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A TENSOR IDENTITY IN QUANTUM FIELD THEORY

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ABSTRACT

Using the methods of spinor calculus, we prove the identity

$$\overline{\Psi}'\left(\chi'\right)\sigma_4^{\mu\nu}\Psi'\left(\chi'\right)=\Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}\overline{\Psi}(\chi)\sigma_4^{\rho\sigma}\Psi(\chi)$$

INTRODUCTION

A great insight into the structure of the Lorentz group, $L_+^{\uparrow}$, is gained by bringing to bear on it the methods of spinor calculus [1]. Let GL (2,C) be the general linear group in two dimensions. Then the special linear group, SL(2,C), is defined by SL(2,C) = {M ∈ GL(2,C) | det M = +1}

There are two inequivalent representations of SL(2,C). The first is the self-representation defined by D(M) = M ∀ M ∈ SL(2,C),
where D(M) is a linear map from SL(2,C) to the automorphism group of a linear vector space F with elements ϕ_A , A = 1,2. The second representation is the complex conjugate self-representation defined by D(M) = M*∀M ∈ SL(2,C)

The representation space in this case is denoted by $\dot{F}$ with elemen $\psi_{\dot{A}}$ $\dot{A}$ = 1,2.

It is found that D(M) = M^{-1T} is an equivalent representation of D(M) = M. The representation space of D(M) = M^{-1T} is denoted by F* with element ϕ^A A=1,2.
Because of the equivalence, there exists a 2x2 matrix

$$\varepsilon^{AB}=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}=\left(\varepsilon_{AB}\right)^{-1}$$

such that

$$\left(M^{-1T}\right)^{A}{}_{B}=\varepsilon^{AC}M_{C}{}^{D}\varepsilon_{DB} \qquad (1)$$

Similarly, it is found that M* and M^{*-1T} are equivalent representations of M with the representation space of M^{*-1T} denoted by $\dot{F}^*$ with elements $\bar{\Psi}^{\dot{A}}$ $\quad \dot{A} = 1,2$.

Hence, there exists a 2x2 matirx

$$\varepsilon^{\dot{A}\dot{B}} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \left(\varepsilon_{\dot{A}\dot{B}}\right)^{-1}$$

such that

$$\left(M^{*-1T}\right)^{\dot{A}}{}_{\dot{B}} = \varepsilon^{\dot{A}\dot{C}}\left(M^{*}\right)_{\dot{C}}{}^{\dot{D}}\varepsilon_{\dot{D}\dot{B}} \qquad (2)$$

Eqs. (1) and (2) are known respectively as representations with undotted and dotted indices.

Under SL(2,C) covariant and contra variant spinors with undotted indices transform respectively as follows:

$$\phi_A = M_A{}^{B}\phi_B \qquad (3a)$$

$$\phi^A = \left(M^{-1T}\right)^{A}{}_{B}\phi^B \qquad (3b)$$

Similarly, covariant and contravariant spinors with dotted indices transform respectively as follows:

$$\bar{\Psi}_{\dot{A}} = \left(M^{*}\right)_{\dot{A}}{}^{\dot{B}}\bar{\Psi}_{\dot{B}} \qquad (4a)$$

$$\bar{\Psi}^{\dot{A}} = \left(M^{*-1T}\right)^{\dot{A}}{}_{\dot{B}}\bar{\Psi}^{\dot{B}} \qquad (4b)$$

In a previous paper [2], the author proved, using the methods of spinor calculus, the following two well-known scalar and vector identities:

$$\bar{\Psi}'\left(x'\right)\Psi'\left(x'\right) = \bar{\Psi}(x)\Psi(x)$$

and

$$\bar{\Psi}'\left(x'\right)\gamma^{\mu}\Psi'\left(x'\right) = \Lambda^{\mu}{}_{\nu}\,\bar{\Psi}(x)\gamma^{\nu}\,\Psi(x)$$

Here $\Psi(x)$ is the Dirac four-spinor in the Weyl (chiral) representation

$$\Psi_a(x) = \begin{pmatrix} \phi_A \\ \bar{\psi}^{\dot{A}} \end{pmatrix} \qquad a = 1,2,3,4 \qquad (5)$$

and (also in the chiral representation)

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu}_{A\dot{B}} \\ \bar{\sigma}^{\mu\dot{A}B} & 0 \end{pmatrix}$$

Here

$$\sigma^{\mu} = \left(\sigma^{0}, \sigma\right) = \left(1, \sigma\right)$$

and

$$\bar{\sigma}^{\mu} = \left(\bar{\sigma}^{0}, \bar{\sigma}\right) = \left(1, -\sigma\right) \quad (7b)$$

where $\sigma = \left(\sigma^{1}, \sigma^{2}, \sigma^{3}\right)$ are the three Pauli matrices

In the present paper we shall prove the second-rank tensor identity

$$\bar{\Psi}'\left(x'\right)\sigma_{4}^{\mu\nu}\Psi'\left(x'\right) = \Lambda^{\mu}{}_{\rho}\Lambda^{\nu}{}_{\sigma}\,\bar{\Psi}(x)\sigma_{4}^{\rho\sigma}\Psi(x) \quad (8)$$

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where

$$\sigma_{4}^{\mu\nu} = \frac{i}{2}\left[\gamma^{\mu}, \gamma^{\nu}\right] \quad (9)$$

and $\Lambda^{\mu}{}_{\nu}$ are elements of the matrix of the restricted Lorentz group $L_{+}^{\uparrow}$

given by $\Lambda^{\mu}{}_{\nu} = \frac{1}{2} Tr\left[M^{+}\bar{\sigma}^{\mu} M \sigma_{\nu}\right]$ (10)

PROOF OF THE TENSOR IDENTITY

We now prove the second-rank tensor identity

$$\bar{\Psi}'\left(x'\right)\sigma_{4}^{\mu\nu}\Psi'\left(x'\right) = \Lambda^{\mu}{}_{\rho}\Lambda^{\nu}\sigma\,\bar{\Psi}(x)\sigma_{4}^{\rho\sigma}\Psi(x)$$

where

$$\sigma_{4}^{\mu\nu} = \frac{i}{2}\left[\gamma^{\mu}, \gamma^{\nu}\right] = \frac{i}{2}\gamma^{\mu}\gamma^{\nu} - \frac{i}{2}\gamma^{\nu}\gamma^{\mu}$$

with

$$\Psi(x) = \begin{pmatrix} \phi A \\ \overline{\Psi}_{\dot{A}} \end{pmatrix}$$

$$\overline{\Psi}(x) = \Psi^{+}(x)\gamma^{0} = \left(\phi^{A*}, \overline{\psi}^{*}_{\dot{A}}\right) \begin{pmatrix} 0 & \sigma^{0}_{A\dot{B}} \\ \overline{\sigma}^{0\dot{A}B} & 0 \end{pmatrix}$$

$$= \left(\overline{\Psi}^{*}_{\dot{A}}\, \overline{\sigma}^{0\dot{A}B}, \phi^{A*}\sigma^{0}_{A\dot{B}}\right)$$

On using

$$\phi^{B} = \overline{\Psi}^{*}_{\dot{A}}\, \overline{\sigma}^{0\dot{A}B} \tag{11a}$$

$$\overline{\Psi}_{\dot{B}} = \phi^{A*}\sigma^{0}_{A\dot{B}} \tag{11b}$$

we obtain

$$\overline{\Psi}(x) = \left(\phi^{B}, \overline{\Psi}_{\dot{B}}\right)$$

Then, by Eq. (9)

$$\overline{\Psi}(x)\sigma_{4}^{\rho\sigma}\Psi(x)$$

$$= \frac{i}{2}\left(\phi^{A}, \overline{\Psi}_{\dot{A}}\right)\begin{pmatrix} 0 & \sigma^{\rho}_{A\dot{B}} \\ \overline{\sigma}^{\rho\dot{A}B} & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^{\sigma}_{B\dot{C}} \\ \overline{\sigma}^{\sigma\dot{B}C} & 0 \end{pmatrix}\begin{pmatrix} \phi_{C} \\ \overline{\Psi}^{\dot{C}} \end{pmatrix}$$

$$- \frac{i}{2}\left(\phi^{A}, \overline{\Psi}_{\dot{A}}\right)\begin{pmatrix} 0 & \sigma^{\sigma}_{A\dot{B}} \\ \overline{\sigma}^{\sigma\dot{A}B} & 0 \end{pmatrix}\begin{pmatrix} 0 & \sigma^{\rho}_{B\dot{C}} \\ \overline{\sigma}^{\rho\dot{B}C} & 0 \end{pmatrix}\begin{pmatrix} \phi_{C} \\ \overline{\Psi}^{\dot{C}} \end{pmatrix} \tag{12}$$

since

$$\overline{\Psi}'(x') = \left(\phi'^{A}, \overline{\Psi}'_{\dot{A}}\right),$$

$$\overline{\Psi}'(x')\sigma_{4}^{\mu\nu}\Psi'(x') =$$

$$\frac{i}{2}\left(\phi'^{A},\overline{\Psi}_{\dot{A}}\right)\begin{pmatrix}0 & \sigma^{\mu}_{A\dot{B}}\\ \overline{\sigma}^{\mu\dot{A}B} & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^{\nu}_{B\dot{C}}\\ \overline{\sigma}^{\nu\dot{B}C} & 0\end{pmatrix}\begin{pmatrix}\phi'c\\ \overline{\Psi}'^{\dot{C}}\end{pmatrix}$$

$$-\frac{i}{2}\left(\phi'^{A},\overline{\Psi}'_{\dot{A}}\right)\begin{pmatrix}0 & \sigma^{\nu}_{A\dot{B}}\\ \overline{\sigma}^{\nu\dot{A}B} & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^{\mu}_{B\dot{C}}\\ \overline{\sigma}^{\mu\dot{B}C} & 0\end{pmatrix}\begin{pmatrix}\phi'c\\ \overline{\Psi}'^{\dot{C}}\end{pmatrix} \tag{13}$$

Initially, let us consider the first term of Eq. (13)

$$\text{i.e.,}\ \frac{i}{2}\left(\phi'^{A},\overline{\Psi}'_{\dot{A}}\right)\begin{pmatrix}0 & \sigma^{\mu}_{A\dot{B}}\\ \overline{\sigma}^{\mu\dot{A}B} & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^{\nu}_{B\dot{C}}\\ \overline{\sigma}^{\nu\dot{B}C} & 0\end{pmatrix}\begin{pmatrix}\phi'c\\ \overline{\Psi}'^{\dot{C}}\end{pmatrix}$$

(On using Eqs. (3b), (4a), (3a), and (4b))

$$=\frac{i}{2}\left(\left(M^{-1T}\right)^{A}{}_{B}\phi^{B},\left(M^{*}\right)_{\dot{A}}{}^{\dot{B}}\overline{\Psi}_{\dot{B}}\right)\begin{pmatrix}0 & \sigma^{\mu}_{A\dot{C}}\\ \overline{\sigma}^{\mu\dot{A}C} & 0\end{pmatrix}\begin{pmatrix}0 & \sigma^{\nu}_{C\dot{D}}\\ \overline{\sigma}^{\nu\dot{C}D} & 0\end{pmatrix}\begin{pmatrix}M_{D}{}^{E}\phi_{E}\\ \left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{E}}\end{pmatrix}$$

$$=\frac{i}{2}\left[(M^{*})_{\dot{A}}{}^{\dot{B}}\overline{\Psi}_{\dot{B}}\overline{\sigma}^{\mu\dot{A}C},\left(M^{-1T}\right)^{A}{}_{B}\phi^{B}\sigma^{\mu}{}_{A\dot{C}}\right]\begin{bmatrix}\sigma^{\nu}_{C\dot{D}}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{E}}\\ \overline{\sigma}^{\nu\dot{C}D}M_{D}{}^{E}\phi_{E}\end{bmatrix}$$

$$=\frac{i}{2}\left[(M^{*})_{\dot{A}}{}^{\dot{B}}\overline{\Psi}_{\dot{B}}\overline{\sigma}^{\mu\dot{A}C}\sigma^{\nu}_{C\dot{D}}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{E}}+\left(M^{-1T}\right)^{A}{}_{B}\phi^{B}\sigma^{\mu}_{A\dot{C}}\overline{\sigma}^{\nu\dot{C}D}M_{D}{}^{E}\phi_{E}\right]$$

$$=\frac{i}{2}\left[(M^{*})_{\dot{A}}{}^{\dot{B}}\overline{\Psi}_{\dot{B}}\overline{\sigma}^{\mu\dot{A}C}M_{C}{}^{D}\right]\left[\left(M^{-1}\right)_{D}{}^{C}\sigma^{\nu}_{C\dot{D}}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{E}}\right]$$

$$+\frac{i}{2}\left[\left(M^{-1T}\right)^{A}{}_{B}\phi^{B}\sigma^{\mu}{}_{A\dot{C}}\left(M^{*-1T}\right)^{\dot{C}}{}_{\dot{D}}\right]\left[\left(M^{*T}\right)^{\dot{D}}{}_{\dot{C}}\overline{\sigma}^{\nu\dot{C}D}M_{D}{}^{E}\phi_{E}\right] \tag{14}$$

We first consider the first term in Eq. (14). Assuming that $\phi_{D}\phi^{D}\neq 1$, this can be rewritten as

$$\frac{i}{2}\left[(M^{*})_{\dot{A}}{}^{\dot{B}}\overline{\Psi}_{\dot{B}}\overline{\sigma}^{\mu AC}M_{C}{}^{D}\phi_{D}\right]\left[\phi^{D}\left(M^{-1}\right)_{D}{}^{C}\sigma^{\nu}_{C\dot{D}}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{E}}\right] \tag{15}$$

From linear algebra one may recall that $F^{*}\left(\dot{F}\cong\dot{F}^{**}\right)$ is the dual space of $F\left(\dot{F}^{*}\right)$

The first bracket in Eq. (15) then becomes

$$(M^{*})_{\dot{A}}{}^{\dot{B}}\delta_{\dot{B}}{}^{\dot{F}}\overline{\Psi}_{\dot{F}}\overline{\sigma}^{\mu\dot{A}C}M_{C}{}^{D}\delta_{D}{}^{E}\phi_{E}$$

(by using the identity

$$2\delta_{\dot{B}}{}^{\dot{F}}\delta_{D}{}^{E} = \overline{\sigma}^{\rho\dot{F}E}\sigma_{\rho D\dot{B}} \tag{16}$$

$$= \frac{1}{2}\left[(M^*)_{\dot{A}}{}^{\dot{B}}\overline{\sigma}^{\rho\dot{F}E}\sigma_{\rho D\dot{B}}\overline{\Psi}_{\dot{F}}\overline{\sigma}^{\mu\dot{A}C}M_{C}{}^{D}\phi_{E}\right]$$

$$= \frac{1}{2}\left[\left(M^{+}\right)^{\dot{B}}{}_{\dot{A}}\overline{\sigma}^{\mu\dot{A}C}M_{C}{}^{D}\sigma_{\rho D\dot{B}}\right]\left[\overline{\Psi}_{\dot{F}}\overline{\sigma}^{\rho\dot{F}E}\phi_{E}\right]$$

$$= \frac{1}{2}Tr\left[M^{+}\overline{\sigma}^{\mu}M\sigma_{\rho}\right]\left[\overline{\Psi}_{\dot{F}}\overline{\sigma}^{\rho\dot{F}E}\phi_{E}\right]$$

(by Eq. (10))

$$= \Lambda^{\mu}{}_{\rho}\overline{\Psi}_{\dot{F}}\overline{\sigma}^{\rho\dot{F}E}\phi_{E} \tag{17}$$

We now consider the second bracket in Eq. (15)
i.e.,

$$\phi^{D}\left(M^{-1}\right)_{D}{}^{C}\sigma^{\nu}{}_{C\dot{D}}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{E}}$$

$$= \left(M^{-1T}\right)^{C}{}_{D}\phi^{D}\sigma^{\nu}{}_{C\dot{D}}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{E}}$$

(On making use of the identity $\sigma^{\nu}{}_{C\dot{D}} = \varepsilon_{CE}\varepsilon_{\dot{D}\dot{F}}\overline{\sigma}^{\nu\dot{F}E}$)

$$= \left(M^{-1T}\right)^{C}{}_{D}\phi^{D}\varepsilon_{CE}\varepsilon_{\dot{D}\dot{F}}\overline{\sigma}^{\nu\dot{F}E}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{E}}$$

$$= \left(M^{-1T}\right)^{C}{}_{D}\delta^{D}{}_{F}\phi^{F}\varepsilon_{CE}\varepsilon_{\dot{D}\dot{F}}\overline{\sigma}^{\nu\dot{F}E}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\delta^{\dot{E}}{}_{\dot{G}}\overline{\Psi}^{\dot{G}}$$

(by Eq. (17))

$$= \frac{1}{2}\left(M^{-1T}\right)^{C}{}_{D}\overline{\sigma}^{\sigma\dot{E}D}\sigma_{\sigma F\dot{G}}\phi^{F}\varepsilon_{CE}\varepsilon_{\dot{D}\dot{F}}\overline{\sigma}^{\nu\dot{F}E}\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}}\overline{\Psi}^{\dot{G}} \tag{18}$$

On using $\left(M^{-1T}\right)^{C}{}_{D} = \varepsilon^{CG}M_{G}{}^{H}\varepsilon_{HD}$ (19a)

and $\left(M^{*-1T}\right)^{\dot{D}}{}_{\dot{E}} = \varepsilon^{\dot{D}\dot{H}}(M^*)_{\dot{H}}{}^{\dot{J}}\varepsilon_{\dot{J}\dot{E}}$ (19b)

Eq. (18) becomes

$$= \frac{1}{2} M_G{}^H \varepsilon^{CG} \varepsilon_{HD} \bar{\sigma}^{\sigma \dot{E} D} \sigma_{\sigma F \dot{G}} \phi^F \varepsilon_{CE} \varepsilon_{\dot{D}\dot{F}} \bar{\sigma}^{\nu \dot{F} E} (M^*)_{\dot{H}}{}^{\dot{J}} \varepsilon^{\dot{D}\dot{H}} \varepsilon_{\dot{J}\dot{E}} \bar{\Psi}^{\dot{G}} \quad (20)$$

Upon using $\sigma^{\sigma}_{H\dot{J}} = \varepsilon_{HD} \varepsilon_{\dot{J}\dot{E}} \bar{\sigma}^{\sigma \dot{E} D}$ (21)

Eq. (20) becomes

$$= \frac{1}{2} M_G{}^H \varepsilon^{CG} \varepsilon^{\dot{D}\dot{H}} \sigma^{\sigma}{}_{H\dot{J}} \sigma_{\sigma F \dot{G}} \phi^F \varepsilon_{CE} \varepsilon_{\dot{D}\dot{F}} \bar{\sigma}^{\nu \dot{F} E} (M^*)_{\dot{H}}{}^{\dot{J}} \bar{\Psi}^{\dot{G}} \quad (22)$$

Upon using $\varepsilon^{CG} \varepsilon_{CE} = \left(\varepsilon^T\right)^{GC} \varepsilon_{CE} = -\delta^G{}_E$

and $\varepsilon^{\dot{D}\dot{H}} \varepsilon_{\dot{D}\dot{F}} = \left(\varepsilon^T\right)^{\dot{H}\dot{D}} \varepsilon_{\dot{D}\dot{F}} = -\delta^{\dot{H}}{}_{\dot{F}}$

we have

$$\phi'^C \sigma^{\nu}_{C\dot{D}} \bar{\Psi}'^{\dot{D}} = \frac{1}{2} M_E{}^H \sigma^{\sigma}{}_{H\dot{J}} \sigma_{\sigma F \dot{G}} \phi^F \bar{\sigma}^{\nu \dot{F} E} (M^*)_{\dot{F}}{}^{\dot{J}} \bar{\Psi}^{\dot{G}}$$

$$= \frac{1}{2} \left[\bar{\sigma}^{\nu \dot{F} E} M_E{}^H \sigma_{\sigma H \dot{J}} \left(M^+\right)^{\dot{J}}{}_{\dot{F}} \right] \phi^F \sigma^{\sigma}{}_{F\dot{G}} \bar{\Psi}^{\dot{G}}$$

On using Eq. (10) we have

$$\phi'^C \sigma^{\nu}_{C\dot{D}} \bar{\Psi}'^{\dot{D}} = \Lambda^{\nu}{}_{\sigma} \phi^F \sigma^{\sigma}{}_{F\dot{G}} \bar{\Psi}^{\dot{G}} \quad (23)$$

Eq. (14) now becomes

$$\frac{i}{2} \left(\Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} \bar{\Psi}_{\dot{F}} \bar{\sigma}^{\rho \dot{F} F} \phi_F \right) \left(\phi^F \sigma^{\sigma}{}_{F\dot{G}} \bar{\Psi}^{\dot{G}} \right)$$

$$= \frac{i}{2} \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} \bar{\Psi}_{\dot{F}} \bar{\sigma}^{\rho \dot{F} F} \sigma^{\sigma}{}_{F\dot{G}} \bar{\Psi}^{\dot{G}} \quad (24)$$

We can rewrite the second term of Eq. (14) as

$$\frac{i}{2} \left[\left(M^{-1T}\right)^A{}_B \phi^B \sigma^{\mu}_{A\dot{C}} \left(M^{*-1T}\right)^{\dot{C}}{}_{\dot{D}} \bar{\Psi}^{\dot{D}} \right] \left[\bar{\Psi}_{\dot{D}} \left(M^{*T}\right)^{\dot{D}}{}_{\dot{C}} \bar{\sigma}^{\nu \dot{C} D} M_D{}^E \phi_E \right]$$

By comparing this with Eq. (15), and bearing in main the steps leading to Eqs. (17) and (23), we find that this second term is

$$= \frac{i}{2} \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} \phi^{A} \sigma^{\rho}_{A\dot{C}} \bar{\sigma}^{\sigma\dot{C}D} \phi_{D} \tag{25}$$

In accordance with the steps leading from the first term of Eq. (13) to Eq. (14), both Eqs. (24) and
(25) now yield

$$\frac{i}{2} \overline{\Psi}'(x') \gamma^{\mu} \gamma^{\nu} \Psi'(x') = \frac{i}{2} \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} \overline{\Psi}(x) \gamma^{\rho} \gamma^{\sigma} \Psi(x) \tag{26}$$

Similarly, the second term of Eq. (13) yields

$$\frac{i}{2} \overline{\Psi}'(x') \gamma^{\nu} \gamma^{\mu} \Psi'(x') = \frac{i}{2} \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} \overline{\Psi}(x) \gamma^{\sigma} \gamma^{\rho} \Psi(x) \tag{27}$$

By comparing Eqs. (14) and (13), Eqs. (26) and (27) yield

$$\overline{\Psi}'(x') \sigma_{4}^{\mu\nu} \Psi'(x') = \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} \overline{\Psi}(x) \sigma_{4}^{\rho\sigma} \Psi(x)$$

which completes the proof of the identity.

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