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EFFECTS OF THE GEOMETRY OF VESSEL ON DETONATIONS

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ABSTRACT:

We examine the effect of geometry of vessel on detonations. It is shown that in a non – uniform vessel, maximum temperature occurs towards the end of the tube. Whereas, in a uniform tube maximum temperature occurs at the centre. Also maximum temperature for diverging or converging channel is greater than that of a uniform vessel.

1. INTRODUCTION

Effects of the Frank-Kamenetskii parameter, sealed activation energy are known for uniform planar, cylindrical and spherical vessel but not much is known on the effects of the parameter in a non-uniform vessel. In this paper, we discuss the non-uniformity effects of the vessel.

Ayeni (1978) investigated problems concerning the one-dimensional mixing and simultaneous reaction of a fuel and oxidizer. Of particular interest is the so called ignition time, before which temporal changes are given as a function of activation energy which can vary over all positive values. He showed that terminal runaway occurs at a finite time in the limit of large activation and a lower bound is derived for that time by means of a comparison theorem. Existence and uniqueness results feature prominently.

Olarewaju et al. (2001) examined the effects of Frank-Kamenetskii parameter on strong detonations in a converging vessel. It was shown that when Frank-Kamenetskii parameters differ by 1/30 there is an appreciable difference in the temperature along the converging vessel.

Olanrewaju (2002) examined a mathematical theory of strong detonations for viscous combustible materials and the effect of certain parameters on detonations. He also gives conditions for existence and uniqueness of solution. Further he examined the effects of the geometry of vessel on detonations.

2. MATHEMATICAL FORMULATION AND METHOD OF SOLUTION

We consider the steady state energy equation

$$pcu\frac{dT}{dX} = \frac{kd^2T}{dX^2} + qBy^a e^{-E/RT} + \mu\left(\frac{\partial u}{\partial X}\right)^2 \tag{2.1}$$

and the continuity equation

$$puA = \text{constant} = c \tag{2.2}$$

with initial and boundary conditions

$$\left.\begin{aligned} T(X,0) &= T_0 \quad ,1 \le X \le 1 \\ T(-1,t) &= T_0 \quad ,t > 0 \\ T(1,t) &= T_0 \quad ,t > 0 \end{aligned}\right\} \tag{2.3}$$

$$\left.\begin{aligned} Let\ \theta &= \frac{E}{RT_0^2}(T - T_0) \\ u' &= u/U, x' = \alpha x \end{aligned}\right\} \tag{2.4}$$

for a converging vessel

$$A = A_0 e^{-\alpha x} \tag{2.5}$$

we obtain the non-dimensional equation after dropping as

$$\left.\begin{aligned} \frac{c e^{\alpha x} d\theta}{A_0 A_1 dx} &= \frac{d^2\theta}{dx^2} + B_1 e^{\theta/1+\epsilon\theta} + \frac{D}{A_1} e^{2\alpha x} \\ \theta(-1) &= 0, \theta(1) = 0 \end{aligned}\right\} \tag{2.6}$$

where

$A_1 = \dfrac{\alpha k p A_0}{c}$ (modified thermal conductivity)

$B_1 = \dfrac{q A Y^a A_0 e^{-E/RT_0}}{\alpha \in T_0 c}$ (Frank-Kamenetskii parameters)

$D = \dfrac{\mu c}{\alpha p A_0 \in T_0}$ (modified dynamic Viscossity)

we now consider a physical situation where

$$\left.\begin{aligned} \frac{c}{A_0 A_1} &= 0(\epsilon^2), B_1/A_1 = \sigma\ (\sigma = 0(1)) \\ \frac{D}{A^1} &= 0(\epsilon) \end{aligned}\right\} \tag{2.7}$$

So (2.6) becomes

$$\left.\begin{aligned} \epsilon^2 \frac{d\theta}{dx} &= \frac{d^2\theta}{dx^2} + \sigma e^{\theta/1+\epsilon\theta} + \epsilon e^{2\alpha x} \\ \theta(\pm 1) &= 0 \end{aligned}\right\} \quad (2.8)$$

we now seek asymptotic solution for (2.8)

let $\theta = \theta_0 + \epsilon\theta_1 + \epsilon^2\theta_2 + ...$

then (2.8) becomes

$$\left.\begin{aligned} 0 &= \frac{d^2\theta_0}{dx^2} + \sigma e^{\theta_0} \\ \theta_0(\pm 1) &= 0 \end{aligned}\right\} \quad (2.9)$$

$$\left.\begin{aligned} 0 &= \frac{d^2\theta_1}{dx^2} + \sigma e^{\theta_0}\theta_1 + e^{2\alpha x} \\ \theta_0(\pm 1) &= 0 \end{aligned}\right\} \quad (2.10)$$

solving (2.9), we obtain

$$\left.\begin{aligned} \theta_0 &= 2\ln(e^{\theta_0 ml2} \sec h(cx)) \\ \text{where } c^2 &= 1/2\delta e^{\theta_0 m} \end{aligned}\right\} \quad (2.11)$$

substituting (2.11) into (2.10), we have

$$\left.\begin{aligned} \frac{d^2\theta_1}{dx^2} + \sigma(e^{\theta_0 ml2} \sec h(cx))^2\theta_1 + e^{2\alpha x} &= 0 \\ \theta_1(\pm 1) &= 0 \end{aligned}\right\} \quad (2.12)$$

by shooting method (2.12) becomes

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 1 \\ x_3 \\ -[e^{2\alpha x_1} + 2.8774(\sec h(0.7818x_1))^2 x_2] \end{pmatrix} \qquad (2.13)$$

Satisfying

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ q \end{pmatrix} \qquad (2.14)$$

$$\text{Where } \sigma = 0.878, \theta_{0m} = 1.187\} \qquad (2.15)$$

See figure. 1 for (2.15).

ANALYSIS OF RESULT

Figure 2 shows the effect of the area of vessel on the reaction. The temperature is skewed to the left when α is negative i.e when the channel converges and it is skewed to the right when the channel diverges.

It is shown that in a non-uniform vessel, maximum temperature occurs towards the end of the tube. Also maximum temperature for diverging or converging channel is greater than that of a uniform vessel (see fig. 2).

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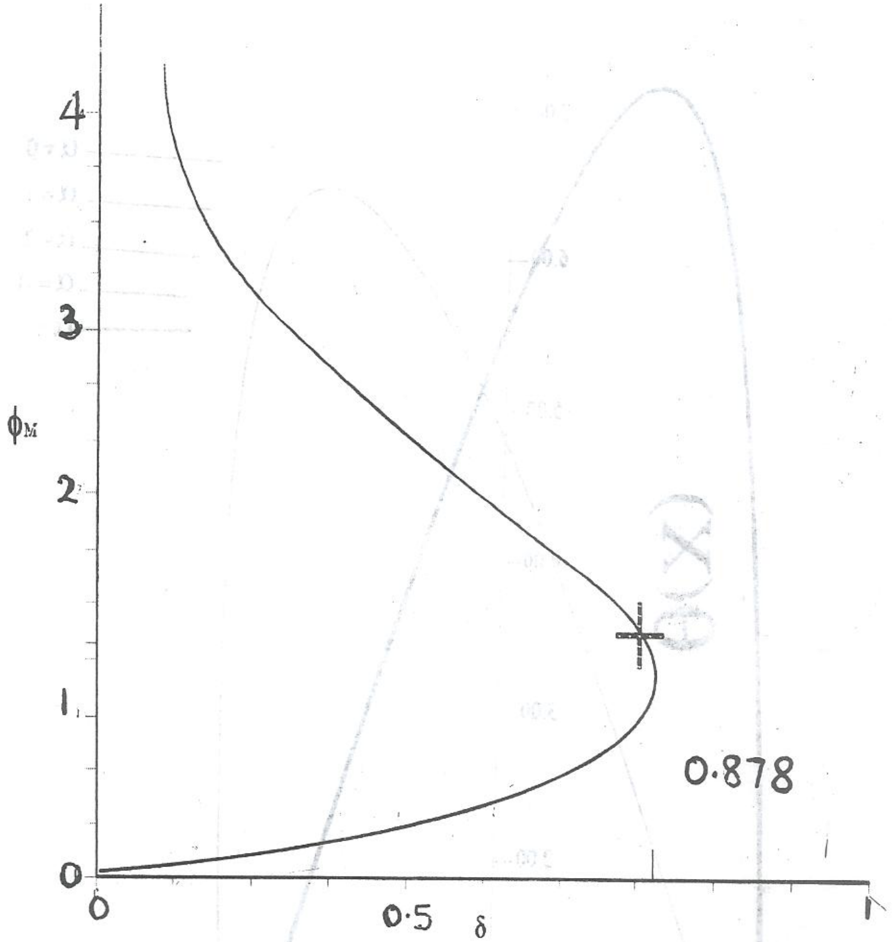

Fig1. Steady – state response for slab with surface maintained at initial uniform temperature

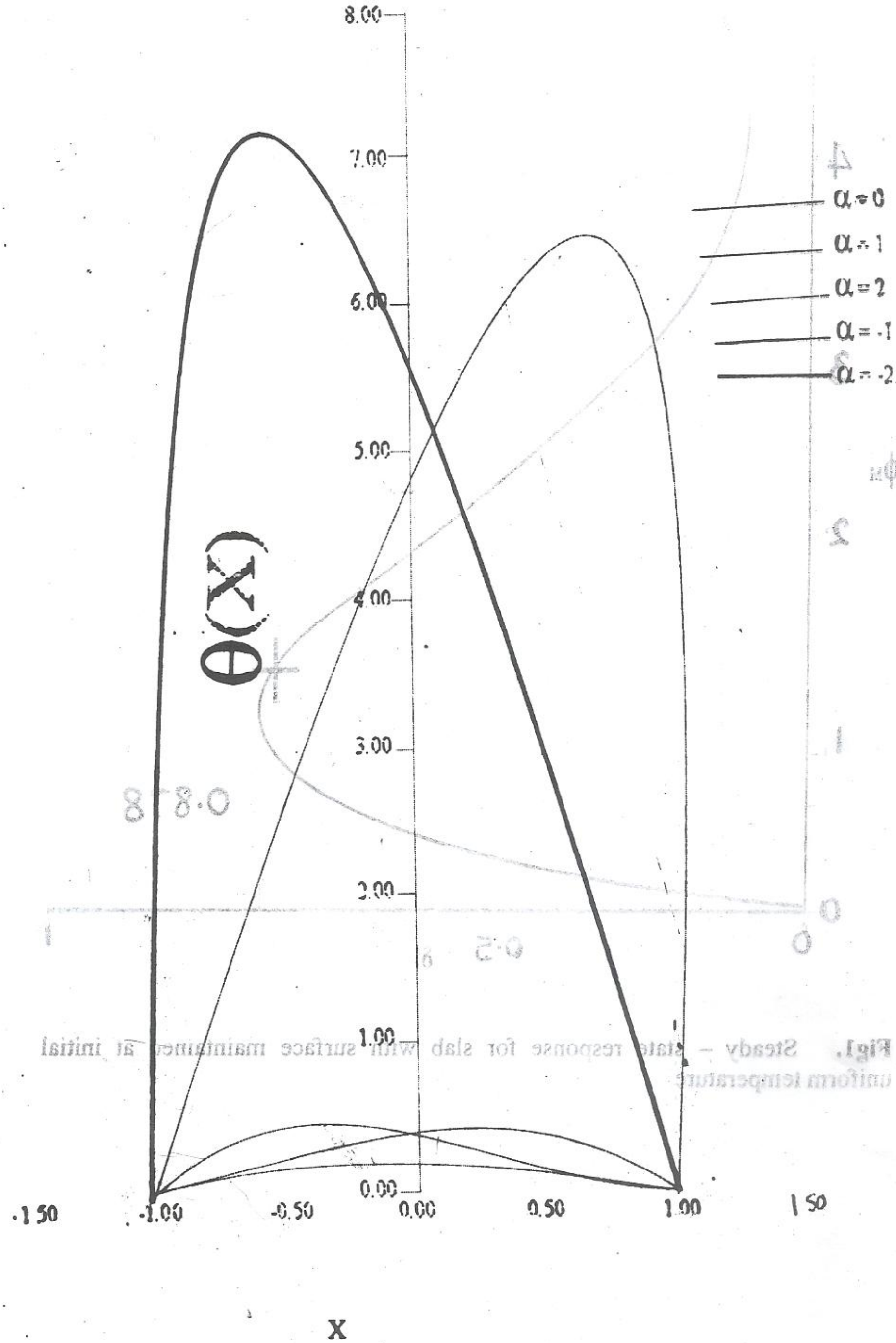

Fig.2. (Graph of θ (X) against for various values of α).