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CRITICAL FIELD/CRITICAL TEMPERATURE CORRELATION IN CONVENTIONAL SUPERCONDUCTORS

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ABSTRACT

In this paper we report a curious analytical relationship between the critical field at absolite zero, $B_c(0)$, and the critical temperature, T_c, for elemental superconductors. We show that the correlation between the two variables is given by the quadratic curve:

$$B_{c_S}(0) = 5.21597T_c + 0.828175T_c^2 \quad (units\ in Tesla),$$

augmented with a sequence of gaussian exponential ripples.

1. INTRODUCTION

In the BCS theory of superconductivity, the crucial pair-interaction matrix element or gap function is obtained through a mean field argument. There are other simplifying assumptions built into the theory. It is expected therefore that the basic results may agree only in outline with experiment, and that detailed refinements of the theory can only come from empirical considerations through the comparison of predictions of the theory with detailed experimental results. A theory based entirely on first principles calculations is not likely to accommodate the mass of parameters that goes into the definition of a typical physical system.

In this paper, we look at the relationship between the critical magnetic field at absolute zero $H_c(0) = B_c(0) / \mu$ for an elemental superconductor and the critical temperature T_c, as seen from the BCS theory, and compare it with known experimental data. We are able to suggest refinements of the BCS formula to match the experimental situation.

2. CORRELATION BETWEEN CRITICAL FIELD AND CRITICAL TEMPERATURE.

From the study of the thermodynamics of a type I superconductor within the BCS framework, we can deduce a relationship between the critical field, $H_c(0) = B_c(0) / \mu$, at absolute zero and the critical Temperature, T_c, where $B(0)$ is the magnetic induction and μ is the permeability. This relationship is (Rickayzen, 1965):

$$H_c(0) = 0.396k \frac{(2m^3 E_F)^{1/4}}{\hbar^3} T_c, \qquad (1)$$

Where E_F = Fermi Energy,
m = electronic mass,
K = Boltzmann's Constant, and
$h = 2\pi\hbar$ is Planck's Constant.

Thus according to this theory, $B_c(0)$ varies linearly with T_c. What we have done in this paper is to examine this relationship empirically by using known data to plot a graph of $B_c(0)$ against T_c and then to fit this with a suitable curve.

The result is shown in Fig. 1 for elemental superconductors. It shows that the data can be fitted quite accurately with the curve:

$$B_C(0) = \alpha\, T_C + \beta T_C^2 + \sum_{i=1}^{5} A(i) \exp\left(- \frac{\left[T_C - D(i) \right]^2}{C(i)} \right) \quad (units\ in\ Tesla),$$

Where the coefficients are given in table 1. On the graph of this function is superimposed the data points given in Table 2.

Table 1 Coefficients for the curve fitting

n	A(n)	D(n)	C(n)
1	46.23998	0.4357916	1.18156E-03
2	5.700129	0.8508219	0.01062147
3	11.84703	1.5459560	0.0708075
4	59.33410	5.6091020	0.8363498
5	4765.580	8.5341260	0.1283618
	α = 5.215972		β = 0.8281747

The ripples due to the sequence of gaussian exponential terms are small for small values of T_c and large for large values of T_c. Thus dividing eqn (2) by T_c will dampen the exponential terms for large T_c, sufficiently enough to produce a linear trend in the graph of $B_c(0) / T_c$ against T_c. Thus one may without loss of generality drop the exponetial terms and we have:

$$B_C(0) = 5.21597 T_C + 0.828175 T_C^2 \quad (units\ in\ Tes'a) \tag{3}$$

If on the other hand one does not ignore the exponential terms completely, then the variation of $B_c(0) / T_c$ with T_c, shown in fig . 2 clearly shows the expected linear trend. From this Figure the linear trend is given by

$$B_c(0) = 9.23[illegible] T_c + 0.[illegible]7 T_c^2 \quad (unit\ in\ Tesla), \tag{4}$$

Which compares quite well with Eq. (3).

Table 2 Critical fields $B_c(0)$ and the corresponding critical temperatures for elemental superconductors. (Data from Ashcroff & Mermin, 1976)

Element	Symbol	T_c K	$B_c(0)$ mTesla
Tungsten	W	0.012	0.1
Iridium	Ir	0.14	1.9
Titanium	Ti	0.39	10.0
Ruthenium	Ru	0.49	6.6
Cadmium	Cd	0.56	3.0
Zirconium	Zr	0.65	4.7
Osmium	Os	0.655	6.5
Zinc	Zn	0.875	5.3
Molybdenum	Mo	0.92	9.8
Gallium	Ga	1.091	5.1
Aluminium	Al	1.196	9.9
Thorium	Th	1.368	16.2
Rhenum	Re	1.698	19.8
Thalium	Tl	2.39	17.1
Indium	In	3.4	29.3
Tin	Sn	3.72	30.5
Mercury	Hg β	3.95	33.9
Mercury	Hg α	4.15	41.1
Tantalum	Ta	4.48	83.0
Lanthanum	La α	4.9	79.8
Vanadium	V	5.3	102.0
Lanthanum	La β	6.06	109.6
Lead	Pb	7.19	80.3
Technetium	Tc	7.77	141.0
Niobium	Nb	9.26	198.0

Correlation between Bc(0) and Tc for elemental superconductors

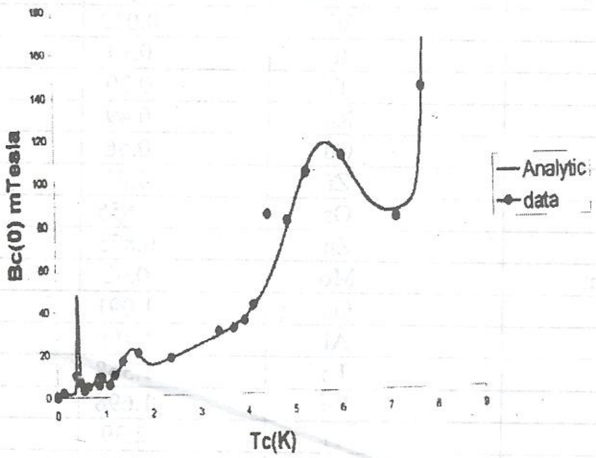

Fig. 1 : Correlation of Critical fields with critical temperatures.
(Fig. 1 is a plot of the analytic function:

$$B_c\ (0) = \alpha\ T_c + \beta\ T_c^2 + \sum_{i=1}^{5} A(i) \exp\left(- \frac{\left[T_c - D(i)\right]^2}{C(i)} \right) \quad (units\ in Tesla)$$

where the coefficients are given in table 1. The data points given in Table 2 are superimposed on this curve).

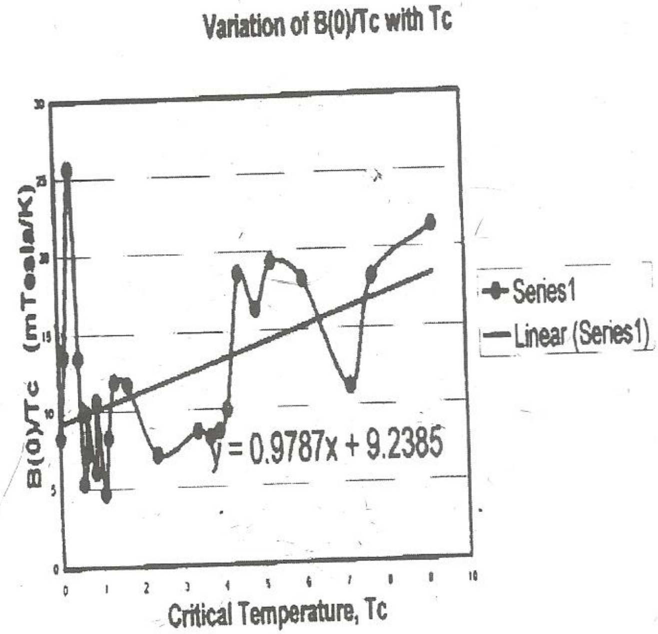

Fig 2. Linear trend in the B_c (0)/T_c vs. T_c curve

3. CONCLUSION

We have shown how an empirical approach could lead to a significant refinement of the BCS theory in respect of the relationship between the critical field B_c (0) and the critical temperature T_c . The value of such a simple empirical relation is that it might carry over to the theory of High Temperature Superconductors.

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