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TRANSFORMATION PROPERTIES OF PSEUDOSCALAR AND PSEUDOVECTOR BILINEAR FORMS

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ABSTRACT

By using spinor calculus the following Lorentz transformation properties of pseudoscalar and pseudovector bilinear forms are proved:

$\overline{\Psi}'(x')\gamma_5\Psi'(x') = \det[\Lambda]\,\overline{\Psi}(x)\gamma_5\Psi(x)$ and

$\overline{\Psi}'(x')\gamma_5\gamma^{\mu}\Psi'(x') = \det[\Lambda]\Lambda^{\mu}{}_{\nu}\,\overline{\Psi}(x)\gamma_5\gamma^{\nu}\Psi(x)$

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INTRODUCTION

Let GL(4,R) be the general linear group on R^4, then the Lorentz group L is defined as follows [Muller-Kirsten 1987]

$$L \equiv 0(1,3) = \{\Lambda \in GL(4,R) \mid \Lambda^T g\Lambda = g\}$$

where g is the metric tensor

g = diag(1,-1,-1,-1)

The proper Lorentz group L_+ is defined by

$$L_+ \equiv S0(1,3) = \{\Lambda \in 0(1,3) \mid \det\Lambda = +1\}$$

while the improper Lorentz group L_- is defined by

$$L_- = \{\Lambda \in 0(1,3) \mid \det\Lambda = -1\}$$

The orthochronous Lorentz transformations $L^{\uparrow}$ are defined by

$$L^{\uparrow} = \{\Lambda \in 0(1,3) \mid \Lambda^0{}_0 \geq 1\}$$

The restricted Lorentz group, synonymously the proper orthochronous Lorentz group $L_+^{\uparrow}$ is defined by

$$L_+^{\uparrow} = L_+ \cap L^{\uparrow}$$

Let $\Psi'(x')$ and $\Psi(x)$ be the four-component Dirac spinors at the space-time points x' *and* x respectively, then Lorentz covariance is expressed by [Bjorken 1964]

$$\Psi'(x') = S(\Lambda)\Psi(x)$$

where the 4 X 4 matrix S satisfies

$$S^{-1}(\Lambda)\gamma^{\mu} S(\Lambda) = \Lambda^{\mu}{}_{\nu}\gamma^{\nu} \qquad (1)$$

and where in the Weyl (Chiral) representation

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix} \qquad (2)$$

with $\sigma^{\mu} = (\sigma^{0}, \sigma) = (1, \sigma)$ (3a)

$\bar{\sigma}^{\mu} = (\bar{\sigma}^{0}, \bar{\sigma}) = (1, -\sigma)$ (3b)

and with $\sigma = \left(\sigma^{1}, \sigma^{2}, \sigma^{3}\right)$ being the three Pauli matrices. Also

$$\Psi_{a}(x) = \begin{pmatrix} \phi_{A} \\ \bar{\Psi}^{\dot{A}} \end{pmatrix} \qquad a = 1,2,3,4 \text{ and } A, \dot{A} = 1,2 \qquad (3c)$$

The improper Lorentz transformation that we shall consider is the space inversion operator

$$S = P = diag(1,-1,-1,-1) = g^{\mu\nu}$$

Eq. (1) then becomes

$$g^{\mu\nu}\gamma^{\nu} = P^{-1}\gamma^{\mu} P \qquad (4)$$

Eq. (4) is satisfied by

$$P = e^{i\phi}\gamma_{0} \qquad (5)$$

where ϕ is a phase factor. We may note the commutation relation

$$\left\{\gamma^{\mu},\gamma^{5}\right\}=0 \tag{6a}$$

from which it follows that

$$\left\{S,\gamma_{5}\right\}=0 \tag{6b}$$

We may note that in the Weyl (chiral) representation

$$\gamma_{5}=\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$$

as a special case of Eq. (6b), we have

$$P\gamma_{5}=-\gamma_{5}P \tag{6c}$$

Covariant and contravariant undotted spinnors transform respectively as follows

$$\phi'_{A}=M_{A}{}^{B}\phi_{B} \qquad A,B=1,2 \tag{7a}$$

$$\phi'^{A}=\left(M^{-1T}\right)^{A}{}_{B}\phi^{B} \qquad A,B=1,2 \tag{7b}$$

where $M\left(M^{-1T}\right)$ is the self-representation (equivalent self -representation) on the representation space $F(F^{*})$ of SL(2,C), where

$$SL(2,C)=\{M\in GL(2,C)\,|\,\det M=+1\}$$

Similarly, covariant and contravariant dotted spinors transform respectively as follows

$$\overline{\Psi}_{\dot{A}}=\left(M^{*}\right)_{\dot{A}}{}^{\dot{B}}\overline{\Psi}_{\dot{B}} \tag{8a}$$

$$\overline{\Psi}^{\dot{A}}=\left(M^{*-1T}\right)^{\dot{A}}{}_{\dot{B}}\overline{\Psi}^{\dot{B}} \tag{8b}$$

where

$M^{*}(M^{*-1T})$ is the self-conjugate (equivalent self-conjugate) representation of SL (2,C) on the representation space $\dot{F}(\dot{F}^{*})$ of SL(2,C).

PROOF OF THE PSEUDOSCALAR BILINEAR FORM IDENTITY

In an earlier paper [Odundun 2000] the following two identities were proved:

$$\overline{\Psi}'(x')\Psi'(x') = \overline{\Psi}(x)\Psi(x)$$

$$\overline{\Psi}'(x')\gamma^{\mu}\Psi'(x') = \Lambda^{\mu}{}_{\nu}\overline{\Psi}(x)\gamma^{\nu}\Psi(x)$$

In another paper [Odundun 2002] the following tensor indentity was proved

$$\overline{\Psi}'(x')\sigma_4^{\mu\nu}\Psi'(x') = \Lambda^{\mu}{}_{p}\Lambda^{\nu}{}_{\sigma}\overline{\Psi}(x)\sigma_4^{p\sigma}\Psi(x)$$

We now prove the following pseudoscalar and pseudovector bilinear form identities

$$\overline{\Psi}'(x')\gamma_5\Psi'(x') = \det[\Lambda]\overline{\Psi}(x)\gamma_5\Psi(x)$$

$$\overline{\Psi}'(x')\gamma_5\gamma^{\mu}\Psi'(x') = \det[\Lambda]\Lambda^{\mu}{}_{\nu}\overline{\Psi}(x)\gamma_5\gamma^{\nu}\Psi(x)$$

We shall start with the proof of the first identity for $\Lambda \in L_+$ and then for $\Lambda \in L_-$ In the latter case it is sufficient to choose Λ as the parity operator

$$S = P = diag(1,-1,-1,-1) = g^{\mu\nu}$$

Let us first note that the index structure of σ^{μ} and $\overline{\sigma}^{\mu}$ are respectively $\sigma^{\mu}_{A\dot{A}}$ and $\overline{\sigma}^{\mu\dot{A}A}$

These two four-matrices are related by

$$\overline{\sigma}^{\mu\dot{A}A} = \varepsilon^{AB}\varepsilon^{\dot{A}\dot{B}}\sigma^{\mu}_{B\dot{B}} \tag{9}$$

and

$$\sigma^{\mu}_{A\dot{A}}\overline{\sigma}^{\mu\dot{B}B} = 2\delta_{A}^{\ B}\delta_{\dot{A}}^{\ \dot{B}} \tag{10}$$

It can be shown that the spinor representation of $\Lambda \in L_{+}^{\uparrow}$ is

$$\Lambda^{\mu}{}_{\nu} = \frac{1}{2}Tr[M^{+}\overline{\sigma}^{\mu}M\sigma_{\nu}] \tag{11}$$

With $\Psi(x)$ given by Eq. (3c) and, nothing Eq. (2), the adjoint spinor

$$\overline{\Psi}(x) = \Psi^{+}(x)\gamma^{0} = \left(\phi^{A^*}, \overline{\psi}^{*}_{\dot{A}}\right)\begin{pmatrix} 0 & \sigma^{0}_{A\dot{B}} \\ \overline{\sigma}^{0\dot{A}B} & 0 \end{pmatrix}$$

$$= \left(\overline{\psi}^{*}_{\dot{A}}\overline{\sigma}^{0\dot{A}B}, \phi^{A^*}\sigma^{0}_{A\dot{B}}\right)$$

On using the identities

$$\phi^{B} = \overline{\psi}^{*}_{\dot{A}}\overline{\sigma}^{0\dot{A}B} \tag{12a}$$

$$\overline{\psi}_{\dot{B}} = \phi^{A^*}\sigma^{0}_{A\dot{B}} \tag{12b}$$

we find that

$$\overline{\Psi}(x) = (\phi^{A}, \overline{\psi}_{\dot{A}}) \tag{12c}$$

We now have

$$\overline{\Psi}(x)\gamma_5\Psi(x) = (\phi^{A}, \overline{\psi}_{\dot{A}})\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \phi_{A} \\ \overline{\psi}^{\dot{A}} \end{pmatrix} = -\phi^{A}\phi_{A} + \overline{\psi}_{\dot{A}}\overline{\psi}^{\dot{A}} \tag{13}$$

Similarly,

$$\overline{\Psi}'(x')\gamma_5\Psi'(x') = -\phi'^{A}\phi'_{A} + \overline{\psi}'_{\dot{A}}\overline{\psi}'^{\dot{A}}$$

(On using Eqs. (7b), (7a), (8a), and (8b))

$$= -\left(M^{-1T}\right)^{A}{}_{B}\phi^{B} M_{A}{}^{C}\phi_{C} + (M^{*})_{\dot{A}}{}^{\dot{B}}\overline{\psi}_{\dot{B}}(M^{*-1T})^{\dot{A}}{}_{\dot{C}}\overline{\psi}^{\dot{C}}$$

$$= -\phi^{B}(M^{-1})_{B}{}^{A} M_{A}{}^{C}\phi_{C} + \overline{\psi}_{\dot{B}}(M^{*T})^{\dot{B}}{}_{\dot{A}}(M^{*-1T})^{\dot{A}}{}_{\dot{C}}\overline{\psi}^{\dot{C}}$$

$$= -\phi^{B}\delta_{B}{}^{C}\phi_{C} + \overline{\psi}_{\dot{B}}\delta^{\dot{B}}{}_{\dot{C}}\overline{\psi}^{\dot{C}} = -\phi^{B}\phi_{B} + \overline{\psi}_{\dot{B}}\overline{\psi}^{\dot{B}}$$

i.e., $\overline{\Psi}'(x')\gamma_5\Psi'(x') = [1]\overline{\Psi}(x)\gamma_5\Psi(x) = \det[\Lambda]\overline{\Psi}(x)\gamma_5\Psi(x)$ (14)

We now consider $\Lambda \in L_{-}$ *i.e.*, $\Lambda = S = P$. Here we shall use the Dirac four-spinor formulation.

Since $\Psi'(x') = S(\Lambda)\Psi(x)$,

$$\overline{\Psi}'(x')\gamma_5\Psi'(x') = \Psi'^{+}(x')\gamma^{0}\gamma_5\Psi'(x') = \Psi^{+}(x)S^{+}\gamma^{o}\gamma_5 S\Psi(x)$$

Since $(\gamma^{0})^{2} = 1$, Eqs. (5) and (6a) yield

$$\Psi^{+}(x)S^{+}\gamma^{o}\gamma_5 S\Psi(x) = \Psi^{+}(x)\gamma^{0}\gamma^{0}\gamma_5\gamma^{0}\Psi(x) = -\Psi^{+}(x)\gamma^{0}\gamma_5\Psi(x)$$

$$= -\overline{\Psi}(x)\gamma_5\Psi(x) = \det[\Lambda]\overline{\Psi}(x)\gamma_5\Psi(x) \quad (15)$$

Hence, for any $\Lambda \in L_{\pm}$, eqs. (14) and (15) give

$$\overline{\Psi}'(x')\gamma_5\Psi'(x') = \det[\Lambda]\overline{\Psi}(x)\gamma_5\Psi(x) \quad (16)$$

PROOF OF THE PSEUDOVECTOR BILINEAR FORM IDENTITY

We now show that

$$\overline{\Psi}'(x')\gamma_5\gamma^{\mu}\Psi'(x') = \det[\Lambda]\Lambda^{\mu}{}_{\nu}\overline{\Psi}(x)\gamma_5\gamma^{\nu}\Psi(x)$$

by first considering $\Lambda \in L_{+}$. By Eq. (12c)

$$\overline{\Psi}(x) = (\phi^{A}, \overline{\psi}_{\dot{A}})$$

$$\therefore \overline{\Psi}(x)\gamma_5\gamma^{\nu}\Psi(x) = \left(\phi^{A}, \overline{\psi}_{\dot{A}}\right)\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & \sigma^{\nu}{}_{A\dot{B}} \\ \overline{\sigma}^{\nu\dot{A}B} & 0 \end{pmatrix}\begin{pmatrix} \phi_{B} \\ \overline{\psi}^{\dot{B}} \end{pmatrix}$$

$$= -\phi^{A}\sigma^{\nu}{}_{A\dot{B}}\overline{\psi}^{\dot{B}} + \overline{\psi}_{\dot{A}}\overline{\sigma}^{\nu\dot{A}B}\phi_{B} \quad (17)$$

With $\Psi'(x') = \begin{pmatrix} \phi'_{A} \\ \overline{\psi}'^{\dot{A}} \end{pmatrix}$

$$\overline{\Psi}'(x') = \Psi'^{+}(x')\gamma^{0} = (\phi'^{B}, \overline{\psi}'_{\dot{B}})$$

Thus

$$\overline{\Psi}'(x')\gamma_5\gamma^{\mu}\Psi'(x') = (\phi'^{B}, \overline{\psi}'_{\dot{B}})\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & \sigma^{\mu}_{B\dot{B}} \\ \overline{\sigma}^{\mu\dot{B}B} & 0 \end{pmatrix}\begin{pmatrix} \phi'_{B} \\ \overline{\psi}'^{\dot{B}} \end{pmatrix}$$

$$= \overline{\psi}'_{\dot{B}}\overline{\sigma}^{\mu\dot{B}B}\phi'_{B} - \phi'^{B}\sigma^{\mu}_{B\dot{B}}\overline{\psi}'^{\dot{B}}$$

$$= -\phi'^{A}\sigma^{\mu}_{A\dot{A}}\psi'^{\dot{A}} + \overline{\psi}'_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}\phi'_{A} \qquad (18)$$

We start by considering the second term of Eq. (18)

$$\overline{\psi}'_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}\phi'_{A} = (M^{*})_{\dot{A}}{}^{\dot{B}}\overline{\psi}_{\dot{B}}\overline{\sigma}^{\mu\dot{A}A}M_{A}{}^{B}\phi_{B}$$

$$= \overline{\psi}_{\dot{B}}(M^{+})^{\dot{B}}{}_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}M_{A}{}^{B}\phi_{B}$$

$$= \overline{\psi}_{\dot{C}}\delta^{\dot{C}}{}_{\dot{B}}(M^{+})^{\dot{B}}{}_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}M_{A}{}^{B}\delta_{B}{}^{C}\phi_{C} \qquad (19)$$

On using Eq. (10), Eq. (19) then becomes

$$\frac{1}{2}\overline{\psi}_{\dot{C}}\overline{\sigma}^{\nu\dot{C}C}\sigma_{\nu B\dot{B}}(M^{+})^{\dot{B}}{}_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}M_{A}{}^{B}\phi_{C}$$

$$= \frac{1}{2}[(M^{+})^{\dot{B}}{}_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}M_{A}{}^{B}\sigma_{\nu B\dot{B}}][\overline{\psi}_{\dot{C}}\overline{\sigma}^{\nu\dot{C}C}\phi_{C}]$$

$$= \frac{1}{2}Tr\left[M^{+}\overline{\sigma}^{\mu}M\sigma_{\nu}\right]\left[\overline{\psi}_{\dot{C}}\overline{\sigma}^{\nu\dot{C}C}\phi_{C}\right]$$

(by Eq. (11))

$$\overline{\psi}'_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}\phi'_{A} = \Lambda^{\mu}{}_{\nu}\overline{\psi}_{\dot{A}}\overline{\sigma}^{\nu\dot{A}A}\phi_{A} \qquad (20)$$

We now consider the first term in Eq. (18)
By Eq.(10)

$$-\phi'^{A}\sigma^{\mu}_{A\dot{A}}\overline{\psi}'^{\dot{A}} = -\phi'^{A}\varepsilon_{AB}\varepsilon_{\dot{A}\dot{B}}\overline{\sigma}^{\mu\dot{B}B}\overline{\psi}'^{\dot{A}}$$

(using Eqs. (7b) and 8b))

$$= -(M^{-1T})^{A}{}_{C}\phi^{C}\varepsilon_{AB}\varepsilon_{\dot{A}\dot{B}}\bar{\sigma}^{\mu\dot{B}B}(M^{*-1T})^{\dot{A}}{}_{\dot{C}}\bar{\psi}^{\dot{C}}$$

$$= -(M^{-1T})^{A}{}_{C}\delta^{C}{}_{D}\phi^{D}\varepsilon_{AB}\varepsilon_{\dot{A}\dot{B}}\bar{\sigma}^{\mu\dot{B}B}(M^{*-1T})^{\dot{A}}{}_{\dot{C}}\delta^{\dot{C}}{}_{\dot{D}}\bar{\psi}^{\dot{D}}$$

(On using Eq. (10))

$$= -\frac{1}{2}(M^{-1T})^{A}{}_{C}\bar{\sigma}^{\nu\dot{C}C}\sigma_{\nu D\dot{D}}\phi^{D}\varepsilon_{AB}\varepsilon_{\dot{A}\dot{B}}\bar{\sigma}^{\mu\dot{B}B}(M^{*-1T})^{\dot{A}}{}_{\dot{C}}\bar{\psi}^{\dot{D}} \tag{21}$$

On using the identities

$$(M^{-1T})^{A}{}_{C} = \varepsilon^{AE}M_{E}{}^{F}\varepsilon_{FC} \tag{22}$$

and

$$(M^{*-1T})^{\dot{A}}{}_{\dot{C}} = \varepsilon^{\dot{A}\dot{E}}(M^{*})_{\dot{E}}{}^{\dot{F}}\varepsilon_{\dot{F}\dot{C}} \tag{23}$$

Eq. (21) becomes

$$-\frac{1}{2}M_{E}{}^{F}\varepsilon^{AE}\varepsilon_{FC}\bar{\sigma}^{\nu\dot{C}C}\sigma_{\nu D\dot{D}}\phi^{D}\varepsilon_{AB}\varepsilon_{\dot{A}\dot{B}}\bar{\sigma}^{\mu\dot{B}B}(M^{*})_{\dot{E}}{}^{\dot{F}}\varepsilon^{\dot{A}\dot{E}}\varepsilon_{\dot{F}\dot{C}}\bar{\psi}^{\dot{D}} \tag{24}$$

Upon using

$$\sigma^{\nu}_{F\dot{F}} = \varepsilon_{FC}\varepsilon_{\dot{F}\dot{C}}\bar{\sigma}^{\nu\dot{C}C} \tag{25}$$

Eq. (24) becomes

$$-\frac{1}{2}M_{E}{}^{F}\varepsilon^{AE}\varepsilon^{\dot{A}\dot{E}}\sigma^{\nu}{}_{F\dot{F}}\sigma_{\nu D\dot{D}}\phi^{D}\varepsilon_{AB}\varepsilon_{\dot{A}\dot{B}}\bar{\sigma}^{\mu\dot{B}B}(M^{*})_{\dot{E}}{}^{\dot{F}}\bar{\psi}^{\dot{D}} \tag{26}$$

Upon using

$$\varepsilon^{AE}\varepsilon_{AB} = (\varepsilon^{T})^{EA}\varepsilon_{AB} = -\delta^{E}{}_{B}$$

and

$$\varepsilon^{AE}\varepsilon_{\dot{A}B} = (\varepsilon^{T})^{EA}\varepsilon_{\dot{A}B} = -\delta^{E}{}_{B}$$

we have $-\phi'^{A}\sigma^{\mu}{}_{A\dot{A}}\overline{\psi}'^{\dot{A}}$

$$= -\frac{1}{2}M_{B}{}^{F}\sigma^{\nu}{}_{F\dot{F}}\sigma_{\nu D\dot{D}}\phi^{D}\overline{\sigma}^{\mu\dot{B}B}(M^{*})_{\dot{B}}{}^{\dot{F}}\overline{\psi}^{\dot{D}}$$

$$= -\frac{1}{2}\left[\overline{\sigma}^{\mu\dot{B}B}M_{B}{}^{F}\sigma_{\nu F\dot{F}}(M^{+})^{\dot{F}}{}_{\dot{B}}\right]\left[\phi^{D}\sigma^{\nu}{}_{D\dot{D}}\overline{\psi}^{\dot{D}}\right]$$

On using Eq. (11) we have

$$-\phi'^{A}\sigma^{\mu}{}_{A\dot{A}}\overline{\psi}'^{\dot{A}} = \Lambda^{\mu}{}_{\nu}\left[-\phi^{D}\sigma^{\nu}{}_{D\dot{D}}\overline{\psi}^{\dot{D}}\right] \quad (27)$$

On combining Eqs. (20) and (27) we obtain

$$\overline{\psi}'_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}\phi'_{A} - \phi'^{A}\sigma^{\mu}{}_{A\dot{A}}\overline{\psi}'^{\dot{A}} = [1]\left[\overline{\psi}'_{\dot{A}}\overline{\sigma}^{\mu\dot{A}A}\phi'_{A} - \phi'^{A}\sigma^{\mu}{}_{A\dot{A}}\overline{\psi}'^{\dot{A}}\right]$$

$$= \det[\Lambda]\Lambda^{\mu}{}_{\nu}(\overline{\psi}_{\dot{B}}\overline{\sigma}^{\nu\dot{B}B}\phi_{B} - \phi^{B}\sigma^{\nu}{}_{B\dot{B}}\overline{\psi}^{\dot{B}})$$

$$i.e.,\ \overline{\Psi}'(x')\gamma_{5}\gamma^{\mu}\Psi'(x') = \det[\Lambda]\Lambda^{\mu}{}_{\nu}\overline{\Psi}(x)\gamma_{5}\gamma^{\nu}\Psi(x) \quad (28)$$

We now consider $\Lambda \in L_{-}$, $i.e.$, $\Lambda = S = P$. Then

$$\overline{\Psi}'(x')\gamma_{5}\gamma^{\mu}\Psi'(x') = \Psi^{+}(x)\gamma^{0}(P^{+}\gamma_{5}\gamma^{\mu}P)\Psi(x)$$

Since γ_5 is diagonal and real in the Weyl representation, Eq. (6c) gives

$$\gamma_{5}P^{+} = -P^{+}\gamma_{5}$$

$$\therefore \overline{\Psi}'(x')\gamma_{5}\gamma^{\mu}\Psi'(x') = -\Psi^{+}(x)\gamma^{0}\gamma_{5}P^{+}\gamma^{\mu}P\Psi(x)$$

$$= -\Psi^{+}(x)\gamma^{0}\gamma_{5}P^{-1}\gamma^{\mu}P\Psi(x) \quad \text{(by Eq. (1))}$$

$$= -\Lambda^{\mu}{}_{\nu}\overline{\Psi}(x)\gamma_{5}\gamma^{\nu}\Psi(x)$$

$$= \det[\Lambda]\Lambda^{\mu}{}_{\nu}\overline{\Psi}(x)\gamma_{5}\gamma^{\nu}\Psi(x) \quad (29)$$

Hence, Eqs. (28) and (29) give for $\Lambda \in L_{\pm}$

$$\overline{\Psi}'(x')\gamma_5\gamma^{\mu}\Psi'(x') = \det[\Lambda]\Lambda^{\mu}{}_{\nu}\overline{\Psi}(x)\gamma_5\gamma^{\nu}\Psi(x).$$

This completes the proof of the pseudovector bilinear form identity.

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