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ON SOME OBSERVABLE WAVE FORMS ON OCEAN BEACHES

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ABSTRACT

This paper considers the theory of the observed shallow water waves. The interest is essentially on the range of periods associated with beach long waves. This range seems to be of environmental interest in the locality. This is so, considering the proportion of ocean energy trapped within this range and often dissipated along the shoreline.

On the whole, the analysis re-enforces the concept of the cnoidal and solitary oscillations as essential members of shallow water processes. It is difficult to observe this considering the complicated wave patterns over beaches. However, spectral decomposition of this pattern is quite revealing.

1. INTRODUCTION

A look over a beach presents patterns of endless moving succession of irregular humps and hollows reaching from horizon to horizon. The processes are so complicated and random that it seems an impossible assignment to form any realistic picture concerning their evolutional patterns.

The formulation concerning these patterns dated beyond the days of Scott Russell (1844), Kortweg D.J and De Vries (1895). For details of the developments in the initial stage of wave theory, one may refer to Benney (1965). A comprehensive account and more recent developments may be obtained from Witham (1973) and Okeke (1999). This study begins by introducing the reader to the basic equations and the non-linear boundary conditions governing the evolutional pattern of shallow water waves. It continues to and touches some aspects of the recent developments in the theory.

2. BASIC EQUATIONS

The x-axis is taken perpendicular to the wave front whilst the y-axis is perpendicular to it. $y = \eta(x,t)$ is the wave profile whilst $y = -h(x)$ is the bottom profile of the water layer (Stoker, 1957).

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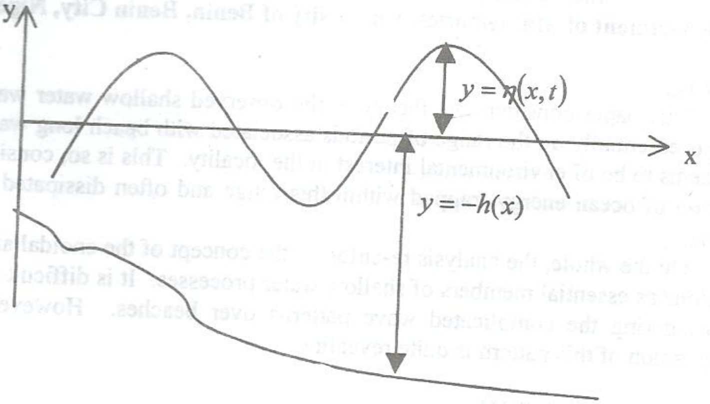

Fig I: Wave profile in beaches

The equation of continuity gives

$$\nabla \cdot \underline{q} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad -h(x) \le y \le \eta(x,t) \quad \underline{q} = \text{particle velocity} \qquad (1)$$

The boundary conditions are:

$$\frac{d(\eta - y)}{dt} = \eta_t + u\eta_x - V = 0 \qquad \text{on } y = \eta(x,t) \qquad (2)$$

$$P(x,y,t) = P = 0 \qquad \text{on } y = \eta(x,t) \qquad (3)$$

$$\frac{d(h+y)}{dt} = u\frac{\partial h}{\partial x} + V = 0 \qquad \text{on } y = -h(x) \qquad (4)$$

Integrating (1), using (2) and (4),

$$\int_{h(x)}^{\eta(x,t)} \frac{\partial u}{\partial x} dy + (\eta_t + u\eta_x)_\eta + (uh_x)_h = 0 \qquad (5)$$

$$\frac{\partial}{\partial x}\int_{-h}^{\eta} u\,dy = \int_{-h}^{\eta} \frac{\partial u}{\partial x} dy + u|_\eta \eta_x + u|_{-h} h_x \qquad (6)$$

$$\frac{\partial}{\partial x}\int_{-h}^{\eta} u dy = -\eta_t \tag{7}$$

To simplify (7), we use the vertical equation of motion, which **reads (in static** equilibrium),

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = -g \tag{8}$$

from which we obtain, using (3)

$$p = \rho g(\eta - y) \tag{9}$$

$$\frac{\partial p}{\partial x} = \rho g \frac{\partial \eta}{\partial x} \tag{10}$$

(10) is independent of y-coordinate. For motion in x-direction,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + V\frac{\partial u}{\partial y} - \frac{1}{\rho}\frac{\partial p}{\partial x} = \frac{du}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

But from (10), $\frac{\partial p}{\partial x}$ is independent of y. Thus, $\frac{\partial u}{\partial t}$ is independent of y-coordinate. Consequently, the entire motion is uniform with depth. This statement is true to the order of $\left(\frac{h_0}{L_0}\right)^2$, h_0 = typical water depth when undisturbed and L_0 = typical wavelength. Thus, the approximation is true provided that $h_0 << L_0$ and $u = u(x,t) + O\left(\frac{h_0}{L_0}\right)^2$, i.e. in shallow water.

Equation (7) now becomes

$$\frac{\partial}{\partial x}[u(\eta + h)] = -\eta_t \tag{11}$$

Together with

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial \eta}{\partial x} \tag{12}$$

$u(x,t)$ and $\eta(x,t)$ in (11) and (12) are connected by the relationship

$$u = 2(c - c_0) \tag{13}$$

where

$$c^2 = g(\eta + h_0) \text{ and } c_0^2 = gh_0$$

concerning internal friction, the decay time, T_d is given by $T_d = \frac{L^2}{8\nu\pi^2}$ where ν is the coefficient related to viscosity and L is the wavelength. The following table identical to those of Kinsman (1965) gives values of T_d with the corresponding wavelengths.

Table 1:

T_d	L
5 seconds	1.8cm (wind waves)
2.3 hours	1.1 meter (3 seconds swell)
2 ½ years	101 meters (10 seconds or more swell)

Thus considering the range of wave periods, which are of geophysical interest in which we are concerned with, the effects of viscous dissipation, will be neglected in the subsequent discussions.

Using (13) in (11) or (12), then,

$$\eta_t + c_0\left(1 + \frac{3}{2}\frac{\eta}{h_0}\right)\eta_x + O\left(\eta^2_{h_0}\right) = 0 \tag{14}$$

If we incorporate the effect of dispersion (frequency), (14) gives

$$\eta_t + c_0\left(1 + \frac{3}{2}\frac{\eta}{h_0}\right)\eta_x + \gamma\eta_{xxx} = 0 \tag{15}$$

$$\gamma = \frac{c_0 h_0}{6}$$

Linearised form of (15)

$$\eta_t + c_0\eta_x + \gamma\eta_{xxx} = 0 \tag{16}$$

(16) is solved with the initial condition

$$\eta(x,0) = f(x)$$

Let $N(k,t) = \int_{-\infty}^{\infty} e^{-ikx}\eta(x,t)dx$

With $\eta(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}N(k,t)dk$

$$F(k) = \int_{-\infty}^{\infty} e^{-ikx}\eta(x,0)dx = \int_{-\infty}^{\infty} e^{-ikx}f(x)dx$$

From (16)

$$N(k,t) = F(k)\exp\left[ikct\left(\frac{k^2h^2}{6}-1\right)\right]$$

If $f(x) = \delta(x)$, a point source with infinite strength, then,

$$F(k) = \int_{-\infty}^{\infty} e^{-ikx}\delta(x)dx = 1$$

$$N(k,t) = \exp\left[ikct\left(\frac{k^2h^2}{6}-1\right)\right]$$

$$\eta(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ikx}e^{\left[ikct\left(\frac{k^2h^2}{6}-1\right)\right]}dk$$

$$= \left(\frac{cth^2}{2}\right)^{-\frac{1}{3}} A_i\left[\left(\frac{cth}{2}\right)^{-\frac{1}{3}}(x-ct)\right] \qquad (17)$$

$A_i(z)$ is called Airy function which is the solution of

$$u'' + zu = 0 \qquad \textbf{(18)}$$

This function models the evolutions of a variety of phenomena such as intensity of light in the neighbourhood of a causic. It also defines the behaviour of wave train near the front. Also,

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$$A_i(z) = \begin{cases} \dfrac{1}{2\sqrt{\pi}} z^{-\frac{1}{4}} \exp\left(-\dfrac{2}{3} z^{\frac{3}{2}}\right), & z \to \infty \\ \dfrac{1}{\sqrt{\pi}} |z|^{-\frac{1}{4}} \sin\left(\dfrac{2}{3}|z|^{\frac{3}{2}} + \dfrac{\pi}{4}\right), & z \to -\infty \end{cases}$$

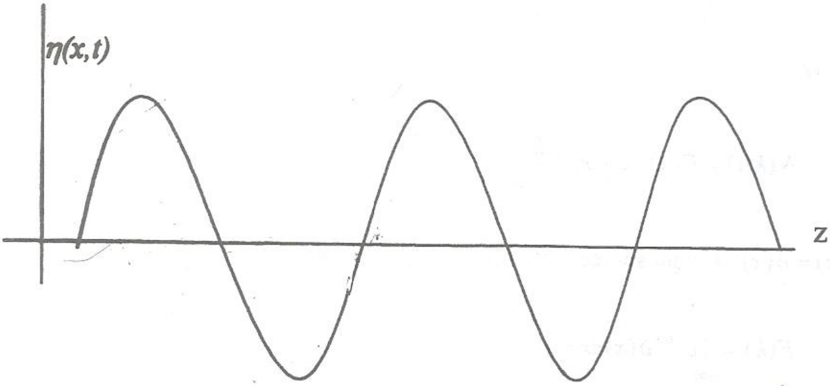

Fig II: Point source evolutions

3. SOLITARY AND CNOIDAL WAVES

It is usually assumed that the wave motion in this consideration is uniform. That is $\chi = x - u_0 t$ where $u_0 = c_0\left(1 + \dfrac{\eta_0}{h_0}\right)$ η_0 being the wave height, $\eta(x,t) = h_0 \xi(\chi)$. In the case of solitary waves, $\xi(\chi) \to 0$., $\xi'(\chi) \to 0$ as $\chi \to \infty$ Equation (15) now takes the form

$$\frac{h_0}{3}\xi'^2 = \xi^2(\alpha - \xi) \tag{19}$$

$$\alpha = 2\left(\frac{u_0}{c_0} - 1\right) \quad or \quad \frac{u_0}{c_0} = \frac{\alpha}{2} + 1$$

Thus, the solution of (19) is

$$\xi = \alpha \sec h^2\left[\left(\frac{3\alpha}{4h_0}\right)(x-ut)\right], \quad \alpha = \frac{\eta_0}{h_0} = \xi(0)$$

With

$$\mu(x,t) = \eta_0 \sec h^2\left[\left(\frac{3\eta_0}{4h^3}\right)^{\frac{1}{2}}(x-u_0t)\right] \tag{20}$$

(20) defines a wave with a single hump and wave number $k = \left(\frac{3\eta_0}{4h^3}\right)^{\frac{1}{2}}$, which depends on wave height η_0 This is one of the striking features of the non-linear waves.

But if $\xi(\chi)$ and $\xi'(\chi)$ do not vanish as $\xi \to \infty$, then,

$$\xi'^2 = f(\xi) \tag{21}$$

Where $f(\xi)$ is a cubic polynomial with three distinct roots, i. e. 0, α, α-β where $0 < \alpha < \beta$. $\beta = \frac{h_0^2}{L_0^2}$

As $\alpha \to 0$,

$$\xi(x) = \frac{\alpha}{2}\left[1 + cos\left(\sqrt{3\beta}\,\frac{\chi}{h_0}\right)\right] \tag{22}$$

Generally,

$$\xi(x) = \alpha c_n^2\left(\frac{3\beta}{4h_0^2}\right)(x-ut) \tag{23}$$

c_n is the Jacobian elliptic function. But (22) and (23) give rise to periodic solutions. For details see Witham, (1973).

4. EFFECTS OF HIGHER ORDER NON-LINEAR TERM

Following Okeke (1997), equation (15) is generalized to give

$$\frac{\partial \eta}{\partial t}+c_0\left(1+\frac{3}{2}\frac{\eta}{h_0}-\frac{3}{8}\frac{\eta^2}{h_0^2}\right)\frac{\partial \eta}{\partial x}=O\left(\frac{\eta^3}{h_0^3}\right) \tag{24}$$

and thus we obtain

$$\frac{\partial \eta}{\partial t}+c_0\left(1+\frac{3}{2}\frac{\eta}{h_0}-\frac{3}{8}\frac{\eta^2}{h_0^2}\right)\frac{\partial \eta}{\partial x}+\gamma\frac{\partial^3 \eta}{\partial x^3}=0 \tag{25}$$

$c_0=\sqrt{gh_0}\,;\quad \gamma=\frac{c_0 h_0}{6}$ as before

Introducing $X=x-u_0 t,\, u_0^2=g(h_0+\eta_0)$. As before, u_0 is the uniform wave speed. (25) transforms to

$$-\frac{u_0}{2}\eta^2+c_0\left(\frac{\eta^2}{2}+\frac{\eta^3}{4h_0}-\frac{\eta^4}{32h_0^2}\right)+\gamma^2\left(\frac{\partial \eta}{\partial X}\right)=\chi_0\eta-\chi_1-\ldots \tag{26}$$

If h_0 is suitably chosen such that

$$\frac{\partial \eta}{\partial \chi}=\frac{\partial^2 \eta}{\partial \chi^2}=0 \tag{27}$$

where wave height at the shelf edge is $\eta=a_0$, then,

$$\chi_0=c_0\chi_{00}=\frac{c_0 a_0^2}{4h_0}\left(3-\frac{a_0}{2h_0}\right)-c_0 a_0\left(\frac{u}{c_0}-1\right) \tag{28}$$

$$\chi_1=c_0\chi_{11}=\frac{c_0 a_0^2}{2}\left(\frac{u_0}{c_0}-1\right)-\frac{c_0 a_0^3}{2h_0}\left(1-\frac{3a_0}{16h_0}\right)$$

(26) takes the form

$$\beta\left(\frac{d\eta}{d\chi}\right)^2=\eta^4-\alpha_1\eta^3+\alpha_2\eta^2+\alpha_3\eta-\alpha_4 \tag{29}$$

$$\beta = 16h_0^2\left(\frac{\gamma}{c_0}\right);\quad \alpha_1 = 8h_0;\quad \alpha_2 = 16h_0^2\left(\frac{u}{c_0}-1\right);\quad \alpha_3 = 32h_0\chi_{00};$$

$$\frac{u}{c_0}-1 = \frac{\eta_0}{2h_0} - \frac{\eta_0^2}{8h_0^2} + O\left(\frac{\eta_0^3}{h_0^3}\right);\quad \alpha_4 = 32h_0^2\chi_{11}$$

It may be of interest to evaluate the contribution to the wave energy **arisen** from the additional term. Thus (26) takes the form

$$-g\eta^2 + g\left(1.5\frac{\eta^3}{h_0} - 0.19\frac{\eta^4}{h_0^2}\right) + \frac{h_0\eta^2}{3} = 3(\chi_0\eta - \chi_1) \qquad \textbf{(30)}$$

where $\eta_0 = 0.774h_0$, $\frac{u}{c_0} - 1 \sim \frac{1}{3}$

Multiplying (30) by ρ (density) and carrying out vertical averaging of terms **in (30)**, we have

$$\rho g\overline{\eta}^2 = \frac{\rho h_0}{3}\overline{\eta}^2 + \rho g\left(1.5\frac{\overline{\eta}^3}{h_0} - 0.19\frac{\overline{\eta}^4}{h_0^2}\right) + \chi_{12} \qquad \textbf{(31)}$$

χ_{12} includes contribution from mean sea elevation. $\overline{\eta}^2$ is proportional **to the kinetic** energy of linear wavefc. n whilst $\overline{\eta}^2$ is related to potential energy. $\overline{\eta}^3$ **arises from** first order non-linear term in (15). $\overline{\eta}^4$ suggests the effect of the **additional second** order non-linear term. If $\eta_0 = 0.774h_0$, the contribution of $\overline{\eta}^4$ to (31) **is on 8%**. This is quite significant, which if neglected can affect the accuracy **of any of the** related data.

Solitary wave solution obtained from (19)

(29) now takes the form

$$\beta\left(\frac{d\eta}{d\chi}\right)^2 = \eta^2(\eta - b)(\eta - d) \qquad \textbf{(32)}$$

where

$$b+d=\alpha_1,\ bd=\alpha_2,\ d=4h_0\left[1\pm\left(2-\frac{u}{c_0}\right)^{\frac{1}{2}}\right]$$

and finally,

$$\eta(x)=\frac{\alpha_2 \sec h^2\left(\frac{R_1}{2}\right)}{A_0+d\sec h^2\left(\frac{R_1}{2}\right)} \tag{33}$$

$$A_0=b-d=\alpha-2d=8h_0\sqrt{2-\frac{u}{c_0}},\quad R_1=2\left[\sqrt{\alpha_2\left(\frac{\chi}{\beta}\right)}\right]$$

α_2 is related to the wave amplitude η_0 through the flow velocity u_0. As $\chi\rightarrow\infty$, $\sec h\left(\frac{R_1}{2}\right)\rightarrow 1$ and $\eta\rightarrow\frac{\alpha_2}{6}$ and as $\chi\rightarrow\infty$, $\sec h\left(\frac{R_1}{2}\right)\rightarrow 0$ and $\eta(x)\rightarrow 0$ $\alpha_2 = 3.68h_0^2$. Hence, the solution in this case still models solitary waveform suggesting a new mathematical form of solitary waves.

Following Okeke (1997) and by assuming that the oscillations no longer vanish at infinity, we obtain a periodic solution of the form,

$$\eta(x,t)=\frac{H[\sin\omega(x-ut)]}{2+\sin\omega(x-ut)} \tag{34}$$

where

$$\alpha_1 H^3-2\alpha_2 H^2-3\alpha H+4\alpha_4=0$$

is obtained

Solution (34) is essentially no cnoidal as in (23) but instead, models a more complex form of periodic motion. This will be illustrated later.

Eventually, we made bold and eventually successful attempts in the study of complete equations (11) and (12). It began by eliminating u from the two equations, that is

$$u=\frac{u_0\eta+A}{h_0+\eta} \tag{35}$$

we obtain a more complete equation where A is a constant.

numerically, if $\eta_0 \in (0.5h_0, 0.85h_0)$; then $a \in (1.1h_0, 1.3h_0)$.

Consequently, the sea level will be depressed between $0.1h_0$ and $0.31h_0$ from its undisturbed state during the passage of solitary waves. This is an interesting outcome of the study.

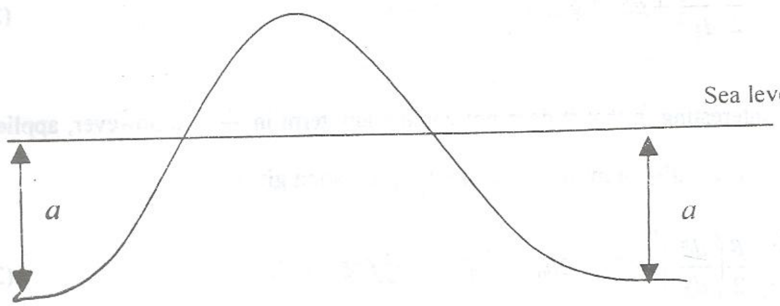

Fig III

A complete solution for a solitary wave mode.

The derivation of cnoidal mode requires a totally different approach. From Okeke (1999), we write

$$R_0\left(\frac{d\xi}{dz}\right)^2 = G(\xi) \tag{43}$$

where

$$R_0 = \frac{\beta}{c_0}$$

and

$$G(\xi) = -\xi^3 + \left(2 + F^2\right)\xi^2 - \left(1 - 2F^2\right)\xi + F^2$$
$$= 4A_1(\xi_0 - \xi)^3 + 6A_2(\xi_0 - \xi)^2 + 4A_3(\xi_0 - \xi) \tag{44}$$

the representation in (44) is one often found useful in the analysis of cubic polynomial in which

$$\beta(h_0+\eta)^2\frac{d^2\eta}{d\chi^2}+g\eta^3+c_3\eta^2+c_2\eta+c_1=0 \qquad (36)$$

$$c_1=\frac{A^2}{2}-u_0h_0A-Bh_0^2,\quad c_2=gh_0^2-u_0^2h_0-2Bh_0,\quad c_3=2gh_0-B-\frac{u_0^2}{2}$$

If $\eta=\xi-h_0$, (36) reduces to

$$\frac{\beta}{2}\frac{d^2\xi}{d\chi^2}+g\xi-\left(gh_0+\frac{u_0}{2}\right)+\frac{u_0^2h_0^2}{2\xi^2}=0 \qquad (37)$$

(37) is interesting in that it does not contain any term in $\frac{1}{\xi}$. It, however, applies to waves which vanish at infinity. Further simplification gives

$$\frac{\beta}{2}\left(\frac{d\xi}{d\chi}\right)^2+\xi^3-\left(2h_0+f^2\right)\xi^2+2h_0^2f^2\xi-f^2h_0=0 \qquad (38)$$

$$f^2=\frac{u_0^2}{g}$$

Using the transformation

$$d\chi=\sqrt{\xi dz} \qquad (39)$$

implies that water surface does not touch the bottom. That is, $\eta > -h_0$; which is quite realistic. Eventually,

$$\frac{Y}{g}\left(\frac{d\xi}{dz}\right)^2=F(\xi) \qquad (40)$$

$$F(\xi)=-\xi^3+\left(2h_0+f^2\right)\xi^2-h_0\left(h_0+2f^2\right)\xi+f^2h^2 \qquad (41)$$

The roots $F(\xi)=0$ are non-zero $-a$, $-a$, b which is now a generalization of (19), which are 0, 0, a. The solutions so obtained are

$$\xi=-a+(a+b)\sec h^2\left[\frac{g}{\gamma}(a+b)^{\frac{1}{2}}z\right] \qquad (42)$$

$$A_1 = \frac{1}{4}, \quad A_2 = \frac{1}{6}\left(2 + F^2 - 3\xi_0\right), \quad A_3 = \frac{1}{4}\left[2\xi_0\left(2 + F^2\right) - 3\xi_0^2 - \left(1 + 2F^2\right)\right]$$

Where ξ_0 satisfies the cubic equation

$$5\xi_0^3 - 3\xi_0^2\left(2 + F^2\right) + \xi_0\left(1 + F^2\right) + F^2 = 0 \qquad \textbf{(45)}$$

(45) is solved numerically for a range of values of $\frac{\eta_0}{h_0}$ as shown in Table II **below.**

Table II: Variation of non-breaking wave parameters with wave **height**

$\frac{\eta_0}{h_0}$	$-\xi_0$	T(seconds)	Wave lengths, L(m)
0.30	0.19801	7.05	80.2
0.41	0.19853	7.31	88.6
0.59	0.19942	7.52	93.3
0.65	0.20103	7.81	96.1
0.71	0.20130	8.01	100.5
0.78	0.20160	8.30	103.3
0.80	0.20207	8.42	107.5
0.85	0.20505	8.85	130.7

An extension of Table II clearly predicts the wave breaking after the height $\eta_0 = 0.85h_0$ is attained.

The periodic solution obtained in this case is of the form

$$\eta(z) = \xi_0' - P_1^2 + P_1^2 c_n^2(\omega z) \qquad \textbf{(46)}$$

c_n is Jacobian elliptic function of the first kind with modulus *k*, where *k* = ***sin θ, θ*** being the modulus angle;

$$\omega^2 = \frac{P_2^2}{4R_0}, \quad P_1^2 = R_0 - 3A_2, \quad P_2 = R_0 + 3A_2$$

The complete period of oscillation is given by

$$T = \frac{4K(k^2)}{\omega} \tag{47}$$

where ω is the complete Jacobian elliptic function given by

$$K(k^2) = \frac{\pi}{2}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; k^2\right) \tag{48}$$

${}_2F_1$ is in fact the familiar hypergeometric function.
Since $0 < k < 1$,

$${}_2F_1\left(\frac{1}{2}, \frac{1}{2}; 1; k^2\right) = 1 + \frac{k^2}{4} + O(k^4) \tag{49}$$

The period range predicted by this model is often associated with ocean swell on beaches.

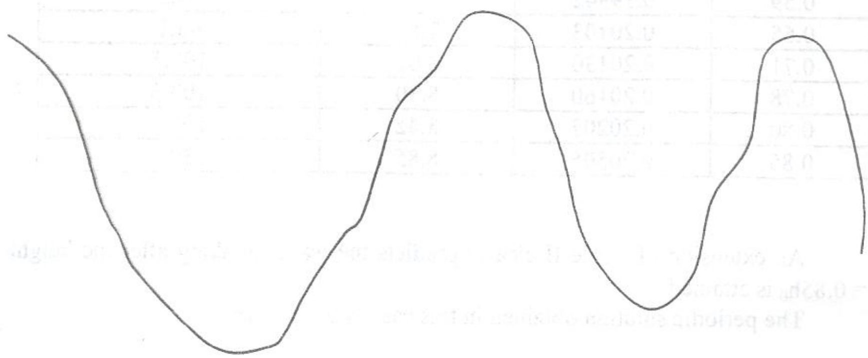

Fig IV: A more complete profile of periodic oscillation

CONCLUSION

The solution (46) is the most complete and is interestingly independent of the wave height in relation to the depth of the water layer. It seems to suggest, therefore, that shallow water waves remain unbreaking before reaching the shoreline. This is expected considering the dominant role dispersion terms plays in the entire present development. This conclusion may appear unconvincing considering the complicated wave patterns observed over beaches. However, harmonic analysis of ocean surfaces suggests the presence of a wide range of interacting frequencies in the system. That part of spectrum associated with swell will satisfy the non-breaking theory.

Further, uniform depth approximation on which this analysis is based is also in reasonable agreement with observation. Usually, when the wavelength of an oscillation is large compared with the changes in the sea-bed profile, the sea-bed is assumed to be uniform with respect to the wave oscillation. Considering the range of wave periods considered, the condition is easily satisfied over most of the shallow beaches.

Also investigated is the wave breaking near the shoreline (Okeke, 1983). It is proved that the part of the depth distribution $h(x)$ involved in the wave breaking near the shoreline is $h^{-1/4}$. This result follows closely solution of equations near the regular singularity.

Finally, the main areas of present interest are:

1. Spectral study of the present derivations.
2. The incorporation of the effects of the uniformly sloping beach in subsequent analysis.
3. The application of oblique incident theory.
4. The role of the present theory in relation to the local beaches.

It may be of interest to mention a research group with sole interest on ocean solitons. Information regarding their activities can be obtained form the Head, School of Ocean Sciences, University of Wales, Anglesey, North Wales, Britain.

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