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ON OPTIMAL DESIGN OF A LOW- PASSAGE ACTIVE FILTER USING BESSEL FUNCTION RESPONSE

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ABSTRACT
At any level, decision-making is essentially a matter of choosing between a number of alternatives. Such choices are totally subjective, being based on our knowledge of the alternatives available and on the sum total of our personal experience and culture. Choice suggests numerical weighting. Using numerical values to quantify the preferred choice reduces intuitive decision making to arithmetical calculation. In dealing essentially with circuit specification in frequency domain, the design problem includes deciding the size order of filter required and computing optimum values of the components. In this presentation, Bessel function response is chosen to determine the filter order and a sum of square objective function is formulated to determine optimum component by applying a conjugate direction algorithm in minimizing the objective function. A measure of the effect of variation in individual component value on circuit response is computed as a measure of tolerance in terms of rejection rate and yield using Gaussian distribution function. This is a didactic presentation of what we are to do in the presence of this problem.

KEY WORDS AND PHRASES: Template specification, Bessel response, conjugate direction algorithm, design objectives function, sensitivity to component variation, Gussian distribution, rejection rate percent, the yield or acceptance, tolerance analysis.

1 INTRODUTION
Our most concern is the application of optimization techniques in the design of filters. The range of disciplines to which optimization has been applied goes very far beyond the confines of electronics and engineering. Lawrence Dixon [7] provides an interesting insight into this breadth, with applications from a wide cross-section of engineering and scientific fields. Specifications for linear frequency-selective networks are often very demanding and mathematical complexities of designing a suitable network are frequently very great.

In 1887, Oliver Heaviside realized that the impedance presented to an alternating current by an inductance L is proportional to ωL. Since the impedance presented by a capacitance C is proportional to 1/ωc, then by proper arrangement of inductors and capacitors it is possible to selectively shunt either the high or low frequencies and by pass the others. A high-pass filter network can be constructed in which the inductors shunt the low frequencies while the high frequencies are transmitted through the capacitor. It was not until 1900 that a successful low-pass filter was constructed by Pupin. A high-pass filter was built by Campbell in 1906. A low pass filter can be constructed by interchanging the position of the inductors and capacitors. There are indeed many situations in which no analytic design method are available, and numerical techniques then provide the only course open to the designer. Hence the earliest published instance of the use of optimization in electronics involved the design of filters. It is interesting to note that the design of filters with completely arbitrary loss factor is one of the situation for which no exact design method exist. The published result showed that computer optimization allowed design solution to be produced, although run-times were rather long. Some author used steepest descent algorithm, which perform poorly, to obtain optimum component values. Lasdon and Warren [13] considered the same problem but applied the powerful Dividon-Flecher-Powell quasi-Newton algorithm with predictable better result. In this work we have employed a conjugate direction quasi-Newton method proposed in Ref [2].

Usually the name associated with the filter accurately describes the type of frequency response for the filter network. There are certain basic terms used to describe the characteristics of all electronic filter network. These terms include pass band, stop band and roll-off point. A majority of the better type of electronic filters are constructed of capacitors, inductors and resistors. Their purpose is to either enhance or attenuate a range of frequencies. [1,4-9,11,14,15-18,20].

Filters can also be described as low-pass or high-pass. In the low-pass filter network, the capacitor offers a low-impedance path for all higher frequencies present in the circuit. The inductance offer low impedance to the lower frequencies since the inductance is in series. Electrical and electronic filter often found as discrete devices include low-pass radio frequency filter used to eliminate unwanted RF single frequencies from being transmitted by any type of broadcast transmitter. The higher-pass uses the same component, as does the low-pass filter. In the higher-pass filter the positions of the inductance and capacitance are reversed. The capacitors are connected in series with the signal path and provide a low-impedance path for higher frequencies. The parallel -connected inductor offer a low-impedance path to low frequencies 1, 14-20].

Filters can also be classified as active or passive according to their components. Passive filter network consists of impedance (resistor, capacitor and inductor) arranged in shunt.

Advantages associated with passive filter include:

(1) it generates little noise of its own due to temperature.
(2) it can be subjected to variety of voltage without fear of situation.
(3) It does not require power supply.

Active filters employ only resistors, capacitors, and some form of active element (amplifier), which introduce some gain into the signals.

Advantages associated with active filter include:

(1) Simplicity in manufacturing
(2) It is practical for use of the lowest possible frequency.
(3) It costs much lower than its equivalent passive counterparts.
(4) It has negligible sensitivity to external electrical field.
(5) It has small physical dimension
(6) It has gain and frequency adjustment flexibility i. e. it is easier to tune or adjust because the operational amplifier is capable of providing a gain and the input is not alternated.
(7) Because of the high input and low input impedance of operational amplifier the active filter does not cause overloading of the source load.

The classical problem of filter design consists in:

(a) Obtaining a realizable network function Hr(s) whose corresponding amplitude and phase function Hr(jω) and arg Hr(jω), satisfy the given template. This is referred to as approximation. Approximation theory is the theory of how to achieve an approximating ideal response by a realizable transfer function. It involves selecting a transfer function, which on one hand satisfies the specification of the filter and on the other can be exactly realized by a practical circuit. The five major approximations are: Butterworth, Chebyshev, Inverse Chebyshev, Elliptic and Bessel. Of specific importance in this paper is Bessel approximation. In cases where linearity of phase in the pass-band is of concern, Bessel filter could be used as low-pass approximation.

(b) Synthesizing a network by performing a sequence of mathematical operation on Hr(s), leading to a network which thus satisfies the original specification. This is referred to as synthesis.

For a given problem specification, the task is to describe what order of filter is; how many transfer function pole there should be (we also need to know whether zeros are required) and where they should be positioned. But the next approach to system specification is to determine without any consideration of the proposed kind of filter format a suitable realizable network function, on which to base the subsequent computation stages.

Interested reader will find useful introduction to filter theory in the literature [10]. Much of the difficulty of approximation or synthesis cycle has been eased by the production of design tables and graphs relating gain and phase specification to standard filter function (Butterworths, Chebyshev, Inverse Chebyshev, Elliptic,) and given component values for standard active and/or passive filter structure which realize these various filter function [see, for example, References 8 and 10]

2 MATHEMATICAL REQUIREMENT OF LOW PASS ACTIVE FILTER DESIGN

The starting point is the specification, which require the skill and experience of the designer and will lead to initial guess problem. The design problem then involves determining the correct numerical value for the component within the circuits. The adjustable component values are referred to as design variable. The realized and specified performance is next compared and a decision is taken based on the result of this comparison. If the realized performance does not satisfy the specification, the design variables are adjusted and this is repeated until a satisfying solution is achieved or until a predetermined accuracy (tolerance) is achieved. The active filter structure has a regular form, which admits of a simple recursive analysis scheme. Therefore given the current set of component values stored in a vector x(say), the analysis procedure would return a vector whose element or entries are the s-plane voltage transfer function coefficients.

Given this fast and very efficient analysis capability, we next consider the formulation of a suitable design objective function. Since the aim is to minimize the "error" or deviation of the specified from the realized component values, we formulate the design objective function $\phi(x)$ as a sum of squares of residuals $f_i(x)$ defined by

$$f_i(\underline{x}) = [a_{si} - a_{ri}(\underline{x})] \quad (2.1)$$

and
$$\phi(x) = \sum_{i=1}^{n} [f_i(\underline{x})]^2 \quad (2.2)$$

where n is the filter order, a_{si} are the specified component values and a_{ri} are the realized component values which are the coefficients of the transfer function. We shall next determine n the filter order from the Bessel function chosen. Using amplitude response function associated with a low-pass o^{th} – order Bessel filter given as function response we know that if the Bessel function of order n is

$$J_n(x) = \left(\frac{x}{2}\right)^n \left\{ \frac{1}{n!} - \frac{1}{(n+1)}! \left(\frac{x}{2}\right)^2 + \frac{1}{(2!)(n+2)'} \left(\frac{x}{2}\right)^4 - \ldots\ldots\ldots \right\}$$

then,

$$J_0(x)=1-\frac{1}{1!}\left(\frac{x}{2}\right)^2+\frac{1}{(2!)^2}\left(\frac{x}{7}\right)^4-\frac{1}{(3!)^2}\left(\frac{x}{2}\right)^6+.............$$

Now

$$\int_0^1 e^{-wt}J_0(t)dt=\int_0^1\left(1-wt+\frac{1}{2}w^2t^2-\frac{1}{6}w^3t^3+\frac{1}{24}w^4t^4+...\right)\left(1-\frac{t^2}{4}+\frac{t^4}{64}-...\right)dt$$

$$=\int_0^1\left(1-wt+\frac{1}{2}wt-\frac{1}{6}w^3t^3+\frac{1}{24}w^4t^4--\frac{t^2}{4}+\frac{wt^3}{4}-\frac{1}{8}w^3t^4-\frac{1}{96}w^4t^8+\frac{t^4}{64}-\frac{wt^5}{64}+\frac{1}{128}w^2t^6\right)dt$$

$$=\int_0^1\left(1-wt+\frac{1}{2}w^2t^2-\frac{1}{6}w^3t^3+.........\right)dt$$

$$=\left\{t-\frac{1}{2}wt^2+\frac{1}{6}w^2t^3-\frac{1}{24}w^3t^4+\frac{1}{126}w^4t^5+..........\right\}_0^1$$

$$=t\left(1-\frac{1}{2}wt+\frac{1}{6}w^2t^2-\frac{1}{24}w^3t^3+\frac{1}{120}w^4t^4+.......\right)\Big\}_0^1$$

for $w=1$

$$1-\frac{1}{2}w+\frac{1}{6}w^2-\frac{1}{24}w^3+\frac{1}{120}w^4$$

$$=1-0.5+0.16667-0.0416667+0.00833\cong 0.63330$$

$$n=10^{0.6333}=4.29\cong 4...$$

Taking n = 4 as the filter order, the objective function then becomes

$$\Phi(x)=\sum_{i-1}^{4}\left(a_{si}-a_{ri}(x)\right)^2$$

A conjugate direction algorithm discussed in [16] was chosen to minimize **(Φ(x)**,

for $x\in R^n$ under the assumption that $\Phi(x)$ is differentiable and $\frac{\partial^2\Phi(x)}{\partial x_i\partial x_j}$, the

Hessian matrix associated with Φ (x) exist and is positive definite.
A program was written in Turbo Pascal and run on COMPAQ 386 and the execution time was 4.3 seconds of CPU time. The results are displayed in table I and Table II.

TABLE I Order = 4. Nominal component values

Iteration	C_1	R_1	C_2	R_2	C_3	R_3	C_4	R_4
1	11.8270	0.9467	20.7600	1.5659	75.2130	1.8698	857640	1.5843
2	12.4048	0.9735	22.3741	1.6055	80.1130	1.9111	69.46	2.0028
3	12.5329	1.0414	23.5630	1.5875	83.2780	2.0098	111.7680	2.0341
4	13.6508	1.0535	26.1620	1.3075	82.3700	2.0588	112.7720	2.2355
5	14.7197	1.0534	25.1800	1.3450	84.0250	1.9636	115.6630	2.3045
6	14.7247	1.0817	25.0711	1.3747	102.5990	1.9121	115.6230	2.4212
7	14.8447	1.2707	24.0710	1.4671	102.599	1.9121	115.6230	2.4212
8	14.4127	1.3175	26.1540	1.6671	105.3070	2.0676	115.5480	2.3212
9	14.5647	1.3405	26.9200	1.7452	108.5990	2.0903	117.7500	2.2099
10	14.6367	1.3333	27.1290	1.7607	109.552	2.1247	117.8040	2.2442
11	15.6107	1.3222	27.2460	1.8408	111.4820	2.4607	117.7000	2.2852
12	16.6777	1.4011	29.2420	1.5419	112.8230	2.5325	119.6038	2.5274
13	18.1007	1.4556	29.5460	1.6230	120.8230	2.6225	120.1270	2.3810
14	18.1237	1.4381	30.1554	1.7040	122.6240	2.6689	120.1270	2.3810
15	18.6157	1.4972	30.0466	1.7360	121.9010	2.6689	120.5420	2.3660

R= Resistors = R_1, R_2, R_3, R_4, C= Capacitors = C_1, C_2, C_3, C_4,

TABLE II Realized component valueswhen order =4

COMPONENTS	REALIZED VALUES AT SOLUTION
C_1	18.6157
C_2	30.0466
C_3	121.9010
C_4	120.5420
R_1	1.4972
R_2	1.7360
R_3	2.6689
R_4	2.3660

3. SENSIVITY ANALYSIS

In practice, component values are not accurate. Manufacturing tolerances of resistors and capacitors for example are usually between ± 0.1%, and ± 20%. Transistor current gain is subject to large variation typically + 100%, -5%. Most integrated circuit components vary in the range ± 5% to ± 20%. Moreover, components vary owing to environmental effects, such as temperature and humidity. This section is concerned with evaluating the effect of these variations on the nominal response of the circuit. The response obtained by analysis of the circuit with the nominal component values, that is, those predicted by the circuit design, is called the nominal response. It is normally a close approximation to the required response given as part of the design specification. The general term sensitivity is applied to all measures of the effect on circuit response of variation in individual component values. The collective effect of simultaneous variation in all component values is called the tolerance.

Let us suppose that the value of a circuit response ϕ which is a function of the n component values $x_1, x_2\ldots, x_n$ be given by

$$\Phi(x) = \Phi(x_1, x_2\ldots, x_n) \tag{3.1}$$

This function ϕ need not be specified. It is normally expected to be the frequency response at a given frequency ϖ_0 or the transient response at a given time t_0, and the value of ϖ_0 or t_0 would appear in the function f as an independent variable. Let increment changes in the component values $x_1, x_2, x_3, \ldots, x_n$ be denoted by $\Delta x_1, \Delta x_2, \ldots \Delta x_n$ and let these cause an incremental change $\Delta\phi$ in ϕ. The Taylor expansion of a function about a point gives the change in value of the function caused by variation of its parameters in terms of the function its gradients at the point, and the parameter increments as[illegible]ng that the function is differentiable. Hence, expanding about the nominal value of response ϕ, the increment $\Delta\Phi$ is given by

$$\Delta\Phi = \sum_{1=1}^{n}\frac{\partial\Phi}{\partial xi}\Delta x_i + \frac{1}{2!}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\Phi}{\partial x_i\,\partial x_j}\Delta x_i\Delta x_j + \frac{1}{3!}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{\partial^3\Phi}{\partial x_i\,\partial x_j\,\partial x_k}\Delta x_i\Delta x_j\Delta x_k + \tag{3.2}$$

which is an infinite series involving the partial derivatives of the function Φ with respect to the component values and is valid if all derivatives exist [1,8,11,14-18]. Neglecting terms involving derivatives of third and higher order on the assumption that changes in the component values are sufficiently small for accuracy to be maintained, and indeed, neglecting also the second-order term we have,

$$\Delta\Phi = \sum_{i=1}^{n} \frac{\partial\Phi}{\partial x_i} \Delta x_i \quad (3.3)$$

Hence, we define the absolute sensitivity S_i of the response $\Phi = \Phi(\underline{x})$ to the component x_i by the first-order partial derivative $\frac{\partial\Phi}{\partial x_i}$.

That is,

$$S_i = \frac{\partial\Phi}{\partial x_i} = \lim_{\Delta x_i \to 0} \frac{\Delta\Phi}{\Delta x_i} \quad (3.4)$$

If we denote the gradient G by $\nabla\Phi$ which is a row vector of first order partial derivatives that is,

$$\nabla\Phi = G^T = \left[\frac{\partial\Phi}{\partial X_1}, \frac{\partial\Phi}{\partial X_2}, \dots, \frac{\partial\Phi}{\partial X_n}\right]^T = [S1, S2 \dots\dots\dots S_n]^T \quad (3.5)$$

where the symbol T denotes transpose.

When the response is evaluated at m sample points, t_1, t_2, ...tm of an independent parameter t (for frequency or time) the response ϕ_i is given by

$$\phi_i = \phi(X_1, X_2, \dots\dots Xn, t_i) = \phi_i(X_1, X_2, \dots\dots, X_n) \quad (3.6)$$

The gradient vector G is most usefully replaced by the m x n gradient matrix S where,

$$S = \begin{pmatrix} \frac{\partial\phi_1}{\partial x_1} & \frac{\partial\phi_1}{\partial x_2} & \dots & \frac{\partial\phi_1}{\partial x_n} \\ \frac{\partial\phi_2}{\partial x_1} & \dots\dots & \dots\dots & \dots\dots \\ \frac{\partial\phi_m}{\partial x_1} & \dots\dots & \dots\dots & \frac{\partial\phi_m}{\partial x_n} \end{pmatrix} \quad (3.7)$$

This is the first-order sensitivity matrix. The Hessian matrix. H

$$H = \left[\frac{\partial^2\phi}{\partial x_i\, \partial x_j}\right], i = 1, \dots, n, j = 1, \dots, n \quad (3.8)$$

is the second-order sensitivity matrix. Unless we can obtain the Hessian as a symbolic transfer function we do not normally use it for responses.
The matrix form of equation (3.2) may be written:

$$\Delta\varphi = G^T \underline{\Delta x} + \frac{1}{2} \underline{\Delta x}^T H \underline{\Delta x}, \tag{3.9}$$

Where Δx is the vector containing the incremental changes in component value defined as

$$\Delta x = [\Delta x_1, \Delta x_2 \dots \Delta x_n]^T \tag{3.10}$$

The relative sensitivity Si of the response $\varphi = \varphi(x_1, x_2, \dots x_n)$ to the component x_i is defined by

$$S_i \frac{\partial(\log_n \varphi)}{\partial(\log_n x_i)} = \frac{x_i}{\varphi}\frac{\partial\varphi}{\partial x_i} = \frac{x_i}{\varphi} S_i \tag{3.11}$$

The semi – relative sensitivity Q_i and Q_r^i of the response $\varphi = \varphi(x_1, x_2, \dots x_1)$ to the component x_i are defined by

$$Q_i = \frac{\partial(\log_n \phi)}{\partial x_i} = \frac{1}{y} S_i \tag{3.12}$$

$$Q_r^i = \frac{\partial\phi}{\partial(\log_n x_i)} = x_i S_i \tag{3.13}$$

4. TOLERANCE ANALYSIS: COMPONENT VARIATION.

It is often sufficiently accurate in tolerance analysis to assume that a component distribution is Gaussian (or Normal) with mean value and standard deviation equal to that of the actual distribution. This assumption may be justified first by the observation of the general form of a Gaussian distribution. Secondly, the combined effect of the distribution of a number of different components on a network response more closely approximates a Gaussian distribution as the number of components increase. The properties of the Guassian distribution must therefore be established in relation to tolerance analysis and design [1].
The Gaussian distribution function is defined by

$$\varphi(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2} dx \tag{4.1}$$

where $\varphi(x)$ is the probability that the sample value lies between $-\infty$ and x. The probability density function $\psi(x)$ obtainable by differentiation of φ (x) as given in equation (4.1) yields

$$\psi(x) = \frac{d\varphi(x)}{dx} = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2} \tag{4.2}$$

The incremental area $\psi(x)dx$ then gives the probability of a sample assuming a value in the interval x to $x + \delta x$. If we assign upper and lower tolerance limits x_u and x_L respectively such that the region $x_L<x<x_u$ is acceptable, then the probability of a sample assuming an acceptable value is given by

$$\int_{x_L}^{x} \psi(x)dx = \int_{x_L}^{x_u} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2 / 2\sigma^2} dx \tag{4.3}$$

This is called the yield [1]. Since the total area under the probability density graph is unity, the probability of a sample assuming an unacceptable value is the rejection ratio = [1 – yield]. Both figures are usually quoted as a percentage. When the tolerance limits are equally distant from published tables, then in terms of a parameter K we have

$$K\sigma = \mu - x_L = x_u - \mu \tag{4.4}$$

Letting $Z = (x - \mu)/\sigma$ in equation (4.3) and taking advantage of symmetry, we get

$$\text{Yield} = 2\int_{0}^{K} \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} dZ \tag{4.5}$$

Evaluating this integral produces the following Table III.

TABLE III. Rejection or acceptability of a component value.

K	Rejection Rate%	Yield
.6270	76.5	23.5
.6654	75.3	24.7
.8035	67.7	32.3
.9724	67.5	32.5
1.2270	60.9	39.1
1.76	47.7	52.3
3.2113	-44.4	144.4
3.5579	-404.5	504.5

TABLE IV. Component distribution.

X_i-μ_{xi}	Ψ (x_i--μ_{xi})
-3.81	0.015
-0.48	0.020
-0.26	0.570
0.15	1.670
-0.12	3.710
3.59	0.060
7.02	0.033
20.35	0.0001

i = 1,......, 2n, n is the filter order.

5. DISCUSSION AND CONCLUSION

When the nominal circuit design has been completed and a final full analysis carried out, the designer should have all possible information about the behaviour of his circuit. However, circuit design is far from being completed. A design may appear to be perfectly acceptable on the basis of nominal behaviour and sensitivity to component variation yet is quite useless in practice. Design will be complete only when tolerances (k in table III) have been assigned, and a full random simulation carried out to assess yield (table III), production costs and behaviour as components vary together.

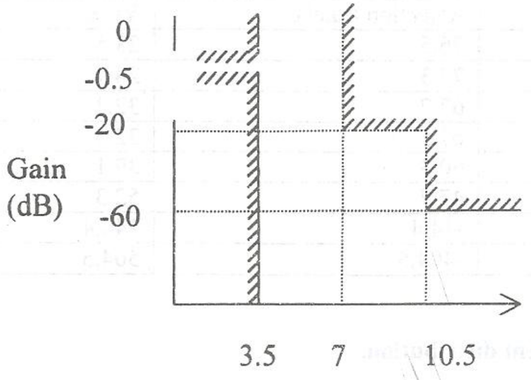

Fig. 1 Amplitude response specification for example filter.

Fig. I shows a template specification for a low pass auti-aliasing filter intended for a particular communication system. It is possible to assess the rejection rate percent of a component value, which is the major problem of tolerance assignment that has not been adequately considered. It is easy to deduce that design based on the specifications proposed in table II is far from being perfect. Sensitivity analysis is the vital link between analysis and design and applications of sensitivity begin with tolerance analysis and if any circuit does not meet the design specification, more information is necessary.

REFERENCES

[1]. P.R. Adby (1981): Applied circuit theory: Matrix and computer methods, English Language Book Society and Ellis Horwood Limited, England pp. 393486.

[2]. F.M. Aderibigbe and T.A. Adewale (2000): A second-order method for minimizing Nonlinear, Nonquadratic functions, Journal of Mathematical Science and Education.

[3]. T.A. Adewale and F.M. Aderibigbe (2000): A New Line Search Technique; Quaestiones Mathematicae, Journal of South African Mathematical Society (to appear)

[4]. L.T. Bruton (1969): Network Transfer function using the concept of frequency dependent resistance IEEE Traus. Circuit theory CT-16, 406-408

[5]. Chen, Chi-Tsong (1970): Linear Systems Theory and Design, Holt Rinehart and Winston, Holt –sanders, Japan, 89.

[6]. S.M.H.C. Collin (1994): Dictionary of Computing, Peter Collin Publishing Company Ltd, U.K.

[7]. L.C.W. Dixon (Ed) (1976): Optimization in Action, Academic Press New York.

[8]. M.S. Ghausi, K.R. Laker (1981): Modern Filter Design, Active RC and Switched Capacitor, Prentice Hall.

[9]. P.E. Gill and W Murray (1974): Newton – type Methods for Unconstrained and Linearly constrained optimization, Mathematical Programme, 7 311-350.

[10]. R.Gregorion, G.C. Temes (1986); Analog MOS Intergrated circuits for Signal Processing , John Wiley and Sons.

[11]. D.M. Himmelblau (1972), Applied Nonlinear Programming Mc Graw –Hill Book Company, New York, 341-392

[12]. R.H.F. Jackson, G.P.Mc Cormick (1985); Poly adic Third –order Lagrangian Tensor structure and Second –order Sensitivity Analysis with Factorable functions, NBSIR 85-3222.

[13]. L.S. Lasdon, A.D. Warrem (1966); Optimal design of Filters with bounded lossy elements IEEE Tran. Circuit Theory, C.T. – 13 175-187.

[14]. J.B. Marrion (1970); Classical Dynamic of Particles and Systems, Academic Presxs, New York 491 – 498.

[15]. R.E. Massara (1991); Optimization Method In Electronic Circuit Design, Longman Scientific Technical Ltd. UK. England 1-34, 71-76, 141-194.

[16]. R.E. Massara (1978); Design Consideration for Low-pass and High-pass all pole FNDR networks Electron Lett.14. (7) 225.-226.

[17]. R.E. Massara, A.Y.Al- Sager (1985); On the Optimal design of FDNR – derived band pass active filters; Prc. 17th European Conference on circuit Theory and Design – ECCTD 621-624.

[18]. C.W. Merrian III (1978); FORTRAM computer Programs, Lexington Books, Toronto.

[19]. J.M. Rollet; (1973); Economical RC – Active lossy ladder filters, Electron Lett. 9(3) 70-72.