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PHASE SYNCHRONIZATION OF COUPLED OSCILLATORS

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ABSTRACT

The behaviours of coupled Oscillators, each of which has periodic motion with. Random natural frequency in the absence of coupling, are investigated Some novel collective phenomena are revealed subject to the boundary conditions applied. By increasing the coupling, a bifurcation tree from high dimensional quasi-periodicity to chaos to quasi periodicity an periodicity is found.

1. INTRODUCTION

The dynamics of systems consisting of a large number of mutually interacting Units is an intriguing problem in many fields. The investigation of coupled, oscillators has attracted c constant interest for many decades [1]. The rich collective behaviors of these systems, such as mutual entertainment, self-synchronization , and so on are observed in many field e.g. coupled laser Josephson junction, arrays, biological and chemical oscillators [2] .In early studies, interest was focused on coupled oscillators of which each is periodic without coupling. Recently, the investigation has been extended to coupled chaotic systems i.e. individual systems are chaotic without coupling. Significant phenomena were found, such as phase synchronization of two mutually coupled chaotic oscillators [3] and clustering and cluster-cluster synchronization of multiple coupled chaotic units for local and global couplings Recently, Zheng et al [4] studied the complex synchronization tree of a system of oscillators in the chaotic region with nearest neighbors interaction, which corresponds to a simplified model of over damped Josephson juctions, which follows the model of Ambegaokar et al [5] . They noticed that these oscillators while chaotic in phase, cluster in time averaged frequency until they completely synchronize in a phase-locked state. In this work, we study the following N coupled oscillators with the nearest coupling,

$$\theta_i = \omega_i + \frac{k}{3}\left[\sin\left(\theta_{i+1} - \theta_i\right) + \sin\left(\theta_{i-1} - \theta_i\right)\right. \qquad (1)$$

i = 1,2……N where k ,Θ_i and w_i are the coupling strength, the angle of the.

2п, and the natural frequency of the I oscillator, respectively model (1) has been extensively investigated in the past several decades. Here in this work, we concentrate on the dynamical behaviour of the system

In equation (1) the periodic boundary condition $\theta_i + N(t) = \theta_i(t)$ is applied

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Zheng et al [4] argued that, without loss of generality, constraining the frequency to

$$\sum_{i=1}^{N} \omega i = 0 \qquad (2)$$

The above system of N oscillators has a critical coupling strength $K = K_c$, such that for $k > k_c$, complete frequency synchronization can be observed where

$\{\theta_i = 0, \quad i = 1......N\}$ and each θ_i is locked to a fixed value For $K < K_c$, no phase locking exists, and $\Theta_i(t)$ are no longer zero.

It is found that if we define an average frequency as

$$\varpi_i = \lim_{T\to\infty} \frac{1}{T}\int_0^T \theta i(t)dt \qquad (3)$$

Synchronization between different oscillators, in the sense of $\varpi_l = \varpi_{j}, l \neq j$, can be observed in the region where strict phase locking of $\Theta i = 0$ is broken . It is interesting to investigate how he various oscillators are led to complete synchronization (phase locking for $K > K_c$) via a sequence of bifurcation by

$$\Omega = \sum_{i=1}^{N} |\varpi_i| \qquad (4)$$

Here in this work, we show that the use of constraint (2) leads to missing out on some information about the system (1). We use the general case

$$\sum_{i=1}^{N} \omega_i = \omega_0$$

We show that the case of complete frequency synchronization can occur even for $\omega_0 \neq 0$, that is there is a critical value of the coupling strength, Ks where frequency synchronization between all oscillators oecurs.

RESULTS AND DISCUSSION

In Figs. 1 and 2 , Natural frequencies are randomly chosen from a normal Gaussian distribution. The actual frequencies can be seen in Fig. 1 and Fig 2 for N = 15, Wo = 0.52 and N = 15, Wo = 1.77 respectively. In computing these figures, initial conditions of θ j (0) are randomly chosen. In fig. 1 and fi . 2, the quantity T in equation 3 is taken sufficiently long in our simulation so that fluctuations due t finite T are invisible.

To get a general idea, we plot ω, define in (3) vs K for N = 15, . By varying K from K = 0 to K>K_c , interesting behavior of transition tree for phase synchronization is clearly shown. First, if two adjacent oscillators (or adjacent clusters of oscillators) have close frequencies, they can be easily synchronized by increasing K. In this case, one always finds two branches merging to a single one. Second , if two non adjacent oscillators or two non adjacent clusters) have close frequencies while the oscillators between them have considerably different frequencies, one can find the non adjacent oscillators can also be synchronized to each other i.e. non local clusters can be formed, and these non clusters can quickly bring the oscillators between them to the synchronized status and form a solidly larger synchronized cluster. The transition may be from two to one or from multiple branches to one.

3. CONCLUSION

These findings greatly enlarge the application perspectives of chaotic. synchronizationThese features of phase dynamics are expected to be observable in practical systems by experiments such as coupled laser arrays, Josephson junction chains and coupled electrical circuits.

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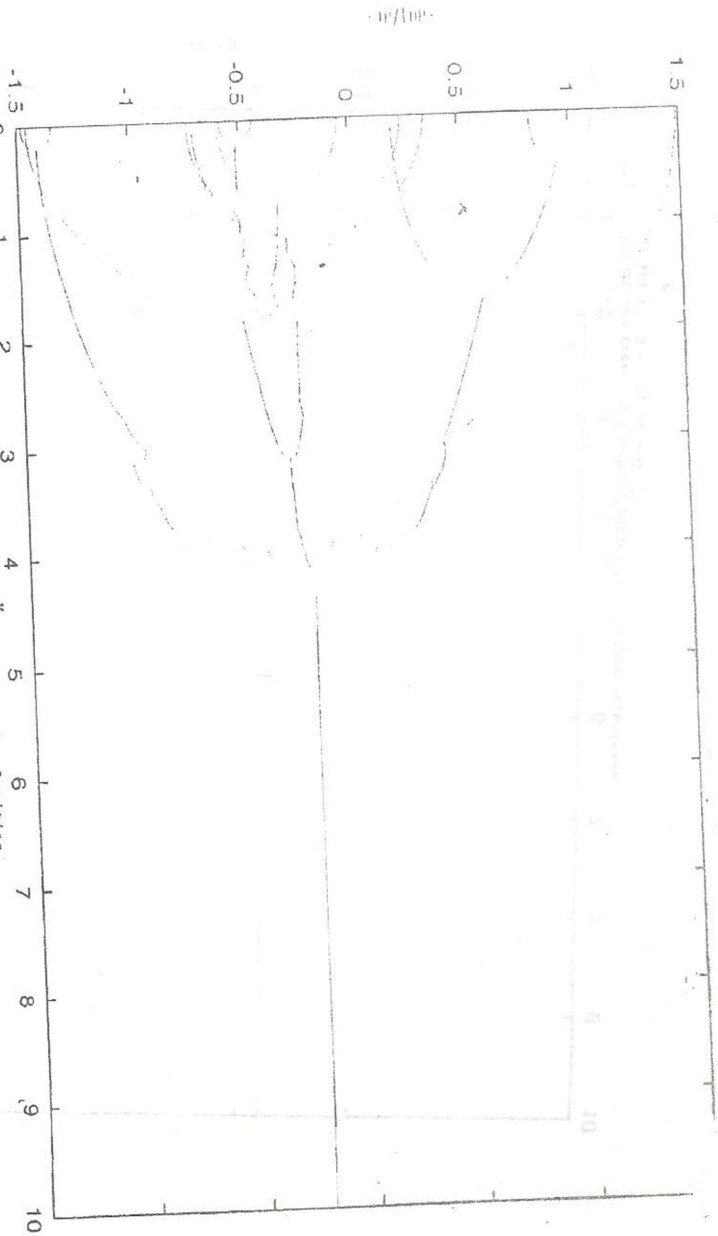

Fig. 1: Transition trees of Synchronization for average d velocities versus K N = 15, W_o = 1.77.

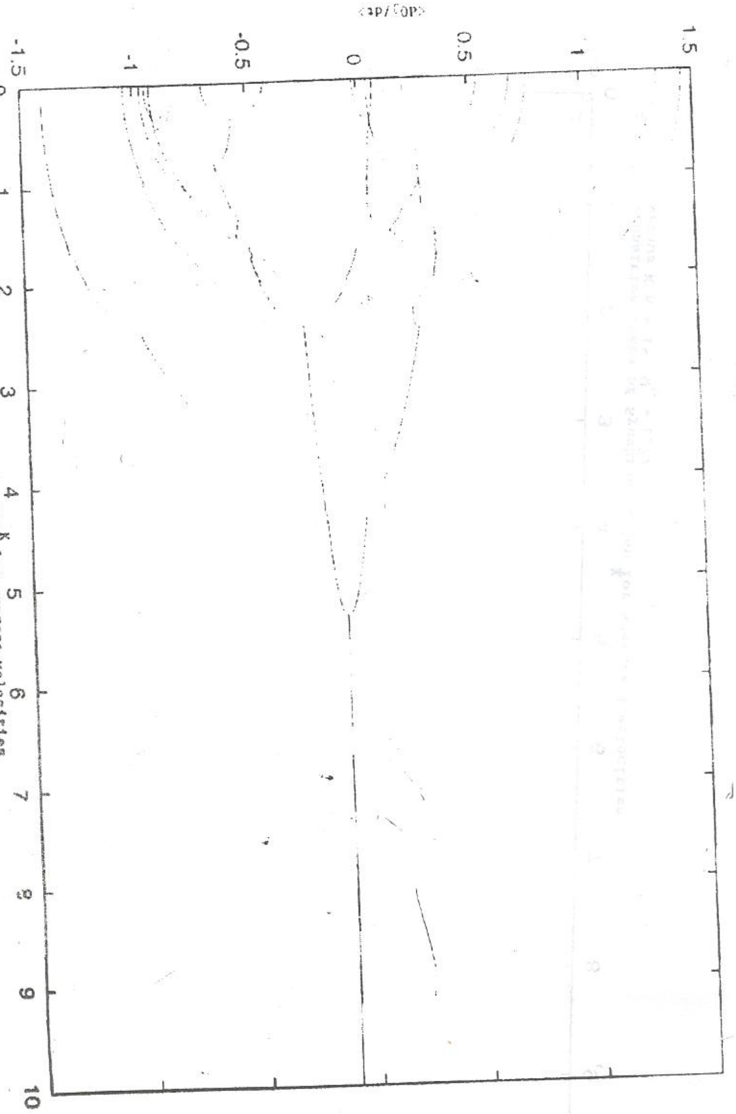

Fig. 2: Transition trees of Synchronization for average velocities versus K. N = 15, wo = 0.52.