J. Nig. Ass. Math. Phys.
Vol. 5 · 2001

THE DYNAMIC RESPONSE OF PLATES ON PASTERNAK FOUNDATION TO DISTRIBUTED MOVING LOAD

J. A. GBADEYAN AND M. S. DADA
Department Of Mathematics, University Of Ilorin, Ilorin, Kwara State, Nigeria

ABSTRACT

The dynamic analysis of the rectangular plate resting on a Pasternak foundation and subjected to uniform partially distributed moving masses is carried out. The effects of shear deformation and rotary inertia are neglected. The governing partial differential equation is transformed into a set of coupled ordinary differential equations that are eventually solved using finite difference technique. It is found that an increase in the area of the distribution of the moving mass causes a reduction in the maximum dynamic deflection. Various values of the dynamic deflection for various values of foundation moduli are obtained and presented in tabular form. Moreover, the critical speeds of the moving masses and forces were calculated. Finally, numerical examples are given and the results compared well with existing ones for the limiting cases in which the area of the load distribution reduces to zero, and also the effect of inertia mass neglected.

1. INTRODUCTION

The emphasis placed on safety performances and reliability of structures such as beams, plates etc has led to the need for extensive research analysis in determining structures response to dynamic loading especially moving loads. As a matter of fact, it is known that such moving loads are likely to produce larger structural deflections and stresses than when the same loads are not dynamic (i.e. when they act statically). As such, the moving loads have a great potential of producing hazard when acting on various structures.

Consequently, engineers, applied mathematicians and applied physicists who are concerned with the design of railway and highway bridges and space station facilities that are likely to be affected by an abrupt change of mass, have carried out and continue to carry out investigations of the response of a variety of structures to moving loads.[1-10].

These moving loads problems can be discussed from two points of view viz. (1) the point of concentrated mass formulations and (ii) the distributed or partially distributed load formulation. The first formulation is the most common one. This is, perhaps due to the fact that it is a simplified version of the second. As a matter of fact, it is a special case of the second if the load distribution interval in the second formulation is assumed small. A considerable amount of work has been carried out on this first version as per the formulation involving structures like beams. Timoshenko [1] studied the case of a concentrated load moving with a constant velocity along a beam neglecting the effect of damping. He obtained a closed form solution to the governing initial boundary value problem and an expression for the critical velocity. The dynamic response of a simply supported beam transversed by a concentrated moving load was determined by Stanisic and Hardin [2]. They developed an interesting technique which, however, cannot easily be applied to various boundary conditions which are of practical interest. An analytical method capable of handling different boundary conditions associated with the moving problems of beam was developed by Akin and Mofid [3]. The problem of the dynamic behaviour of elastic beam subjected to moving concentrated mass was also studied by Sadiku and Leipholz [4]. Gbadeyan and Oni [5] presented a more versatile technique which can be used to determine the dynamic behaviour of beams having arbitrary end supports. Considerable amount of work has also been done on the behaviour of elastic plates traversed by moving concentrated masses. The elegant technique developed for the beams in [5] was also extended to non_Mindlin rectangular plates in the same paper. Earlier, Stanisic et al [2] showed that the natural frequency of plates traversed by moving concentrated forces is greater than that of plates subjected to moving concentrated masses. A finite element analysis of the problem for non-uniform elastic plates was carried out in [6]. Recently, the dynamic analysis of a rectangular plate continuously supported by an elastic foundation transvered by moving concentrated masses was carried out by Gbadeyan and Oni [7]. Although, the above-completed works on concentrated loads are impressive, they do not represent the reality of the problem formulation as concentrated masses do not exist physically. Thus, for practical application, it is useful to consider moving load problem involving distributed moving load as opposed to concentrated moving loads. To this end, an analysis of dynamic behavior of Bernoulli beam carrying uniform partially distributed moving masses was carried out in [8]. It was shown that the inertia effect of the moving mass is of importance in the dynamical behaviour of such structures. The work in [8] was extended in [9] by considering the vibration of a Timoshenko moving masses. Most recently, Gbadeyan et al [10] considered the vibration of non-Mindlin rectangular plate subjected to uniform partially distributed moving loads. It is found that the magnitude of the distribution of the load varies directly as the deflection for the moving force problem while it varies inversely as the deflection for the moving mass problem.

This present paper is concerned with the behaviour of a rectangular non-Mindlin plate continuously resting on an elastic foundation and traversed by a uniform partially distributed moving load. An illustrative example involving simply supported plate is presented. Numerical analysis is also carried out.

2. THEORETICAL ANALYSIS

Consider a thin isotropic elastic plate which is referred to x,y,z system of rectangular co-ordinates, with the origin O of the x,y,z system at the corner of the plane of the plate as shown in figure 1. The transverse displacement $w(x,y,t)$ at time t of the rectangular plate on a subgrade satisfies the partial differential equation [7]

$$D\left\{\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right\}+m_p\frac{\partial^2 w}{\partial^2 t}=f(x,y;t)-F_f(x,y;t) \qquad (2.1)$$

where the flexural rigidity of the plate is defined by

$$D=\frac{1}{12}Eh^2(1-v^2)^{-1}$$

h is the thickness of the plate
v is the Poisson ratio of the plate
E is Young's modulus of the plate
m_p is the mass density per unit are of the plate
$F_f(x,y,z)$ is the foundation reaction and
f(x,y,z) is the applied surface moving load on the plate
The foundation reaction and the transverse displacement w(x,y,t) are related as follows

$$F_f(x,y,z)=-\left(G\nabla^2 w-Kw-m_f w\right)$$

where ∇^2 is the two dimensional Laplace operator,
m_f is the mass of the subgrade,
G is the shear modulus of the foundation, and
K is the foundation stiffness

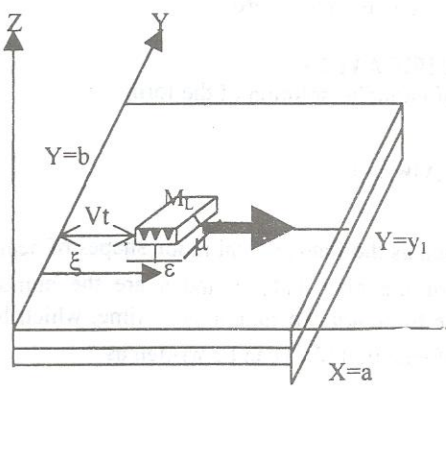

Figure 1

The expression [10] for the applied surface-moving load is

$$f(x,y,t)=\frac{1}{\mu\varepsilon}\left[M_l g-M_l\frac{d^2w}{dt^2}\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]\left[H\left(y-y_1+\frac{\mu}{2}\right)-H\left(y-y_1-\frac{\mu}{2}\right)\right] \tag{2.2}$$

where

$$\frac{d^2w}{d^2t}=\frac{\partial^2 w}{\partial t^2}+2u\frac{\partial^2 w}{\partial x\partial t}+u^2\frac{\partial^2 w}{\partial x^2} \tag{2.3}$$

H is the Heaviside unit function, M_l is the mass of the load which is assumed to be in contact with the plate during the course of the motion, u is the velocity of the load, the dimensions of the load is μ by ε and $\xi=ut+\varepsilon/2$.
The governing equation for the model therefore becomes

$$D\left\{\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right\}+(m_p+m_f)\frac{\partial^2 w}{\partial t^2}-G\frac{\partial^2 w}{\partial x^2}-G\frac{\partial^2 w}{\partial y^2}+Kw$$
$$=\frac{1}{\mu\varepsilon}\left[-M_l g-M_l\frac{d^2w}{dt^2}\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]\left[H\left(y-y_1+\frac{\mu}{2}\right)-H\left(y-y_1-\frac{\mu}{2}\right)\right] \tag{2.4}$$

The expression for the concentrated force is obtained by taking the limit of the expression for the force f(x,y,t) as ε and μ tend to zero.

3. ANALYTICAL SIMPLIFICATION

Assuming a separation of variables solution of the form

$$w(x,y,t)=\sum_{r=1}^{M}\sum_{s=1}^{N}T_{rs}(t)w_r(x)w_s(y) \tag{3.0}$$

Where $w_r(x)$ and $w_s(y)$ are chosen as the fundamental mode shapes of beams having the same boundary conditions of the plate [11], r and s are the number of the contributed modes and $T_{rs}(t)$ are the unknown functions of time, which have to be calculated. The right hand side of equation (2.2) can be written as

$$\frac{1}{\mu\cdot\varepsilon}\left[-M_l g-M_l\frac{d^2w}{dt^2}\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]$$
$$\left[H\left(y-y_1+\frac{\mu}{2}\right)-H\left(y-y_1-\frac{\mu}{2}\right)\right]\sum_{r=1}^{M}\sum_{s=1}^{N}\psi_{rs}(t)w_r(x)w_s(y) \tag{3.1}$$

Substituting equation (2.3) into equation (3.1) and multiply the result by *Wn(x) Wm(y)* yields

$$\frac{1}{\mu\varepsilon}\left[-M_l g w_n(x)w_m(y)-M_l w_n(x)w_m(y)w_n(x)w_m(y)\sum_{r=1}^{M}\sum_{s=1}^{N}\{T''_{rs}(t)w_r(x)w_s(y)\right.$$
$$+2uT'_{rs}(t)w'_r(x)w_s(y)$$
$$\left.+u^2T_{rs}(t)w''_r(x)w_s(y)\}\right]\left[H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right]\left[H\left(y-y_1+\frac{\mu}{2}\right)-H\left(y-y_1-\frac{\mu}{2}\right)\right]$$
$$=w_n(x)w_m(y)\sum_{r=1}^{M}\sum_{s=1}^{N}\psi_{rs}(t)w_r(x)w_s(y) \tag{3.2}$$

Taking the double integrals of both sides of equation (3.2) along the length and width of the plate gives

$$\frac{-M_l g}{\mu\varepsilon}\int_0^a w_n(x)B(x,\varepsilon)dx\int_0^b w_m(y)B(y,\varepsilon)dy$$
$$-\frac{M_l}{\mu\varepsilon}\sum_{r=1}^{M}\sum_{s=1}^{N}\left\{T''_{rs}(t)\int_0^a w_n(x)w_r(x)B(x,\varepsilon)dx\int_0^b w_m(y)w_s(y)B(y,\varepsilon)dy\right.$$
$$+2uT'_{rs}(t)\int_0^a w'_r(x)w_n(x)B(x,\varepsilon)dx\int_0^b w_m(y)w_s(y)B(y,\varepsilon)dy$$
$$\left.+u^2T_{rs}(t)\int_0^a w''_r(x)w_n(x)B(x,\varepsilon)dx\int_0^b w_m(y)w_s(y)B(y,\varepsilon)dy\right\}$$
$$=\sum_{r=1}^{M}\sum_{s=1}^{N}\psi_{rs}(t)\int_0^a w_r(x)w_n(x)dx\int_0^b w_m(y)w_s(y) \tag{3.3}$$

Where

$$B(x,\varepsilon)=H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\text{ and}$$

$$B(y,\mu) = H\left(y - y_1 + \frac{\mu}{2}\right) - H\left(y - y_1 - \frac{\mu}{2}\right)$$

Henceforth,

$$B(x,\varepsilon) = B \text{ and } B(y,\mu) = B(y)$$

By delta integral properties,

$$\delta(x - x_0) = \begin{cases} 0, & x \neq x_0 \\ \infty, & x = x_0 \end{cases} \tag{3.4}$$

$$\frac{d}{dx} H(x - x_0) = \delta(x - x_0) \tag{3.5}$$

Evaluating the first integral in equation (3.3) by parts and using the delta integral properties (3.4) and (3.5), we obtained

$$\frac{1}{\varepsilon}\int_0^a w_n(x) B dx = \frac{1}{\varepsilon}\int_0^a \left[w_n\left(\xi + \frac{\varepsilon}{2}\right) - w_n\left(\xi - \frac{\varepsilon}{2}\right)\right] d\xi \tag{3.6}$$

$$= w_n(\xi) + \left(\frac{\varepsilon}{2}\right)^2\left(\frac{1}{3!}\right) w_n''(\xi) + \left(\frac{\varepsilon}{2}\right)^4\left(\frac{1}{5!}\right) w_n^{iv}(\xi) + \cdots$$

$$\cong w_n(\xi) + \left(\frac{\varepsilon}{2}\right)^2\left(\frac{1}{3!}\right) w_n''(\xi) \tag{3.7}$$

Where Taylor series expansion has been used.
Similarly.

$$\frac{1}{\varepsilon}\int_0^a w_p(x) w_n(x) B dx \cong w_p(\xi) w_n(\xi) + \left(\frac{\varepsilon}{2}\right)^2\left(\frac{1}{3!}\right)\left[w_p(\xi) w_n(\xi)\right]''$$

$$= w_p(\xi) w_n(\xi) + \frac{\varepsilon^2}{24}\left[w_n(\xi) w_p''(\xi) + 2w_n'(\xi) w_p'(\xi) + w_n''(\xi) w_p(\xi)\right] \tag{3.8}$$

Also.

$$\frac{1}{\mu}\int_0^b w_m(y) B(y) dy = \frac{1}{\mu}\int_{y_1 - \frac{\mu}{2}}^{y_1 + \frac{\mu}{2}} w_m(y) dy = A_m \tag{3.9}$$

And

$$\frac{1}{\mu}\int_0^b w_m(y)w_s(y)B(y)dy = \frac{1}{\mu}\int_{y_1-\frac{\mu}{2}}^{y_1+\frac{\mu}{2}} w_m(y)w_s(y)B(y)dy = A_{mr} \tag{3.10}$$

Applying the orthogonal properties of the characteristic functions $w_r(x)$ and $w_s(y)$ and using equations (3.7), (3.9) and (3.10) in equation (3.3), the equation becomes

$$\psi_{rs}(t) = M_l g A_m\left[w_n(\xi)\frac{\varepsilon}{24}w_n^{ii}(\varepsilon) - M_l\sum_{r=1}^{N}\sum_{s=1}^{M}A_{ms}\left[T_{rs}^{ii}(t)\left[w_r(\varepsilon)w_n(\varepsilon)+\frac{\varepsilon^2}{24}\left\{w_n(\varepsilon)\frac{\varepsilon}{24}w_r(\varepsilon)\right\}^{ii}\right]\right]+\right.$$
$$\left. 2uT_{rs}^{i}(t)\left[w^{i}{}_r(\varepsilon)w_n(\varepsilon)+\frac{\varepsilon^2}{24}\left\{w_n(\varepsilon)w_r^{i}(\varepsilon)\right\}^{ii} + u^2T_{rs}(t)\left\{w_r^{ii}(\varepsilon)w_n(\varepsilon)+\frac{\varepsilon^2}{24}\left\{w_n(\varepsilon)w_r^{ii}(\varepsilon)\right\}^{ii}\right\}\right]\right] \tag{3.11}$$

Where the normalized constant is defined as follows

$$\int_0^a w_r^2(x)dx = \int_0^b w_s^2(y)dy = 1$$

Considering equation (3.1) equation (2.4) becomes

$$D\left\{\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right\}+(m_p+m_f)\frac{\partial^2 w}{\partial^2 t}-G\frac{\partial^2 w}{\partial x^2}-G\frac{\partial^2 w}{\partial y^2}+Kw = \sum_{r=1}^{M}\sum_{s=1}^{N}\psi_{rs}(t)w_r(x)w_s(y) \tag{3.12}$$

using equation (3.0) in equation (3.12) yields

$$\sum_{r=1}^{M}\sum_{s=1}^{N}\left[DT_{rs}(t)\left\{w_r^{iv}(x)w_s(y)+2\cdot w_r^{ii}(x)w_s^{ii}(y)+w_r(x)w_s^{iv}(y).\right\}+(m_p+m_f)T_{rs}^{ii}(t)w_r(x)w_s(y)\right.$$
$$\left.-GT_{rs}(t)\left\{w_r^{ii}(x)w_s(y)+w_r(x)w_s^{ii}(y)\right\}+KT_{rs}(t).w_r(x)w_s(y)-\psi_{rs}(t)w_r(x)w_s(y)\right]=0 \tag{3.13}$$

For arbitrary x and y, equation (3.13) becomes

$$DT_{rs}(t)\left\{w_r^{iv}(x)w_s(y)+2\cdot w_r^{ii}(x)w_s^{ii}(y)+w_r(x)w_s^{iv}(y).\right\}+(m_p+m_f)T_{rs}^{ii}(t)w_r(x)w_s(y)$$
$$-GT_{rs}(t)\left\{w_r^{ii}(x)w_s(y)+w_r(x)w_s^{ii}(y)\right\}+KT_{rs}(t).w_r(x)w_s(y)-\psi_{rs}(t)w_r(x)w_s(y).=0 \tag{3.14}$$

The equation of motion describing the free vibration of a plate on elastic foundation is

$$D\left\{\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right\}-G\frac{\partial^2 w}{\partial x^2}-G\frac{\partial^2 w}{\partial y^2}+Kw+\omega^2\left(m_p+m_f\right)w=0 \qquad (3.15)$$

Where ω is the circular frequency expressed in radian/unit time.

The substitution of w(x,y,t) as expressed in equation (3.0) into equation (3.15) yields

$$\begin{aligned}&D\{w_r^{iv}(x)w_s(y)+2\cdot w_r^{ii}(x)w_s^{ii}(y)+w_r(x)w_s^{iv}(y).\}\\ &-G\{w_r^{ii}(x)w_s(y)+w_r(x)w_s^{ii}(y)\}+Kw_r(x)w_s(y)=\lambda_{rs}w_r(x)w_s(y)\end{aligned} \qquad (3.16)$$

where $\lambda_{rs}=\omega(m_p+m_f)$

Substituting equation (3.16) into equation (3.14) gives

$$\{\lambda_{rs}T_{rs}(t)+(m_p+m_f)T_{rs}^{ii}(t)\}w_r(x)w_s(y)-\psi_{sr}(t)w_r(x)w_s(y)=0 \qquad (3.17)$$

Hence

$$\lambda_{rs}T_{rs}(t)+(m_p+m_f)T_{rs}^{ii}(t)=\psi_{sr}(t) \qquad (3.18)$$

Considering the expression for Ψ_{rs} in equation (3.11), equation (3.14) finally implies

$$\begin{aligned}\lambda_{rs}T_{rs}(t)+(m_p+m_f)T_{rs}^{ii}(t)=M_l gA_{m}\Bigg[&w_n(\xi)\frac{\varepsilon}{24}w_n^{ii}(\varepsilon)-M_l\sum_{r=1}^{N}\sum_{s=1}^{M}A_{ms}\Big[T_{rs}^{ii}(t)\Big[w_r(\varepsilon)w_n(\varepsilon)\\ &+\frac{\varepsilon^2}{24}\left\{w_n(\varepsilon)\frac{\varepsilon}{24}w_r(\varepsilon)\right\}^{ii}\Big]+2uT_{rs}^{i}(t)\Big[w_r^{i}(\varepsilon)w_n(\varepsilon)+\frac{\varepsilon^2}{24}\{w_n(\varepsilon)w_r^{i}(\varepsilon)\}^{ii}\\ &+u^2T_{rs}(t)\{w_r^{ii}(\varepsilon)w_n(\varepsilon)\}+\frac{\varepsilon^2}{24}\{w_n(\varepsilon)w_r^{ii}(\varepsilon.)\}^{ii}\Big\}\Bigg]\end{aligned} \qquad (3.19)$$

These coupled ordinary differential equations are to be solved subject to the various boundary conditions for the plate. The kernel for the required boundary condition is substituted into the coupled ordinary differential equations and solved for the solutions of the unknown function of time.

4. SIMPLY SUPPORTED RECTANGULAR PLATES

The normalized deflection curves (kernels) for simply supported plates are

$$w_r(x)w_s(y) = c\sin\left\{\frac{r\pi x}{a}\right\}\sin\left(\frac{s\pi y}{b}\right) \tag{4.1}$$

r = 1,2,3,..., s = 1,2,3, ...

Where c is evaluated from the equation

$$\int_0^a\int_0^b w_r^2(y)w_s^2(y)dxdy = c^2\int_0^a\int_0^b \sin^2\left\{\frac{r\pi x}{a}\right\}\sin^2\left(\frac{s\pi y}{b}\right)dxdy = 1 \tag{4.2}$$

Such that

$$c = \frac{2}{\sqrt{ab}} \tag{4.3}$$

Substituting equation (4.1) into equation (3.16) yields the eigen values

$$\lambda_{rs} = D\pi^4\left\{\frac{r^2}{a^2}+\frac{s^2}{b^2}\right\}^2 + G\pi^4\left\{\frac{r^2}{a^2}+\frac{s^2}{b^2}\right\}^2 + K \tag{4.4}$$

Following the procedure in section three, the exact governing equations are obtained since the kernel could be directly integrated without the use of Taylor's series expansion.

Substituting equation (4.1) into equation (3.3), we obtain

$$\psi_{rs}(t) = \frac{M_l gc}{\mu\varepsilon}\int_0^a \sin\left(\frac{n\pi x}{a}\right)B\{x,\xi)dx\int_0^b \sin\left(\frac{m\pi y}{b}\right)B(y)dx$$

$$-\frac{M_l c^2}{\mu\varepsilon}\sum_{r=1}^{M}\sum_{s=1}^{N}T_{rs}^{ii}(t)\int_0^b \sin\left\{\frac{m\pi y}{b}\right\}\sin\left(\frac{s\pi y}{b}\right)B(y)dy\int_0^a \sin\left\{\frac{r\pi x}{a}\right\}\sin\left(\frac{n\pi x}{a}\right)B\{x,\xi)dx$$

$$+2uT_{rs}^{i}(t)\int_0^a \frac{r\pi}{a}\cos\left\{\frac{r\pi x}{a}\right\}\sin\left(\frac{n\pi x}{a}\right)B\{x,\xi)dx\int_0^b \sin\left\{\frac{m\pi y}{b}\right\}\sin\left(\frac{s\pi y}{b}\right)B(y)dy$$

$$+u^2T_{rs}(t)\int_0^a\left\{-\left(\frac{r\pi}{a}\right)^2\right\}\sin\left\{\frac{r\pi x}{a}\right\}\sin\left(\frac{n\pi x}{a}\right)B\{x,\xi)dx\int_0^b \sin\left\{\frac{m\pi y}{b}\right\}\sin\left(\frac{s\pi y}{b}\right)B(y)dy$$

(4.5)

By direct integration and further simplification of equation (4.5) we have

$$\psi_{rs}(t) = -\frac{2aM_l gc}{n\pi\varepsilon} A_m \sin\left(\frac{r\pi\xi}{a}\right)\sin\left(\frac{n\pi\varepsilon}{2a}\right)$$

$$-\frac{M_l c^2}{\varepsilon}\sum_{r=1}^{M}\sum_{s=1}^{N} A_{ms}\left[T_{rs}''(t)\left\{\frac{a}{\pi(r-a)}\cos\left\{\frac{(r-n)\pi\xi}{a}\right\}\sin\left\{\frac{(r-n)\pi\xi}{2a}\right\}-\right.\right.$$

$$\left.\frac{a}{\pi(r+n)}\cos\left(\frac{(r+n)\pi\xi}{a}\right)\sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}+2uT_{rs}'(t)r\pi\left\{\frac{1}{\pi(r+n)}\sin\left\{\frac{(r+n)\pi\xi}{a}\right\}\sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right.$$

$$\left.\left.-\frac{1}{\pi(r-n)}\sin\left\{\frac{(r-n)\pi\xi}{a}\right\}\sin\left\{\frac{(r-n)\pi\xi}{2a}\right\}\right\}\right\}$$

$$+u^2T_{rs}(t)\left\{-\frac{(r\pi)^2}{a}\right\}\left\{\frac{1}{\pi(r-n)}\cos\left\{\frac{(r-n)\pi\xi}{a}\right\}\sin\left\{\frac{(r-n)\pi\xi}{2a}\right\}\right.$$

$$\left.\left.-\left\{\frac{1}{\pi(r+n)}\cos\left\{\frac{(r+n)\pi\xi}{a}\right\}\sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}\right\}\right] \tag{4.6}$$

For r ≠ n
Where,

$$A_m = \frac{2b}{m\mu\pi}\sin\left\{\frac{m\pi\mu}{2b}\right\}\sin\left\{\frac{s\pi y_1}{b}\right\} \tag{4.6a}$$

$$A_{ms} = \frac{b}{(m-s)\mu\pi}\sin\left\{\frac{(m-s)\pi\mu}{2b}\right\}\cos\left\{\frac{(m-s)\pi y_1}{b}\right\} - \frac{b}{(m+s)\mu\pi}\sin\left\{\frac{(m+s)\pi\mu}{2b}\right\}\cos\left\{\frac{(m+s)\pi y_1}{b}\right\} \tag{4.6b}$$

for m ≠s

$$A_{ms} = \frac{1}{2} - \frac{b}{(m+s)\mu\pi}\sin\left\{\frac{(m+s)\pi\mu}{2b}\right\}\cos\left\{\frac{(m+s)\pi y_1}{b}\right\} \text{ for m = s} \tag{4.6c}$$

$$\psi_{rs}(t) = -\frac{2aM_l gc}{n\pi\varepsilon} A_m \sin\left(\frac{r\pi\xi}{a}\right)\sin\left(\frac{n\pi\varepsilon}{2a}\right)$$

$$-\frac{M_l c^2}{\varepsilon}\sum_{r=1}^{M}\sum_{s=1}^{N}A_{ms}\left[T_{rs}^{ii}(t)\frac{1}{2}\left\{\varepsilon-\frac{2a}{\pi(r+n)}cos\left\{\frac{(r+n)\pi\xi}{a}\right\}sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}\right.$$
$$+2uT_{rs}^{i}(t)r\pi\left\{\frac{1}{\pi(r+n)}Sin\left\{\frac{(r+n)\pi\xi}{a}\right\}sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}$$

$$\left.+u^2T_{rs}(t)\left\{-\frac{(r\pi)^2}{a}\right\}\left\{\frac{\varepsilon}{2a}-\frac{1}{\pi(r+n)}\cos\left\{\frac{(r+n)\pi\xi}{a}\right\}\sin\left\{\frac{(r+n)\pi\xi}{a}\right\}\right\}\right]_{for\ r=n} \quad \textbf{(4.7)}$$

Considering equations (4.6) and (4.7), equation (3.17) becomes

$$\lambda_{rs}T_{rs}(t)+(m_p+m_f)T_{rs}^{ii}(t)=-\frac{2aM_l gc}{n\pi\varepsilon}A_m\sin\left(\frac{r\pi\xi}{a}\right)\sin\left(\frac{n\pi\varepsilon}{2a}\right)$$

$$-\frac{M_l c^2}{\varepsilon}\sum_{r=1}^{M}\sum_{s=1}^{N}A_{ms}\left[T_{rs}^{ii}(t)\left\{\frac{a}{\pi(r-n)}cos\left\{\frac{(r-n)\pi\xi}{a}\right\}sin\left\{\frac{(r-n)\pi\xi}{2a}\right\}\right.\right.$$
$$\left.-\frac{a}{\pi(r+n)}cos\left\{\frac{(r+n)\pi\xi}{a}\right\}sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}$$

$$+2uT_{rs}^{i}(t)r\pi\left\{\frac{1}{\pi(r+n)}\sin\left\{\frac{(r+n)\pi\xi}{a}\right\}\sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}-\frac{1}{\pi(r-n)}\sin\left\{\frac{(r-n)\pi\xi}{a}\right\}\sin\left\{\frac{(r-n)\pi\xi}{2a}\right\}\right\}$$

$$+u^2T_{rs}(t)\left\{-\frac{(r\pi)^2}{a}\right\}\left\{\frac{1}{\pi(r-n)}cos\left\{\frac{(r-n)\pi\xi}{a}\right\}sin\left\{\frac{(r-n)\pi\varepsilon}{2a}\right\}\right.$$
$$\left.\left.-\frac{1}{\pi(r+n)}cos\left\{\frac{(r+n)\pi\xi}{a}\right\}sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}\right]$$

$$T_{rs}(0)=T_{rs}\left(\frac{a}{u}\right)=0,\ \text{for } r\neq n \quad \textbf{(4.8)}$$

And

$$\lambda_{rs}T_{rs}(t)+(m_p+m_f)T_{rs}^{ii}(t)=-\frac{2aM_l gc}{n\pi\varepsilon}A_m\sin\left(\frac{r\pi\xi}{a}\right)\sin\left(\frac{n\pi\varepsilon}{2a}\right)$$

$$-\frac{M_l c^2}{\pi\varepsilon}\sum_{r=1}^{M}\sum_{s=1}^{N}A_{ms}\left[T_{rs}^{ii}(t)a\left\{\frac{\pi\varepsilon}{2a}-\frac{a}{\pi(r+n)}cos\left\{\frac{(r+n)\pi\xi}{a}\right\}sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}\right.$$

$$+2uT_{rs}^{i}(t)r\pi\left\{\frac{1}{\pi(r+n)}\sin\left\{\frac{(r+n)\pi\xi}{a}\right\}\sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}$$

$$+u^2T_{rs}(t)\left\{-\frac{(r\pi)^2}{a}\right\}\left\{\frac{\pi\varepsilon}{2a}-\frac{1}{\pi(r+n)}\cos\left\{\frac{(r+n)\pi\xi}{a}\right\}\sin\left\{\frac{(r+n)\pi\xi}{2a}\right\}\right\}\Bigg]$$

$$T_{rs}(0)=T_{rs}\left(\tfrac{a}{u}\right)=0,\ \text{for } r=n \qquad (4.9)$$

5. RESULTS AND DISCUSSION

For the numerical work the coupled differential equations (4.8) and (4.9) are solved using the central difference formula of finite difference method. The various parameters have been defined as follows: Poisson ratio v = 0.2, E = 2.109 x 10^7kgm^{-2}, u = 1.5ms^{-1} and the dimension of the plate is taken to be a = 0.914m by b = 0.457 for the purpose of comparison. The areas (Ad1, Ad2 and Ad3) of the distribution of the moving load are taken as ε x μ. Ad1 = 980mm^2, Ad2 = 245mm^2 and Ad3 = 0.005mm^2.

The deflection curve at the mid span of the plate is shown in Table 1 with the moving masses and moving forces for values of K = 20 and G = 4. As the distribution tends towards concentration (concentrated moving load), the displacement profile tends to symmetry and the line of symmetry is at the centre. The behaviours of point load compares well with that in [7]. The moving force table represents the results obtained when the inertia effect of the load is neglected.

The distribution of the moving load is proportional to the deflection of the plate. This is evident in table 2 that shows the dynamic deflection of the plate at a specified time. The more the area of the distribution of the moving masses, the less the dynamic deflection. The deflection of the concentrated moving load agrees with the research work in [2, 5].

The presence of the elastic foundation reduces the deflection of the plate. This is seen in tables 3a and 3b that show the displacement profile of different values of the subgrade's shear moduli G and K. It is found that the variation in deflection for concentrated moving mass is larger compared with distributed moving mass for various values of foundation moduli. Also the result reveals that an increase in G or K, decreases the deflection at every point of the plate. The effect of G on the plate's displacement is more pronounced than that of the foundation reaction modulus K.

The critical speed of the moving load is obtained in table 4. It is found that critical speed of the moving mass and the moving force are approximately 25m/s and 26m/s respectively.

6. CONCLUSIONS

The structure of interest is a rectangular plate on a non-winkler elastic foundation under the influence of a uniform partially distributed moving load. The governing equation is analytically simplified to form coupled ordinary differential equations. Finite difference technique was adopted in solving the differential equations for the simply supported plate. The results show that the presence of the foundation moduli reduce the deflection of the plate. Also the area of the distribution of the load has significant effect on the displacement amplitude.

TABLE 1: Variation of the deflection at the middle of the plate

Time in sec (ts)	Moving mass deflection (mm) for Ad1	Moving mass deflection (mm) for Ad2	Moving mass deflection (mm) for Ad3
0	0	0	0
0.038083	-0.60916732417	-0.29757086961	0.02751096037
0.076166	-1.07100424808	-0.78607934958	-0.50447045007
0.114249	-1.50422174734	-1.26263401172	-0.98740310197
0.152332	-1.87679421898	-1.67850691980	-1.43665870852
0.190415	-2.18162523848	-2.03341017142	-1.83331688412
0.228498	-2.40248088487	-2.31130328229	-2.16158231802
0.266581	-2.53010637250	-2.50001215193	-2.40745289496
0.304664	-2.55766076347	-2.59083950503	-2.55977405822
0.342747	-2.48459543146	-2.57873500259	-2.61133110066
0.380830	-2.31402454586	-2.46796665853	-2.55963295697
0.418913	-2.05396619358	-2.25565063098	-2.40713363580
0.456996	-1.71588573804	-1.96022791724	-2.16110910627
0.495079	-1.31393522549	-1.58523276919	-1.83270824114
0.533162	-0.86401523136	-1.16117449512	-1.43596916037
0.571245	-0.38272237805	-0.68989700515	-0.98652234996
0.609328	0.10743021909	-0.19739908724	-0.50366289449
0.647411	0.67548648116	0.35556695148	0.02838824416
0.685494	0	0	0
t(s)	moving force	moving force	moving force
0	0	0	0
0.038083	-0.61223322583	-0.29873772652	0.02769261624
0.076166	-1.07579355677	-0.80403098889	-0.50779217899
0.114249	-1.50496871775	-1.26628146290	-0.99247119998
0.152332	-1.87593824009	-1.68054633510	-1.44075330061
0.190415	-2.17484247452	-2.03020182386	-1.83356646520
0.228498	-2.39014229457	-2.30181822610	-2.15592480358
0.266581	-2.51362209276	-2.48500695591	-2.39542617927
0.304664	-2.54048493916	-2.57267897748	-2.54288735408
0.342747	-246973008423	-2.56149399297	-2.59261775498
0.380830	-2.30406599524	-2.45186624650	-2.54273271888
0.418913	-2.04986147729	-2.24802970276	-2.39511701310
0.456996	-1.71687525334	-1.95778656925	-2.15545922907
0.495079	-1.31791507927	-1.59233006604	-1.83297819434
0.533162	-0.86831659862	-1.16566536611	-1.44005587983
0.571245	-0.38514994553	-0.69407682387	-0.99169636559
0.609328	0.10910832330	-0.19856727392	-0.50697364093
0.647411	0.66584864574	0.35432973144	0.02857488858
0.685494	0	0	0

Table 2: Dynamic displacement of the plate at time t = 0.23s.

X(m)	Ad1	Ad2	Ad3
0	0	0	t0
0.0914	-0.0007422	-0.0007141	-0.00066785
0.1828	-0.0014119	-0.0013583	-0.00127032
0.2742	-0.0019433	-0.0018696	-0.00174845
0.3656	-0.0022845	-0.0021978	-0.00205543
0.4570	-0.0024020	-0.0023109	-0.00216121
0.5484	-0.0022845	-0.0021978	-0.00205543
0.6398	-0.0019433	-0.0018696	-0.00174845
0.7312	-0.0014119	-0.0013583	-0.00127032
0.8226	-0.0007422	-0.0007141	-0.00066785
0.9140	0	0	0

Table 3a Variation of K and G with dynamic displacement at the middle of the plate

Time in sec. (ts)	Displacement due to Ad1. G = 0, K =,0	Displacement due to Ad1. G = 0.9, K =0	Displacement due to Ad1. G = 4, K =0
0	0	0	0
0.038083	-0.60918088944	-0.6091546546	-0.60907527506
0.076166	-1.07100159222	-1.07098361665	-1.07083879384
0.114249	-1.50425012895	-1.50418374598	-1.50398840219
0.152332	-1.87678285536	-1.87674561810	-1.87650295188
0.190415	-2.18195401691	-2.18158305363	-2.18128564659
0.228498	-2.40246761393	-2.40243170584	-2.40210615093
0.266581	-2.53015389186	-2.53005851196	-2.52971111737
0.304664	-2.55769763845	-2.55759314088	-2.55726114601
0.342747	-2.48463332163	-2.48453378587	-2.48420752215
0.380830	-2.31405937713	-2.31396435147	-2.31366389764
0.418913	-2.05399721285	-2.05392700441	-2.05364696616
0.456996	-1.71591069442	-1.71585111878	-1.71561883511
0.495079	-1.31396273946	-1.31390940520	-1.31374021053
0.533162	-0.86402754356	-0.86399318040	-0.86387639988
0.571245	-0.38272811542	-0.38271291259	-0.382661168554
0.609328	0.10743167868	0.10742449204	0.10741505782
0.647411	0.67549781161	0.67546759461	0.67536942700
0.685494	0	0	0
t(s)	k=0 G=0	K = 0.2, G = 0	K = 20, G = 0
0	0	0	0
0.038083	-0.60918088944	-0.60918001691	-0.60916732417
0.076166	-1.07100159222	-1.07100159197	-1.07100424808
0.114249	-1.50425012895	-1.50425002153	-1.50422174734

Time in sec. (ts)	Displacement due to Ad1. G = 0, K = 0	Displacement due to Ad1. G = 0.9, K =0	Displacement due to Ad1. G = 4, K =0
0.152332	-1.87678285536-	-1.87678254920-	-1.87679421898-
0.190415	2.18195401691	2.18195370718	2.18162523848
0.228498	-2.40246761393	-2.40246759407	-2.40248088487
0.266581	-2.53015389186	-2.53015368870	-2.53010637250
0.304664	-2.55769763845	-2.55769738194	-2.55766076347
0.342747	-2.48463332163	-2.48463318728	-2.48459543146
0.380830	-2.31405937713	-2.31405899605	-2.31402454586
0.418913	-2.05399721285	-2.05399701826	-2.05396619358
0.456996	-1.71591069442	-1.71591047984	-1.71588573804
0.495079	-1.31396273946	-1.31396267940	-1.31393522549
0.533162	-0.86402754356	-0.86402749496	-0.86401523136
0.571245	-0.38272811542	-0.38272805749	-0.38272237805
0.609328	0.10743167868	0.10743157435	0.10743021909
0.647411	0.67549781161	0.67549772195	0.67548648116
0.685494	0	0	0

TABLE 3b: Variation of K and G with dynamic displacement at the middle of the plate

Time in sec (ts)	Displacement due to Ad3. G = 0, K = 0	Displacement due to Ad3. G = 0.9, K = 0	Displacement due to Ad3. G = 4, K = 0
0	0	0	0
0.038083	0.02751183938	0.02752207341	0.02750187982
0.076166	-0.50447802894	-0.50468436545	-0.50439214847
0.114249	-0.98741784885	-0.98387550872	-0.98725074315
0.152332	-1.43668022160	-1.48036530596	-1.43643644603
0.190415	-1.83334441089	-1.75020226740	-1.83303247353
0.228498	-2.16161487052	-2.17310597587	-2.16124611967
0.266581	-2.40748921079	-2.40542766649	-2.40707694336
0.304664	-2.55981290362	-2.56010804858	-2.55937678383
0.342747	-2.61136992674	-2.61118343769	-2.61090987881
0.380830	-2.55967499751	-2.55958497975	-2.55929646829
0.418913	-2.40715272761	-2.40707003736	-2.40643036077
0.456996	-2.16114427366	-2.16105522434	-2.16082511074
0.495079	-1.83273633713	-1.83266330654	-1.83241804092
0.533162	-1.43598209732	-1.43592425010	-1.43573909093
0.571245	-0.98663984172	-0.98660626063	-0.98647300227
0.609328	-0.50366390062	-0.50364335835	-0.50357532246
0.647411	0.02838877283	0.02838637174	0.02837849900
0.685494	0	0	0

Table 4: Load's velocity variation with maximum deflection

U(m/s)	Moving mass deflection (m)	Moving force deflection (m)
3	0.00259884874222	0.00252759249832
6	0.00287634768845	0.00253889856246
12	0.00457809516455	0.00274664658207
18	0.01128019700310	0.00297187418724
22	0.08523278218104	0.00315728725821
23	0.13230967145247	0.00316682097558
25	0.02303864634032	0.00316744000806
26	0.04111638422853	0.00315729199288
27	0.01235713822511	0.00310527746928
28	0.00486152569957	0.00302956142400

REFERENCES

[1]. S.Timoshenko, D.H. Young and W. Weaver (1974) Vibration Problems in Engineering. New York: John Wiley; 4th edition.

[2]. M.M. Stanisic and J.C. Hardin (1969) On response of beams to an arbitrary number of moving masses. Journal of the Franklin Institute. 287, 115 – 123.

[3]. J.E. Akin and M. Mofid (1989) Numerical solution for response of beams with moving mass. J. of structural engineering. Vol 115, No. 1.

[4]. S. Sadiku and H.H.E. Leipholz (1989) On the dynamics of elastic systems with moving concentrated masses. Ingenieur Achiv 57, 223 – 242.

[5]. J.A. Gbadeyan and S.T. Oni (1995) Dynamic behaviour of beams and rectangular plates under moving loads. J. of sound and vibration. 182(5)677 – 695.

[6]. R.E. Rossi, P.A.A. Laura and R.H. Gutierrez (1990) A note on transverse vibrations of a Timoshenko beam of non-uniform thickness clamped at one end and carrying a concentrated mass at the other side. J. of sound and vibration. 143, 491 – 502.

[7]. J.A. Gbadeyan and S.T. Oni (1992) Dynamic response to moving concentrated masses of elastic plate on a non-winkler elastic foundation. J. of sound and vibration. 154(2) 343 – 358.

[8]. E. Esmailzadeh and M. Ghorashi (1995) Vibration analysis of beams transversed by uniform partially distributed moving masses. J. of sound and vibration 184(1) 9 – 17.

[9]. E. Esmailzadeh and M. Ghorashi (1997) Vibration analysis of Timoshenko beams subjected to a travelling mass. J. of sound and vibration 199(4) 615 – 828.

[10]. J.A. Gbadeyan , M.S. Dada and A.S. Idowu (2000) A vibration analysis of non-Mindlin rectangular plates transversed by uniformly distributed moving masses. Nig. J. of Mathematics and Applications NJMA. Vol. 12, 62 – 70.

[11]. A.W. Leissa (1969) Vibration of plates (NASA SP 160) Washington, D.C. U.S. Government Printing Office.