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EXCITATION OF LOWER HYBRID WAVES BY ELECTRON BEAMS IN A GENERALIZED LORENTZIAN DISTRIBUTION

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ABSTRACT

A theory of lower hybrid wave instability by electron beams with generalized Lorentzian (kappa) drift distributions is presented. The system is one where a super-thermal electron beam, with temperature T_b and drift velocity v_b, streams through plasma containing electrons and ions, with temperatures T_e and T_i, respectively. The electrons and ions are both modeled by isotropic kappa distributions. The electrons appear as magnetized, and therefore treated using the drift kinetic equation, while the ions appear as un-magnetized and hence obey the electrostatic Vlasov equation. The frequency of the wave mode, ω_0, and the linear growth rate of the excitation, γ , are then derived, under any arbitrary temperature ratios, T_i/T_e, T_b/T_e, but equal spectral index, κ , for the three particle distributions functions.

This study would be useful in computing beams generated instabilities in space and laboratory plasmas.

1. INTRODUCTION

Satellite observations have established the presence of very low frequency (VLF) hiss in the earth's ionosphere (Gurnnet and Frank, 1972). This wave, which is in the lower hybrid frequency range, is very closely associated with electron precipitation in the auroral zones (Barrinton et al., 1971; Hoffman & Laaspere, 1972; Laaspere & Johnson, 1972). Anderson et al. (1981) also recorded similar occurrences for the solar wind region, the magnetosphere and the earth's bow shock. The observed excitations (or amplifications) of the lower hybrid waves, both in space and laboratory, have been studied extensively theoretically by invoking the electron beam plasma instabilities to calculate the growth rates; Papadopolous & Palmadesso, (1975) and Maggs, (1976). Chang and Coppi (1981) even went ahead to show that lower hybrid waves lead to ion acceleration. All the above authors, however, use Maxwellian distribution functions for their particle species in spite of the abundant evidences that, space in particular, consist of supra and super particles. In fact, very often, kinetic theoretical studies of waves in space and laboratory plasmas have centered on the investigation of plasmas near thermal equilibrium thereby assuming the Maxwellian or near Maxwellian distributions for the particles. Thus, disregarding the fact that, space plasmas contain high-energy particles, i.e. those particles whose velocities are in excess of the thermal speed, $v = (T/m)^{1/2}$, and whose distributions are often of the power-law of the form $v^{-\sigma}$, where σ is a real number (Mace 1996).

Recently, Thorne & Summers (1991) derived the landau damping for high and for low frequency electrostatic waves using the generalized Lorentzian distribution. Zhaoyue et al. (1991) have also studied the dispersion relation and calculated the growth rates, of ion-acoustic instability, driven by drifting electrons under numerous physical conditions. In addition, exhaustive studies were made by Summers & Thorne (1991) and Mace & Hellberg (1994) on the properties of the so-called modified plasma dispersion function $Z^*(\xi\ )$ which replaces the dispersion function, $Z(\xi\ )$, tabulated by Fried and Conte when kappa distribution is being used. In this paper we have re-visited the beam plasma instability theory and applied the generalized Lorentzian distribution functions as the particle distributions. Our results approach well-known expression for the lower hybrid modes and thier growth rates obtained from Maxwellian distributions. This is expected, because as the spectral index, , of kappa distribution approaches infinity, it becomes the two distribution functions become equal (Summers & Thorne, 1991).

2. THEORY

Let us consider a beam of electrons with density n_b, velocity, v_b, and thermal spread T_b, streaming, parallel to a magnetic field B_0z, in a plasma of density n (n >> n_b), electron thermal velocity T_e, and ion thermal velocity T_i. We assume a case of excitation of waves almost perpendicular to the magnetic field (i.e. k >> $k_{\|}$). We also consider the case where $\Omega_i << \omega << |\Omega_e|$ and $\rho^{-2}_i << k^2 << \rho^{-2}_e$ where k is the wave vector, Ω_i and Ω_e are the ion and electron gyro-frequencies and ρ_i and ρ_e are the ion and electron gyro-radii, respectively. For these modes, the plasma ions appear as un-magnetized while the plasma electrons as magnetized. The relevant dispersion relation can be obtained in analytic form using a Lorentzian ion distribution f^i

$$f^i{}_{\kappa}(v) = \frac{n_i\Gamma(\kappa+1)}{\pi^{3/2}\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\theta_i^{\ 3}}\left[1+\frac{v^2}{\kappa\theta_i^{\ 2}}\right]^{-(\kappa+1)} \tag{1}$$

and a model two-Lorentzian distribution for the electrons f^e

$$f^e{}_{\kappa}(v) = \frac{n_e\Gamma(\kappa+1)}{\pi^{3/2}\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\theta_e^{\ 3}}\left[1+\frac{v^2}{\kappa\theta_i^{\ 2}}\right]^{-(\kappa+1)} +$$

$$\frac{n_b\Gamma(\kappa+1)}{\pi^{3/2}\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\theta_b^{\ 3}}\left[1+\frac{v_x^2+v_y^2+(v_z-\tilde{v}_b)^2}{\kappa\theta_b^{\ 2}}\right]^{-(\kappa+1)} \tag{2}$$

where thermal velocity θ is

$$\theta_\alpha = \left(\frac{\kappa - \frac{3}{2}}{\kappa} \frac{2k_B T_\alpha}{m_\alpha} \right)^{\frac{1}{2}}$$

= i, e and b for ions, electrons and beams, respectively. κ is the spectral index, m is mass, T is temperature and k_B is the Maxwell – Boltzman constant.
The first order perturbed expansion of the Vlasov equation

$$\frac{\partial f^i}{\partial t} + \mathbf{v}.\nabla f^i + \frac{q}{m_i}\mathbf{E}.\frac{\partial f^i}{\partial \mathbf{v}} = 0 \tag{3}$$

gives the ion's contribution to the dispersion relation, where q is the charge and E, the electric field . On the other hand, the magnetization of the electrons forces us to use the drift kinetic equation to describe the electron dynamics (Hasegawa and Chen 1975):

$$\frac{\partial f^e}{\partial t} + \nabla_\perp(\mathbf{v}_\perp.\nabla f_0^e) + v_z \nabla_z f^e + \frac{q}{m}\mathbf{E}_z.\frac{\partial f^e}{\partial v_z} = 0 \tag{4}$$

The perpendicular drift velocity is given as,

$$\mathbf{v}_\perp = \frac{\mathbf{ExB}}{B^2} + \frac{m}{qB^2}E_\perp$$

By applying first order perturbation expansion to the distribution function equation (4) we would have the electron's part of the dispersion relation.
Therefore, the sum of equations (3) and (4) is

$$1 = \frac{\omega_i^2}{n_i k^2}\iiint \frac{\partial f^i/\partial v}{v - \omega/k} d^3v + \frac{\omega_b^2}{n_b k^2}\iiint \frac{\partial f^i/\partial v}{v_z - \omega/k} d^3v + \frac{\omega_e^2}{n_e k^2}\iiint \frac{\partial f^e/\partial v}{v - \omega/k} d^3v + \frac{m_e}{qB}E_\perp \tag{5}$$

where ω_α = (4 nq ^{2}/m)$^{1/2}$ is the plasma frequency and $\mathbf{ExB}/B^2 = 0$ for an electrostatic field. Finally, the dispersion relation is found to be

$$\xi Z_\kappa^* \xi + 1 - \frac{1}{2\kappa} + \frac{1}{\tau_i^2}\left[1 - \frac{1}{2\kappa} + \frac{\xi}{\varepsilon\tau_i} Z_\kappa^*\left(\tfrac{\xi}{\varepsilon\tau_i}\right)\right] +$$

$$\frac{1}{\bar{n}_b \tau_b^2}\left[1 - \frac{1}{2\kappa} + \frac{\xi}{\tau_b}(\xi - \tilde{v}_d) Z^*{}_\kappa(\xi - \tilde{v}_b)\right] = -\frac{\kappa - \frac{3}{2}}{\kappa}\tilde{k}^2\left(1 + \frac{\omega_e}{\Omega_e}\right) \tag{6}$$

In evaluating equation (5) to get (6), we have made use of the following normalizations:

$$\xi = \omega/k\,\sigma_e,\ \tau_\alpha = (T_\alpha/T_e)^{1/2},\ \epsilon = (m_e/m_i)^{1/2},\ \bar{n}_b = \frac{n}{n_b},\ \tilde{v}_b = v_b/\theta_b$$

and $\tilde{k} = k\lambda_D$; where the electron Debye length is

$$\lambda_n = \left(\frac{T_e}{4\pi n_0 q^2}\right)^{1/2} = \left(\frac{\theta_e}{\omega_e}\right)\left(\frac{\kappa}{2\kappa - 3}\right)^{1/2}$$

The properties of the modified dispersion function

$$Z_\kappa^*(\xi) = \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\kappa^{\frac{3}{2}}\Gamma(\kappa-\frac{1}{2})}\int_{-\infty}^{\infty}\frac{ds}{(s-\xi)(1+s^2/\kappa)^{\kappa+1}}$$

have been described thoroughly in Summers & Thorne (1991) and in Mace & Hellberg (1994).

In the realistic limits, ξ ,ξ /$\mathbf{e}_T$ >>1 and $\left|\xi - \tilde{v}_b\right| << 1$, we apply the approximate expansions of the Z^*_κ function for the large and the small arguments (Thorne & Summers 1991):

for x >> 1

$$Z_\kappa^*(x) \approx i\pi^{\frac{1}{2}}\frac{\kappa!\kappa^{\kappa-\frac{1}{2}}}{\Gamma(\kappa-\frac{1}{2})(x)^{2(\kappa+1)}} - \frac{(2\kappa-1)}{2\kappa}\frac{1}{x}$$

and for x << 1

$$Z_\kappa^*(x) \approx i\pi^{\frac{1}{2}}\frac{\kappa!}{\kappa^{\frac{3}{2}}\Gamma(\kappa-\frac{1}{2})} - \frac{(2\kappa-1)(2\kappa+1)}{2\kappa^2}x$$

Consequently, on solving equation (6) in the above limits and also assuming ω= ω₀ + i**γ** , where ω₀ >> , **γ** the normalized wave mode is found to be

$$\frac{\omega_0}{\omega_e} \approx \frac{\sqrt{2}(\kappa-\frac{3}{2})^{1/2}(A\tilde{k}^2 + B\tilde{k})}{C} \qquad (7)$$

where

$$A = \tau_e^3\tau_i^4((\kappa-\tfrac{1}{2})\omega_i^2 - (\kappa-\tfrac{3}{2}))$$

$$B = n_b v_b \varepsilon(\kappa-\tfrac{1}{2})\tau_i^3 + \tau_e^3(\kappa-\tfrac{1}{2}-\tfrac{1}{2}\tau^2+\kappa\tau_i^2 - \varepsilon(\kappa-\tfrac{1}{2})\tau_i^3 + (\kappa-\tfrac{1}{2})\tau_i^4))\Omega_e$$

$$C = n_b\kappa^{1/2}\tau_i^3\varepsilon((\kappa - -\tfrac{1}{2}) + (\kappa-\tfrac{1}{2})\tau_e\tau_i)\Omega_e^2$$

The growth rate is calculated as

$$\frac{\gamma}{\omega_e} \approx \frac{(\kappa-\frac{3}{2})^{3/2}\tilde{k}^2\varepsilon^4\kappa^{\kappa+1/2}\omega_e^2\kappa!}{\sqrt[3]{2}\kappa^{1/2}\tau_i^3(\varepsilon-(\kappa+\frac{1}{2})+(\kappa-\frac{1}{2})\tau_e\tau_i)\omega_0^2\Gamma(\kappa+\frac{1}{2})} \qquad (8)$$

The generalized Lorentzian distribution approaches the Maxwellian for k and therefore equations (7) and (8) approximately become:

$$\frac{\omega_0}{\omega_e} \approx \frac{\varepsilon}{1+\delta^2} \qquad (9)$$

and

$$\frac{\gamma}{\omega_e} \approx -\omega_0 \left(\frac{\sqrt{\pi}\,\overline{n}_b \omega_e}{\widetilde{k}\,(1+\delta^2)} \right) \tag{10}$$

where $\epsilon = (k / k)^{1/2} \approx (m_e/m_i)^{1/2}$ and $\delta^2 = \frac{\omega_e^{\ 2}}{\Omega_e^2} + \overline{n}_b \frac{\omega_e^2}{\widetilde{k}^2} \operatorname{Re} Z$,

(Papadoupolous & Palmadesso, 1975)

3. CONCLUSIONS

Even when the particle species are of high, but non-relativistic, energy, Maxwellian distributions have commonly been used to carry out theoretical laboratory and space plasmas studies. These anomalies were pointed out by some authors, (Summers & Thorne 1991; Zhaoyue 1991; Mace 1996;), who have now recalculated some well-known results with the appropriate kappa distribution functions. In our case we considered one- and two- generalized Lorentzian (kappa) distributions for the ions and the electrons, respectively, to recalculate results of beam plasma interaction involving high-energy particles. The new expressions obtained for the wave mode and growth rate of the beam excited lower hybrid waves should be used in relevant theoretical or experimental computations.

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