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EFFECT OF FRANK-KAMENETSKII PARAMETER ON STRONG DETONATIONS IN A CONVERGING VESSEL

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ABSTRACT

We examine strong detonations in a converging vessel. It is shown that when Frank-Kamenetskii parameters differ even only by 1/30 there is an appreciable difference in the temperature along the converging vessel.

1. INTRODUCTION

"When a quiescent combustible gas mixture contained in an open tube is ignited by a spark at one end of the tube, a combustion wave spreads through the gas. If the open tube containing the combustible gas mixture is sufficiently long then after traveling a distance on the order of ten tube diameters, the deflagration wave begin to accelerate markedly. This high speed wave is a detonation" (see William 1985).

The above experiment shows that velocity is important in strong detonations. Equally important in combustion theory is the so-called Frank- Kamenetskii parameter. Frank-Kamenetskii considered auto-ignition in large volumes of combustible materials where the temperature satisfies the equation.

$$\frac{d^2T}{dx^2} = -De^{-\frac{E}{RT}} \quad |x| < 1, T(\pm 1) = T_0 \tag{1.1}$$

By letting

$$\theta = \frac{E}{RT_0^2}(T - T_0), \quad \in = \frac{RT_0}{E} \tag{1.2}$$

we obtain

$$\frac{d^2\theta}{dx^2} = -\delta\, e^{\frac{\theta}{(1+\in\theta)}} \quad \theta(\pm 1) = 0 \tag{1.3}$$

where

$$\delta = D\frac{E}{RT_0^2}e^{-\frac{E}{RT_0}}$$

The parameter δ is called the Frank- Kamenetskii parameter. Notice that average velocity of the gas mixture is embedded into D as $\in \Rightarrow 0$, we obtain

$$\theta = 2\ln\left[e^{\frac{\theta m}{2}}\sec h(cx)\right] \text{ with}$$

$$C^2 = \tfrac{1}{2}\delta e^{\theta m}$$

where $\theta_m = \theta(0)$ and C is a constant.

Thus δ has profound effect on θ(x) (See figure 1). What is not very clear is its influence on a strong detonation. This is precisely the motivation for the present investigation.

2. MATHEMATICAL FORMULATION

We consider a converging vessel and the momentum equation becomes

$$\rho u A = const = c \qquad (2.1)$$

The energy equation is

$$\rho c u \frac{dT}{dx} = k\frac{d^2T}{dx^2} + qBy^a e^{-\frac{E}{RT}} + \mu\left(\frac{\partial u}{\partial x}\right)^2 \qquad (2.2)$$

And the species equation is

$$\rho u \frac{\partial y}{\partial x} = D\frac{d^2 y}{dx^2} + qAy^a e^{-\frac{E}{RT}} \qquad (2.3)$$

Where

ρ	=	density of reactants
u	=	velocity of reactants
A	=	cross-sectional area of the vessel
T	=	temperature of the reaction
k	=	thermal conductivity of the medium
q	=	heat release per unit per mass
y	=	pre-mixed reactants
a	=	order of reaction
B	=	pre-exponential factor
R	=	universal gas constant
μ	=	dynamic viscosity of the reactant
D	=	diffusion coefficient
C	=	heat capacity
E	=	activation energy

3. METHOD OF SOLUTION

For a converging vessel we take $A = A_0 e^{-x}$, A_0 is a constant.

$$\text{Let } \theta = \frac{E}{RT_0^2}(T - T_0)$$

$$u^1 = \frac{u}{v}, \quad x^1 = \frac{x}{L}$$

2.2 becomes

$$e^x \frac{dT}{dx} = \frac{A_0 k}{c}\frac{d^2T}{dx^2} + \frac{qA_0 By^a}{c} e^{-E/RT} + A_0 \mu \left(\frac{\partial u}{\partial x}\right)^2 \quad (3.1)$$

$$\frac{\partial u}{\partial x} = \frac{c}{\rho A_0} e^x \quad (3.2)$$

Squaring (3.2) and substitute it into (3.1) and non dimensionalising we obtain

$$e^x \frac{d\theta}{dx} = A_1 \frac{d^2\theta}{dx^2} + Be^{\frac{\theta}{1+\epsilon\theta}} + De^{2x} \quad (3.3)$$

We solve (3.3) using

$$\theta(0) = 0, \theta(1) = \alpha(0.3) \quad (3.4)$$

Where

α is a constant and

$$A_1 = \frac{kA_0 \in T_0}{c^2 L}$$

$$B = \frac{qBy^a A_0 L}{c \in T_0} e^{-E/RT_0}$$

$$D = \frac{\mu c L}{\rho A_0}$$

where B = Frank-Kamenetskii parameter

By using shooting method

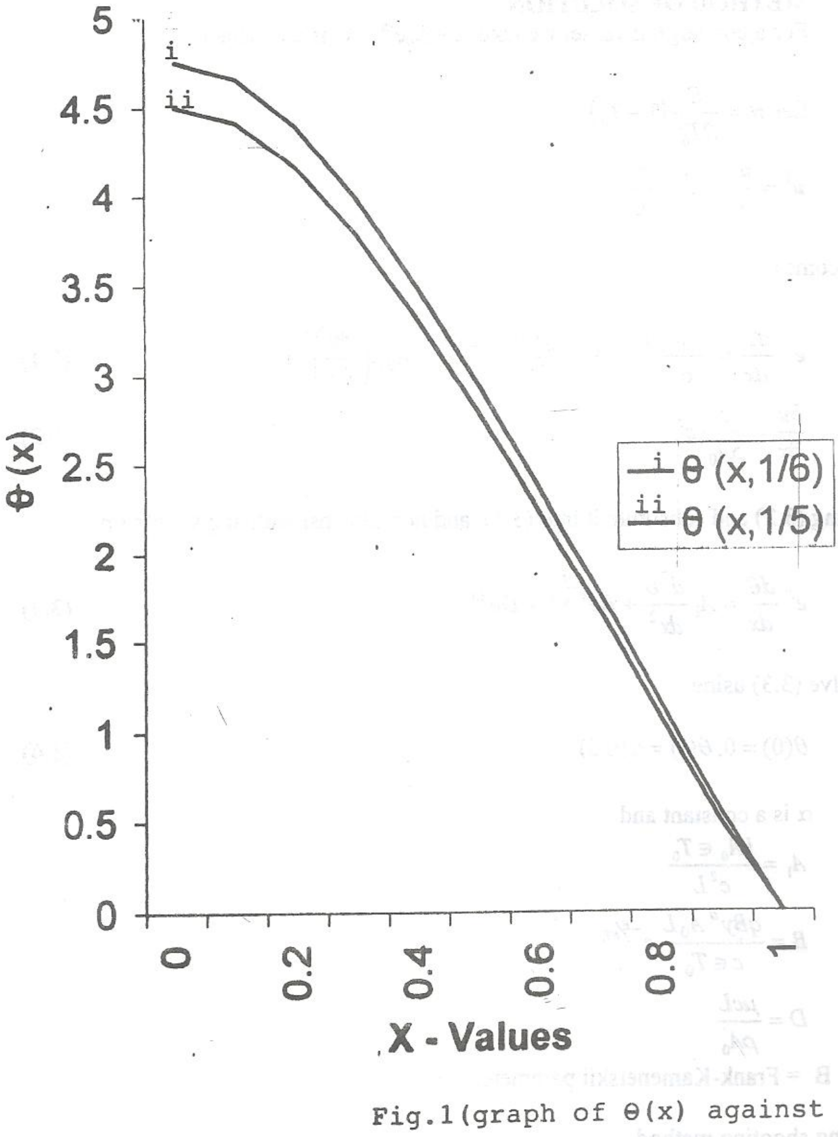

Fig.1(graph of Θ(x) against x)

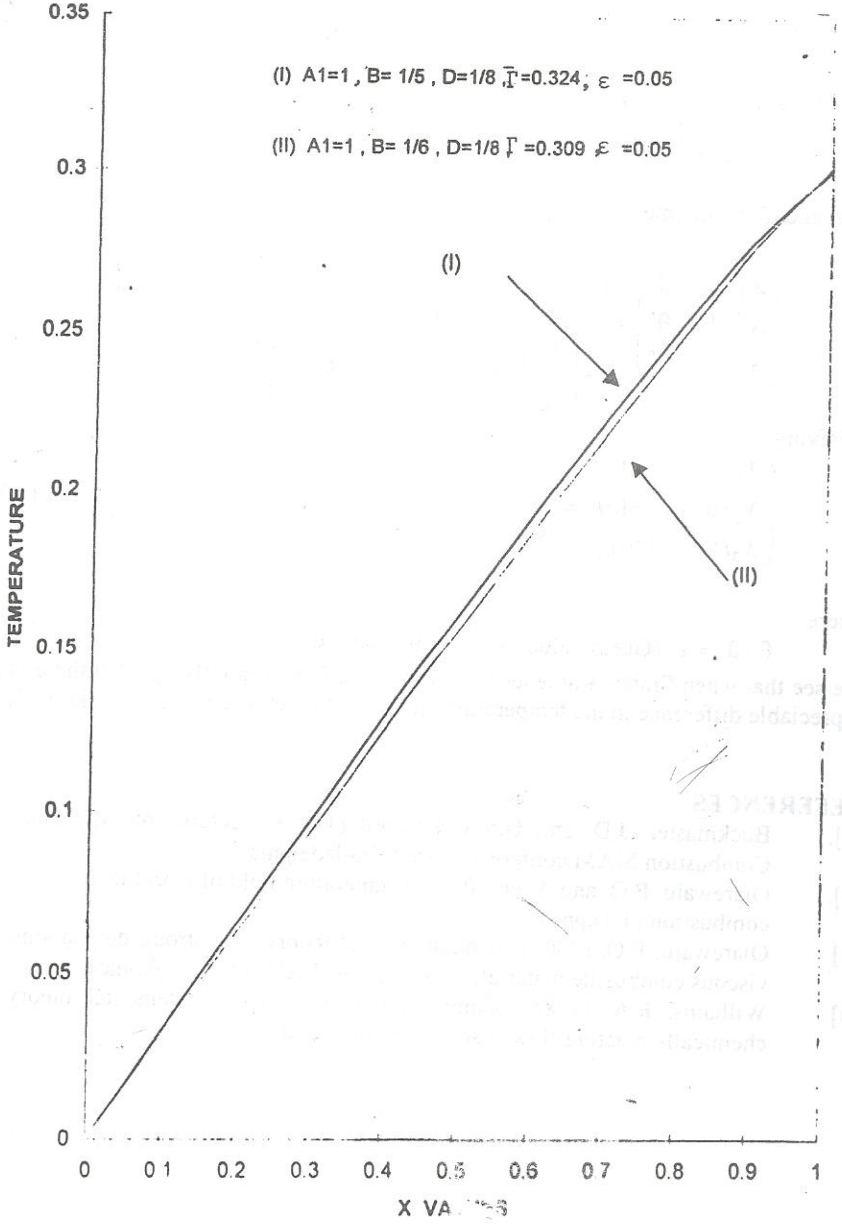

fig 2 graph of θ (x) against x

Let $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} X \\ \theta \\ \theta' \end{pmatrix}$

(2.1) and (3.2) becomes

$$\begin{pmatrix} X_1' \\ X_2' \\ X_3' \end{pmatrix} = \begin{pmatrix} 1 \\ \theta' \\ \theta'' \end{pmatrix} = \begin{pmatrix} 1 \\ X_3 \\ \frac{1}{A_1}\left[e^{X_1} X_3 - Be^{\frac{X_2}{1+\epsilon X_2}} - De^{2X_1} \right] \end{pmatrix} \tag{3.6}$$

satisfying

$$\begin{pmatrix} X_1(0) \\ X_2(0) \\ X_3(0) \end{pmatrix} = \begin{pmatrix} 0 \\ \theta(0) \\ \theta'(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \Gamma \end{pmatrix} \tag{3.7}$$

where

$\theta'(0) = \Gamma$ (Guess value for shooting method)

We see that when Frank- Kamenetskii parameter differ even only by 1/30 there is an appreciable difference in the temperature along the converging vessel (see figure 2).

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