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UNSTEADY BACKWARD POWER-LAW FLOW NEAR A MOVING WALL

R. O. AYENI AND B. I. OLAJUWON
Department of Pure and Applied Mathematics
Ladoke Akintola University of Technology, Ogbomoso, Nigeria

ABSTRACT

In this paper we investigate the effect of a moving wall on the velocity field of a power-law fluid. We show that when the fluid is dilatant (n > 1), momentum penetration is finite.

1. INTRODUCTION

Since Bird (1959) investigated an unsteady pseudoplastic flow near a moving wall, there has been much interest ion non-Newtonian flows. Recently, Hassanien et al, (1998) examined flow and heat transfer in a power-law fluid over a non-isothermal stretching sheet. They studied steady flow and showed that the friction and heat transfer rate results exhibit strong dependence on the fluid parameters.

In this paper we return to a key interest of Bird – that is, the extent of momentum penetration. Bird presented velocity profiles against the reduced variable r. In particular he gave r_1 for various values of n for which the fluid velocity has fallen to 1% of the velocity of the moving wall.

In this paper we examined what has been neglected, the momentum penetration of the backward power-law flow.

2. MATHEMATICAL FORMULATION

The unsteady power-law near a moving wall (see Bird (1959)) is

$$\rho \frac{\partial u}{\partial t} = \frac{-\partial}{\partial y}\tau_{xy} \tag{2.1}$$

where

$$\tau_{xy} = -m\left(-\frac{\partial u}{\partial y}\right)^{n-1}\frac{\partial u}{\partial y} \tag{2.2}$$

ρ is the density, u is the x-component of velocity, τ_{xy} is the stress tensor, n is the power index, m is the coefficient of viscosity, while the space variable y measures the distance from the wall. Bird solved equation (2.1) with the boundary and initial conditions $u \equiv V$ at $y \equiv 0$; $u \equiv 0$ at $y = \infty$.

As in backward heat equation (2.1) becomes

$$\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial y}\tau_{xy} \tag{2.3}$$

We retain (2.2) and our boundary and initial conditions becomes

$$u = V, \quad \frac{\partial u}{\partial y} = -k \quad \text{at } y = 0,\ k > 0 \tag{2.4}$$

$$u = 0,\ t < 0 \tag{2.5}$$

3. EXISTENCE OF SOLUTION

Let $$r = (n+1)n^{-1}y\left(\frac{\rho}{mt}V^{n-1}\right)^{\frac{1}{n+1}} \tag{3.1}$$

$$\phi = \frac{u}{V} \tag{3.2}$$

ϕ_n is the dimensionless velocity and is a function of r alone

Then (2.1) – (2.5) imply

$$\frac{d^2\phi}{dr^2}\left(\frac{d\phi_n}{dr}\right)^{n-1} - (n+1)n^{-1}r\frac{d\phi_n}{dr} = 0 \tag{3.3}$$

$$\phi_n = 1 \text{ and } \frac{d\phi_n}{dr} = -\alpha(\alpha > 0) \text{ at } r = 0 \tag{3.4}$$

For n > 1, equations (3.3) may be integrated twice using (2.2), (2.4) and (2.5) to obtain

$$\phi_n(r) = 1 - \int_0^r \left[\frac{(n-1)(n+1)^n}{2n}\right]^{\frac{1}{n-1}} \left[r^2 + B_n\right]^{\frac{1}{n-1}} dr \tag{2.10}$$

where

$$B_n = \frac{2n(\alpha)^{n-1}}{(n-1)(n+1)^n} \tag{2.11}$$

Clearly, (2.10) exists and is non-negative for each n for $0 \leq r \leq r_1$ for example, when $n = 2$

$$\phi_2(r) = 1 - \frac{3}{4}r^3 - 0.52r$$

When $\alpha = 1.18$ and $r_1 = 0.89$, $\phi_2(r_1) = 0.01(=1\%)$. Note that $\phi(0.90) = -0.01$. Thus, there exists r_1 such that $\phi(r_1) = 0$ for $0.89 < r_1 < 0.90$.

4. MOMENTUM PENETRATION

Equations (3.3) and (3.4) imply that

$$\frac{d^2\phi_n}{dr^2} \leq 0, \quad \frac{d\phi}{dr} < 0; \quad \phi_n(0) < 0 \tag{4.1}$$

Since $\phi_n(r)$ is concave and $\phi_n'(r)$ starts off negative, it must remain negative for $t > 0$. It follows that $\phi_n(r)$ must vanish at some finite r_1. That is, for all n, the momentum penetration is finite.

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