}{2}\alpha + \left(1 - \frac{1}{2}\alpha r\right)\xi\bar{n}^2\right]^{-1}\left[\left(1+\xi\bar{n}^2\right)^4 + A^2\left(1+\xi r\bar{n}^2\right)^2\right] \tag{9}$$

where $\bar{n}$ is the value of n that gives the minimum buckling load. The buckling mode is given by

$$w = a_{1n}\sin x \sin \bar{n} y \tag{10}$$

We therefore expand the imperfection $\overline{w}$ in a series of eigenfunctions ϕ_j that

$$\overline{w} = \mu\sum_{j=1}^{\infty}\bar{a}_j\phi_j \quad , \mu << 1 \quad , \quad \bar{a}_j = O(1) \tag{11}$$

Thus we are assuming the imperfection in the shape of the buckling mode. We expand w, f, and μ in powers of η where $\eta^2 = (\lambda - \lambda_c)$. We then have

$$\begin{pmatrix} w \\ f \end{pmatrix} = \sum_{j=1}^{x} \begin{pmatrix} w_j \\ f_j \end{pmatrix} \eta^{j/2}, \quad \lambda\mu = \bar{\lambda} \sum_{j=1}^{x} \mu_j \eta^{j/2} \tag{12}$$

We substitute (11) - (12) into (1) - (3) and equate like powers of η and we have the following sequence of equations

$$0(\varepsilon^k): L(w_k f_k) = \frac{1}{2}\alpha w_{k-2,xx} + \bar{\xi}\left(1 - \frac{1}{2}\alpha r\right) w_{k-2,yy}$$

$$- K(\bar{\xi}) H\left[\sum_{j=1}^{k-1}\left(\bar{S}(w_{k-j}, f_j) + \bar{S}\left(\mu_j \sum_p \bar{a}_p \phi_p, f_{k-j}\right)\right)\right]$$

$$- \mu_k \bar{\lambda}\left[\frac{1}{2}\alpha \sum_p \bar{a}_p \phi_{p,xx} + \bar{\xi}\left(1 - \frac{1}{2}\alpha r\right) \sum_p \bar{a}_p \phi_{p,yy}\right]$$

$$+ \mu_{k-2}\left[\frac{1}{2}\alpha \sum_p \bar{a}_p \phi_{p,xx} + \bar{\xi}\left(1 - \frac{1}{2}\alpha r\right) \sum_p \bar{a}_p \phi_{p,yy}\right]$$

$$M(w_k, f_k) = -\left(1 + \xi \bar{n}^2\right)^2 H\left[\frac{1}{2}\sum_{j=1}^{k-1}\left(\bar{S}(w_{k-j}, w_j) + \bar{S}\left(\mu_j \sum_p \bar{a}_p \phi_p, w_{k-j}\right)\right)\right] \tag{13}$$

$$w_k = w_{k,xx} = f_k = f_{k,xx} = 0 \quad on \quad x = 0, \pi \tag{14}$$

where

$$L(w, f) \equiv \bar{\nabla}^4 w - K(\xi)\left(f_{xx} + \xi r f_{yy}\right) + \bar{\lambda}\left[\frac{1}{2}\alpha w_{xx} + \xi\left(1 - \frac{1}{2}\alpha r\right) w_{yy}\right]$$

$$M(w, f) \equiv \bar{\nabla}^4 f - \left(1 + \xi \bar{n}^2\right)^2 \left(w_{xx} + \xi r w_{yy}\right) \tag{15}$$

For k= 1 in (13) ϕ_p is a complementary function. Hence a necessary condition for existence of solution is that

$$\mu_1 = 0, \quad \bar{a}_p \neq 0 \tag{16}$$

and

$$\begin{pmatrix} w_1 \\ f_1 \end{pmatrix} = \begin{pmatrix} a_p \\ b_p \end{pmatrix} \phi_p \qquad p = m,k \quad b_p = -\left(1 + \xi r \bar{n}^2\right) a_p \tag{17}$$

Using (16) we have

$$\bar{S}(f_1, w_1) = -(1+\xi r\bar{n}^2)p^2 a_p^2(\cos 2x + \cos 2py) \tag{18}$$

$$\bar{S}(w_1, w_1) = -p^2 a_p^2(\cos 2x + \cos 2py)$$

The (w_2, f_2) problem is inhomogenous. We therefore apply the solvability condition

$$\int_0^{\pi}\int_0^{2\pi}\left[K(\xi)(1+\xi r\bar{n}^2)R_2 - (1+\xi\bar{n}^2)^2 R_1\right]\sin x \sin \bar{n} y\, dy\, dx = 0 \tag{19}$$

(where R_1, R_2 are the right hand sides of (13a,b) respectively) to the problem and we have

$$\left\langle A^2 H\left\{ \begin{array}{l} p^2 a_p^2\left[1+\frac{1}{2}(1+\xi r\bar{n}^2)\right](\cos 2x + \cos 2py) \\ \quad + \bar{\lambda}\mu_2(1+\xi\bar{n}^2)^2\left(\sum_p\left[\frac{1}{2}\alpha + \xi p^2\left(1-\frac{1}{2}\alpha r\right)\bar{a}_p \phi_p\right]\right) \end{array}\right\}.\phi_j\right\rangle = 0 \tag{20}$$

which for $a_p \neq 0$ implies that

$$\mu_2 = 0 \tag{21}$$

where we have used (18).

Using (17) and (21) in the equation for k = 2 gives the problem

$$L(w_2, f_2) = -K(\xi)H\bar{S}(w_1, f_1)$$

$$M(w_2, f_2) = -\frac{1}{2}(1+\xi\bar{n}^2)^2 H\bar{S}(w_1, w_1) \tag{22}$$

$$w_2 = f_2 = w_{2,xx} = f_{2,xx} = 0 \quad on \quad x = 0\pi$$

whose solution is

$$w_2 = \sum_{lodd}\left[a_1 + b_1\cos 2my + b_2\cos 2ky + b_3\cos(m-k)y + b_4\cos(m+k)y\right]\sin lx$$

$$f_2 = \frac{\left(1+\xi\bar{n}^2\right)^2 H}{16\pi^2}\left(2x^2+\cos 2x-1\right)\left(m^2 a_m^2 + k^2 a_k^2\right) \tag{23}$$

$$+\sum_{l\,odd} \sin lx\left[-\frac{\left(1+\xi\bar{n}^2\right)^2}{l^2}a_1 + c_1\cos 2my + c_2\cos 2ky + c_3\cos(m-k)y + c_4\cos(m+k)y\right]$$

where

$$a_l = \frac{4HA^2\left\{m^2a^2m\left[1-2l^2\left(1+\xi\bar{n}^2\right)^{-2}c_m\right]+k^2a_k{}^2\left[1-2l^2\left(1+\xi\bar{n}^2\right)^{-2}c_k\right]\right\}}{\pi^3 l\left(l^2-4\right)\left(l^4-\frac{1}{2}a\bar{\lambda}\,l^2+A^2\right)}$$

$$b_1 = \frac{HA^2m^2}{\pi^2}\left[\left(l^2-4\xi rk^2\right)a_m{}^2-2\left(1+\xi\bar{n}^2\right)^{-2}\left(l^2+4\xi m^2\right)^2 a_m b_m\right]\Delta_m{}^{-1}$$

$$b_2 = \frac{HA^2m^2}{\pi^2}\left[\left(l^2-4\xi rk^2\right)a_k{}^2-2\left(1+\xi\bar{n}^2\right)^{-2}\left(l^2+4\xi k^2\right)^2 a_k b_k\right]\Delta_k{}^{-1}$$

$$c1 = \frac{HA^2m^2}{\pi^2}\left[2a_m b_m\left(l^2-4\xi rkm^2\right)-K(\xi)^{-1}a_m{}^2\left\{\begin{matrix}\left(l^2-4\xi m^2\right)^2+\\ \bar{\lambda}\left[4\xi\left(1-\frac{1}{2}ar\right)m^2-\frac{1}{2}al^2\right]\end{matrix}\right\}\right]\Delta_m{}^{-1}$$

$$c_2 = \frac{HA^2k^2}{\pi^2}\left[2a_k b_k\left(l^2-4\xi rk^2\right)-K(\xi)^{-1}a_m{}^2\left\{\begin{matrix}\left(l^2+4\xi k^2+\right)\\ \bar{\lambda}\left[4\xi\left(1-\frac{1}{2}ar\right)k^2-\frac{1}{2}al^2\right]\end{matrix}\right\}\right]\Delta_k{}^{-1} \tag{24}$$

where

$$\Delta_j = \left(\left(l^2+4\xi j^2\right)^2+\bar{\lambda}\left[4\xi\left(1-\frac{1}{2}\alpha r\right)j^2-\frac{1}{2}\alpha l^2\right]\right)\left(l^2+4\xi j^2\right)^2+A^2\left(l^2-4\xi j^2\right)^2 \quad (2$$

and

$$b_3 = \frac{HA^2}{2\pi^2}\left[(m-k)^2-(m+k)^2\cos 2x\right]\left\{\frac{\left(1+\xi\bar{n}^2\right)^{-2}\left[l^2+\xi(m-k)^2\right]^2}{\left(a_m b_k+a_k b_m\right)+\left[l^2-\xi r(m-k)^2\right]a_m a_k}\right\}\bar{\Delta}_{m-k}{}^{-}$$

$$b_4 = \frac{HA^2}{2\pi^2}\left[(m-k)^2 - (m+k)^2\cos 2x\right]\left\{\begin{array}{l}\left(1+\xi\bar{n}^2\right)^{-2}\left[l^2 + \xi(m+k)^2\right]^2 \\ \left(a_m b_k + a_k b_m\right) + \left[l^2 - \xi r(m+k)^2\right]a_m a_k\end{array}\right\}\bar{\Delta}_{m+k}^{-1} \quad (26)$$

$$c_3 = \frac{HA^2}{2\pi^2}\left\{\left[(m-k)^2 - (m+k)^2\cos 2x\right]\left(\begin{array}{l}\left[l^2 - \xi r(m-k)^2\right]\left(a_m b_k + a_k b_m\right) \\ -K(\xi)^{-1} a_m a_k\left(\left[l^2 - \xi(m-k)^2\right]^2 - \bar{\lambda}\left[\frac{1}{2}\alpha l^2 - \xi\left(1-\frac{1}{2}\alpha r\right)(m-k)^2\right]\right.\end{array}\right.\right.$$

$$c_4 = \frac{HA^2}{2\pi^2}\left\{\left[(m-k)^2 - (m+k)^2\cos 2x\right]\left(\begin{array}{l}\left[l^2 - \xi r(m+k)^2\right]\left(a_m b_k + a_k b_m\right) \\ -K(\xi)^{-1} a_m a_k\left(\left[l^2 - (m+k)^2\right]^2 - \bar{\lambda}\left[\frac{1}{2}\alpha l^2 - \xi\left(1-\frac{1}{2}\alpha r\right)(m+k)^2\right]\right.\end{array}\right.\right.$$

where

$$\bar{\Delta}_p = \left(l^2 + \xi p^2\right)^2\left\{\left(\left(l^2 + \xi p^2\right)^2\right) - \bar{\lambda}\left[\frac{1}{2}\alpha l^2 - \xi\left(1-\frac{1}{2}\alpha r\right)p^2\right] - A^2\left(l^2 + \xi r p^2\right)\left(l^2 - \xi r l^2\right)\right\} \quad (27)$$

Application of the solvability condition (19) to the problem (13) for k = 3 gives

$$\begin{aligned}&\left\langle\left(1+\xi\bar{n}^2\right)^2\left[\frac{1}{2}\alpha w_{1,xx} + \bar{\xi}\left(1-\frac{1}{2}\alpha r\right)w_{1,yy}\right],\phi_j\right\rangle + A^2 H\left\langle\bar{S}\left(w_1, f_2\right) + \bar{S}\left(w_2, f_1\right) - \left(1+\xi r\bar{n}^2\right)\bar{S}\left(w_1, w_2\right),\phi_j\right\rangle \\ &-\left(1+\xi\bar{n}^2\right)^2\mu_3\bar{\lambda}\left\langle\left[\frac{1}{2}\alpha\sum_p \bar{a}_p\phi_{p,xx} + \bar{\xi}\left(1-\frac{1}{2}\alpha r\right)\sum_p \bar{a}_p\phi_{p,yy}\right],\phi_j\right\rangle = 0, \quad p = m,k \quad j = m,k\end{aligned} \quad (28)$$

which are the equations for μ_3

Evaluating (28) leads us to the equations

$$\begin{aligned}\bar{\lambda}\mu_3\bar{a}_p &= a_p + Q a_p^3 + R a_k^2 a_p + S\delta_{3p,k} a_p^2 a_k \\ \bar{\lambda}\mu_3\bar{a}_k &= a_k + Q_1 a_k^3 + R_1 a_p^2 a_k\end{aligned} \quad (29)$$

where

$$Q = \frac{A^2 H\left(1+\xi\frac{r}{n^2}\right)^2}{\left[\frac{1}{2}\alpha + \xi\left(1-\frac{1}{2}\alpha r\right)p^2\right]}\left\{\begin{array}{l}\frac{3Hp^4\left(1+\xi\bar{n}^2\right)}{8\pi^2} + \frac{8}{\pi}\sum_{l\,odd} p^2\left\{\left[l^2\left(\rho_m - 1\right) - \left(1+\xi\bar{n}^2\right)^2\right]a^p{}_l\right\}\left\{l\left(l^2-4\right)\right\}^{-1} \\ +\frac{8}{\pi}\sum_{l\,odd}\left[\left(1-\rho_m\right)c^p{}_l - d^p{}_l\right]\left[\left(4p^2 - l^2p^2\right)\left\{l\left(l^2-4\right)\right\}^{-1}\right]\end{array}\right\}$$

$$R=\frac{A^{2}H\left(1+\xi\overset{r}{n}{}^{2}\right)^{-2}}{\left[\frac{1}{2}a+\bar{\xi}\left(1-\frac{1}{2}ar\right)p^{2}\right]}\left\{\begin{array}{l}\frac{3Hp^{2}k^{2}\left(1+\xi\bar{n}^{2}\right)^{2}}{8\pi^{2}}+\frac{8}{\pi}\sum\limits_{l\,odd}p^{2}\left\{\left[l^{2}\left(\rho_{m}-1\right)-\left(1+\bar{\xi}\bar{n}^{2}\right)^{2}\right]a^{k}{}_{l}\right\}\left\{l\left(l^{2}-4\right)\right\}^{-1}\\ +\frac{8}{\pi}\sum\limits_{l\,odd}\left[-\left(1-\rho_{k}\right)g^{p}{}_{k}-h^{p}{}_{k}\right]\left[\begin{array}{l}\left[k^{2}l^{2}+\left(p-k\right)^{2}+k\left(p-k\right)\right]\\ +\left[\left[-\left(1-\rho_{k}\right)j^{p}{}_{k}-k^{p}{}_{k}\right]\right]\left[\begin{array}{l}\left(p+k\right)k-\left(p+k\right)\\ +\end{array}\right]\\ \left[-\left(1-\rho_{k}\right)j^{p}{}_{k}-k^{p}{}_{k}\right]\left(l^{2}p^{2}\right)\left\{l\left(l^{2}-4\right)\right\}^{-1}\end{array}\right]\end{array}\right\} \tag{30}$$

$$R_{1}=\frac{A^{2}H\left(1+\xi\overset{r}{n}{}^{2}\right)^{-2}}{\left[\frac{1}{2}a+\bar{\xi}\left(1-\frac{1}{2}ar\right)k^{2}\right]}\left\{\begin{array}{l}\frac{3Hk^{4}k^{2}\left(1+\xi\bar{n}^{2}\right)^{2}}{8\pi^{2}}+\frac{8}{\pi}\sum\limits_{l\,odd}k^{2}\left\{\left[l^{2}\left(\rho_{k}-1\right)-\left(1+\bar{\xi}\bar{n}^{2}\right)^{2}\right]a^{k}{}_{l}\right\}\left\{l\left(l^{2}-4\right)\right\}^{-1}\\ +\frac{8}{\pi}\sum\limits_{l\,odd}\left[\left(1-\rho_{k}\right)c^{k}{}_{l}-d^{k}{}_{l}\right]\left[\left(4k^{2}-l^{2}k^{2}\right)\left\{l\left(l^{2}-4\right)\right\}^{-1}\right]\end{array}\right\}$$

$$S=\frac{-8A^{2}H\left(1+\bar{\xi}\bar{n}^{2}\right)^{-2}}{\pi\left[\frac{1}{2}a+\bar{\xi}\left(1-\frac{1}{2}ar\right)p^{2}\right]}\sum_{l\,odd}4l^{2}pk\left[d_{l}^{p}-\left(1-\rho_{m}\right)c^{p}{}_{l}\right]\left\{\left(l^{2}-4\right)\right\}^{-1}$$

Where $\overset{r}{q}_{s}$ are as defined in Oyesanya (1998).

$\bar{a}_{k}=0$ *(29) implies that* $a_{k}=0$ *or* $1+Q_{1}a_{p}^{2}+R_{1}a_{k}^{2}=0$

For $a_{k}=0$ *implies from (28) that*

$$\bar{\lambda}\mu_{3}a_{p}=a_{p}+Qa_{p}^{3} \tag{31}$$

Substituting this into (12), differentiating with respect to a_{p}, setting $d\lambda/da_{p}=0$ and solving give the result for the limit load for this case as

$$\left(\bar{\lambda}-\lambda_{s}\right)^{3/2}=\frac{3}{2}\left(-3Q\right)^{1/2}\lambda_{s}\mu\bar{a}_{p} \tag{32}$$

The case $a_{k}\neq 0$ implies that

$$a_{k}^{2}=-R_{1}^{-1}\left(1+Q_{1}a_{p}^{2}\right),\quad R_{1}^{-1}\left(1+Q_{1}a_{p}^{2}\right)<0 \tag{33}$$

Substituting this into (29a) gives

$$R_3 a_p + Q_2 a_p^3 + S a_p^2 \delta_{3p,k} f\left(\bar{\lambda}, \bar{\xi}, a_p\right) = \bar{\lambda}\mu_3 \bar{a}_p \tag{34}$$

where

$$R_3 = 1 - \frac{R}{R_1}, f\left(\left(\bar{\lambda}, \bar{\xi}, a_p\right)\right) = \left(\frac{1 + Q_1 a_p^2}{R_1}\right)^{1/2}, Q_2 = Q - \frac{RQ_1}{R_1} \tag{35}$$

Substituting (33) into (12), differentiating with respect to a p and setting $d\lambda / da_p = 0$ lead to the result for $3p \neq k$

$$\left(\bar{\lambda} - \lambda_s\right)^{3/2} = \frac{3}{2R_3^{3/2}}\left(-3Q_2\right)^{1/2} \lambda_s \mu \bar{a}_p, \quad \frac{Q_2}{R_3} < 0 \tag{36}$$

For 3p = k, a_p (=x) is given by

$$\alpha x^6 + \beta x^4 + \gamma x^2 + \delta = 0$$

$$\alpha = 9Q_1 R_1^{-1}\left(S^2 Q_1 R_1^{-1} - Q_2^2\right), \beta = -3R_1^{-1}\left(4S^2 R_1^{-1} + 2Q_1 Q_2 R_3 + 3Q_2^2\right)$$

$$\gamma = R_1^{-1}\left(4S^2 R_1^{-1} - R_3\left(Q_1 R_3 + 6Q_2\right)\right) \qquad \delta = \frac{R_3}{R_1} \tag{37}$$

Taking the value of a_p (=x) that satisfies (37) as a_p^s the relation for the limit load for this case is given by

$$\left(\bar{\lambda} - \lambda_s\right)^{3/2} = \left[R_3 a^s{}_p + Q_2 a^s{}_p{}^3 + S a^s{}_p{}^2 f\left(\bar{\lambda}, \bar{\xi}, k\right)\right]^{-1} \bar{\lambda}\mu_3 \bar{a}_p \quad a_k \neq 0, \; 3p = k \tag{38}$$

Thus for the case that leads to secondary bifurcation and hence serious instability we have

$$\left(\bar{\lambda} - \lambda_s\right)^{3/2} = \begin{cases} 3/2\left(-3Q\right)^{1/2} \lambda_s \mu \bar{a}_p & : a_k = 0 \\ 3/2R_3\left(-3Q_2 / R_3\right)^{1/2} \lambda_s \mu \bar{a}_p & : a_k \neq 0, 3p \neq k \end{cases}$$

$$\left[R_3 a^s{}_p + Q_2 a^s{}_p{}^3 + S a^s{}_p{}^2 f\left(\bar{\lambda}, \bar{\xi}, k\right)\right]^{-1} \bar{\lambda}_3 \mu \bar{a}_p : \quad a_k = 0, \quad 3p = k \qquad (39)$$

CONCLUDING REMARKS

In this analysis we have derived an asymptotic formula for the buckling of the toroidal shell segment with modal imperfection under a multimode situation for which secondary bifurcation have been observed. We note that since Q| a_k=0 = b for $\bar{\lambda} = \lambda_p$ the simple mode case result is

$$\left(\lambda_p - \lambda_s\right)^{3/2} = \frac{3}{2}(-3b)^{1/2} \lambda_s \mu_s \bar{a}_p \qquad (40)$$

as in Oyesanya (1990). Our result shows that the reduction in load is of the order of $(\mu a_p)^{2/3}$ which agrees with most results of imperfection sensitivity problems in the shape of the buckling mode. As noted in the literature, generally, imperfection is stochastic. We therefore in a forthcoming communication investigate the toroidal shell segment under random imperfection.

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