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ON THE EXISTENCE AND UNIQUENESS OF A POWER – LAW FLUID FLOWING IN A CYLINDER.

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ABSTRACT:

In this paper we examine the flow of power – law fluid in a cylindrical vessel. We investigate the appropriate conditions for a similarity. Of particular interest are the questions of existence and uniqueness of solution and we include the criteria for both.

1.0 INTRODUCTION

In the existing literature, Ayeni and Olajuwon [1] investigated the effect of a moving wall on the velocity field of a power-law fluid. The resulf showed that when the fluid is dilatant (n > 1), the momentum penetration of the backward power-law flow is finite.

.Elena and Paola [2] examined the flow of a Bingham fluid in contact with a Newtonian fluid. They discussed the steady state solutions with a particular regard for the asymptotic behaviour of the solution. In anothe interesting paper, Howell, et al., [3] examine the momentum and heat transfe occurring in the laminar boundary layer on a continuously moving anc stretching dimensional surface in non-Newtonian power-law fluid. Their resul include situations when velocity is non-linear and when the surface is stretchin linearly.

In this paper, we examine the appropriate conditions for a similarit solution of a power-law fluid flowing in a cylinder with particular regard fo existence and uniqueness of solution.

2.0 MATHEMATICAL FORMULATION.

The governing momentum equation for the flow of a power-law flui flowing in a cylinder is

$$p\frac{\partial w}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) \qquad (2$$

where

$$\tau_{rz} = m\left(-\frac{\partial w}{\partial r}\right)^{n} \qquad \textbf{(2.2)}$$

with boundary conditions

$w(1,t) = 1 \qquad t > 0 \quad 1 \le r < \infty$

$w(\infty, t) = 0 \qquad t > 0$

3.0 SIMILARITY SOLUTION.

We define an independent variable η in the form

$$\eta = \frac{At}{r^{\alpha}} \tag{3.1}$$

Such that

$$w = f(\eta) \tag{3.2}$$

Where w is the dimensionless velocity and it is a function of η alone. Using equation (2.2), (3.1) and (3.2) in equation (2.1)

We have

$$\frac{Ap}{r^{\alpha}} = -\frac{m}{r^{n+1}}\left(\alpha\eta\frac{df}{d\eta}\right)^{n}$$

$$+\frac{mn}{r^{n+1}}\left(\alpha\eta\frac{df}{d\eta}\right)^{n-1}\cdot\left[\alpha^{2}\eta^{2}\frac{d^{2}f}{d\eta^{2}}+\alpha(\alpha+1)\eta^{2}\frac{df}{d\eta}\right] \tag{3.3}$$

Setting

$$\alpha = n+1 \tag{3.4}$$

Equation (3.3) can be written as

$$Ap\frac{df}{d\eta} = -m\left(\alpha\eta\frac{df}{d\eta}\right)^{n}$$

$$+mn\left(\alpha\eta\frac{df}{d\eta}\right)^{n-1}\left[\alpha^{2}\eta^{2}\frac{d^{2}f}{d\eta^{2}}+\alpha(\alpha+1)\eta^{2}\frac{df}{d\eta}\right] \tag{3.5}$$

with $\qquad f(1) = 1, \quad f(\infty) = 0$

Remark: Analytical similarity solution is possible for

$$\alpha = n+1 \text{ and } \eta = \frac{At}{r^{\alpha}}$$

Transforming equation (3.5) to initial value problem by means of shooting method. Equation (3.5) becomes.

$$Ap\frac{df}{d\eta} = -m\left(\alpha\eta\frac{df}{d\eta}\right)^{n}$$

$$+mn\left(\alpha\eta\frac{df}{d\eta}\right)^{n-1}\left[\alpha^{2}\eta^{2}\frac{d^{2}f}{d\eta^{2}} + \alpha(\alpha+1)\eta^{2}\frac{df}{d\eta}\right] \qquad (3.6)$$

with $\qquad f(1) = 1, \quad f^{1}(1) = \Gamma$

where Γ is unknown and is to be determined such that the boundary conditions are satisfied.

Change equation (3.6) into system of linear equation as follows:

Let

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} \eta \\ f \\ f^{1} \end{pmatrix} \qquad (3.8)$$

$$\begin{pmatrix} X_1^{1} \\ X_2^{1} \\ X_3^{1} \end{pmatrix} = \begin{pmatrix} 1 \\ X_3 \\ \dfrac{ApX_3^{2-n}}{mn\alpha^{n+1}X_1^{n+1}} + \dfrac{X_3}{n\alpha X_1} - \left(\dfrac{\alpha+1}{\alpha}\right)X_3 \end{pmatrix} \qquad (3.9)$$

With the conditions

$$\begin{pmatrix} X_1(1) \\ X_2(1) \\ X_3(1) \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ \Gamma \end{pmatrix} \qquad (3.10)$$

$$1 \le X_1 \le \infty, \quad 1 \le X_2 \le L, \quad \Gamma \le X_3 \le M$$

Where L, M are positive constants

THEOREM: For every, $0 < n < 1$ and for which (3.10) holds and $\alpha = n + 1$ problem (3.9) has a unique solution.

Proof: Problem (3.9) can be written as

$$\begin{pmatrix} X_1^1 \\ X_2^1 \\ X_3^1 \end{pmatrix} = \begin{pmatrix} f_1(X_1, X_2, X_3) \\ f_2(X_1, X_2, X_3) \\ f_3(X_1, X_2, X_3) \end{pmatrix} = \begin{pmatrix} 1 \\ X_3 \\ \dfrac{ApX_3^{2-n}}{mn\alpha^{n+1} X_1^{n+1}} + \dfrac{X_3}{n\alpha X_1} - \left(\dfrac{\alpha+1}{\alpha}\right) X_3 \end{pmatrix}$$

Clearly, for every $0 < n < 1$, *and* $1 \le X_1 \le \infty$

$$\frac{\partial fi}{\partial X_j} \quad i, j = 1, 2, 3, \text{ is bounded.}$$

Thus, $f_1\ (X_1, X_2, X_3)\ i = 1,2,3,$ are lipschitz continuous. Hence, there exists a unique solution of the problem (3.9).

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